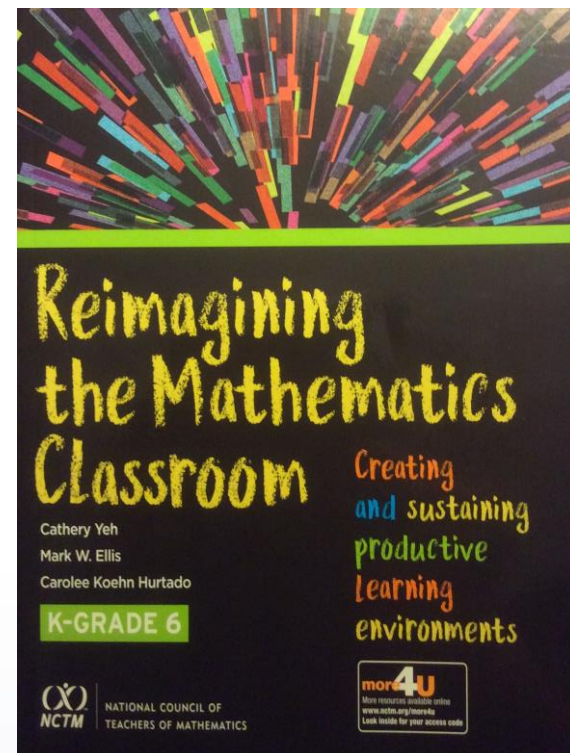


Promoting Mathematical Discourse = Deeper Mathematics Learning



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National Council of Teachers of Mathematics April 2017

Agenda

- Math Talk
- Routines and Habits to Support Mathematical Discourse
 - Analysis of Student Discourse
 - Engagement Across Participation Structure
 - Individual Engagement
 - Small-Group Discourse
 - Whole-Class Discussion
- Learning Together- Collaborative Professional Learning Opportunities

Math Talk Agreements

- Mental math problem
- Everyone quiet during think time
- Silent signals
 - Start with fist against your upper chest
 - Turn thumb clockwise to indicate a *strategy* but not yet a solution
 - Quietly raise thumb when have a solution
 - Try to find another strategy, holding up index finger and so on for other strategies

Today's Math Talk

$$2.8 \times 15$$

Extend Your Understanding

Using ideas from our Math Talk and working with a partner, come up with at least two (2) strategies for finding:

$$4.9 \times 30$$

Reflect and Discuss

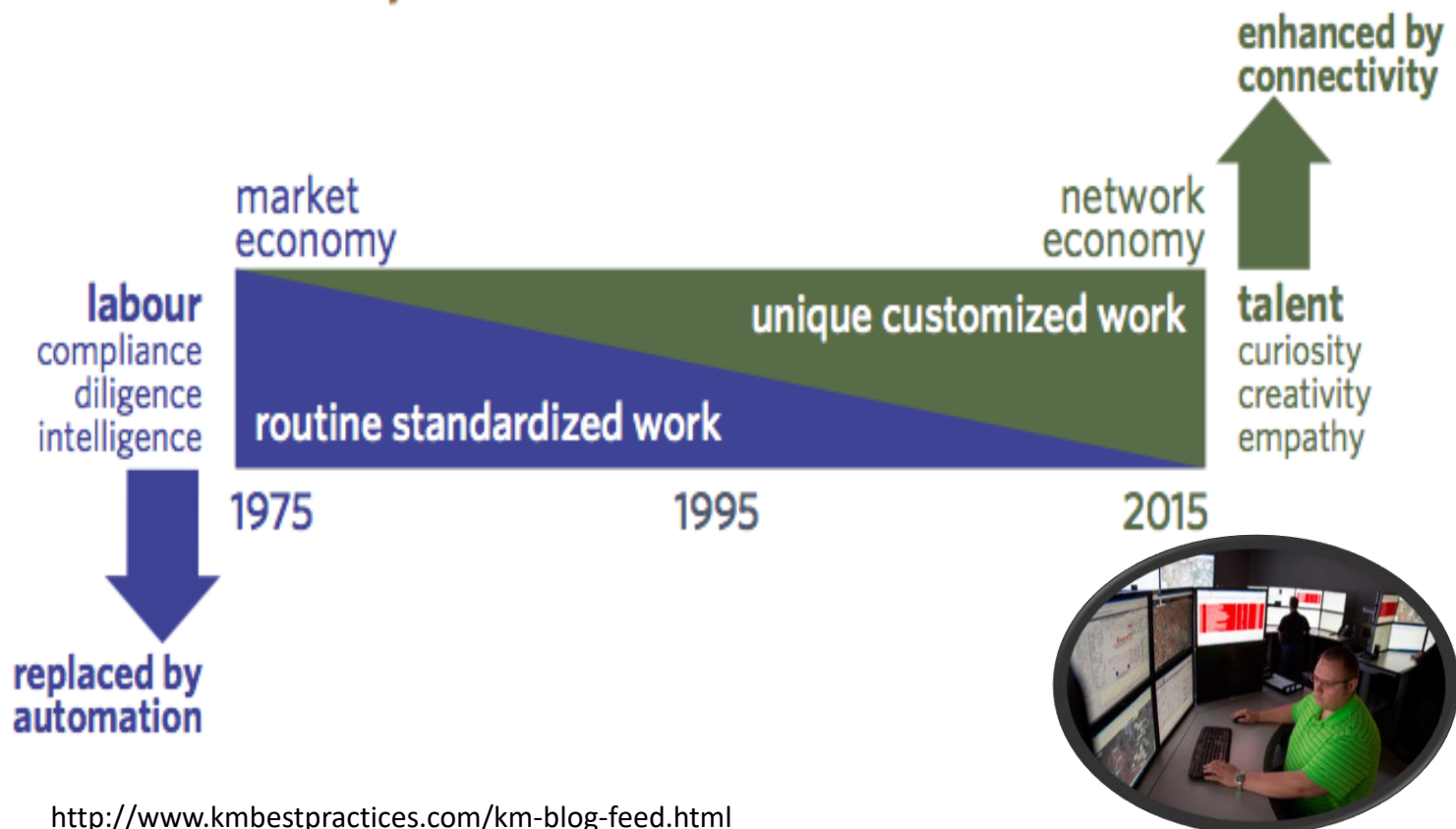
1. What was the role of the teacher during the Math Talk?
2. What was the role of students during the Math Talk?
3. **How was this different from “traditional” 20th century math lessons?**

Levels of Classroom Discourse (*Principles to Actions*, p. 32)

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Why New Standards and Instructional Shifts?

The network era shift



<http://www.kmbestpractices.com/km-blog-feed.html>

<https://www.nytimes.com/2017/02/19/business/energy-environment/oil-jobs-technology.html>

21st Century Expectations for Mathematics Learning

- Understand the concepts behind the calculations
- Coherent connections within and across grade levels
- Skill in reasoning and communicating mathematically
- Flexibility to solve non-routine problems

Does this Matter for College?



Students entering college should have certain approaches, attitudes, and perspectives including:

- A view that mathematics makes sense—perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized;
- Ease in using mathematical knowledge to solve unfamiliar problems;
- Ability to find patterns, make conjectures, and test those conjectures;
- ***Ability to discuss and write coherently about mathematical ideas.***

[CSU/UC Math Competencies](#)

For Deep Learning, Order Matters!

- Traditional routine:

Instruction → Problem Solving

- Research-based routine:

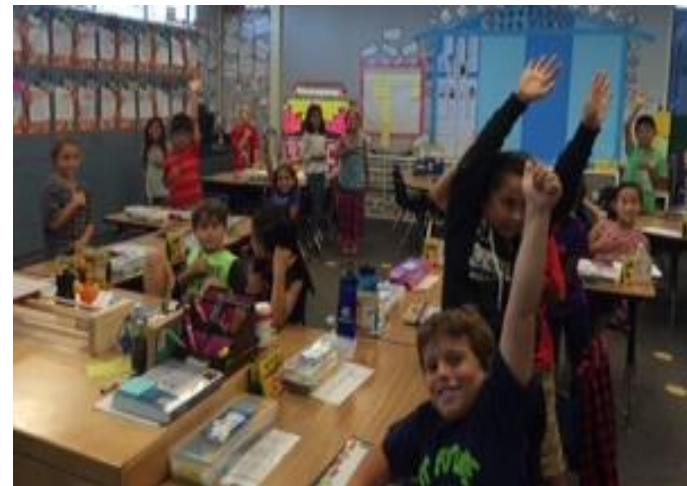
Problem Solving → Instruction

- ***When students activate prior knowledge, engage in a task that has just enough challenge and share their thinking with others, they identify their own understanding, recognize areas for further growth and learn to continually extend and refine their knowledge. This primes them for the teacher to bring guidance in making sense of new learning.***

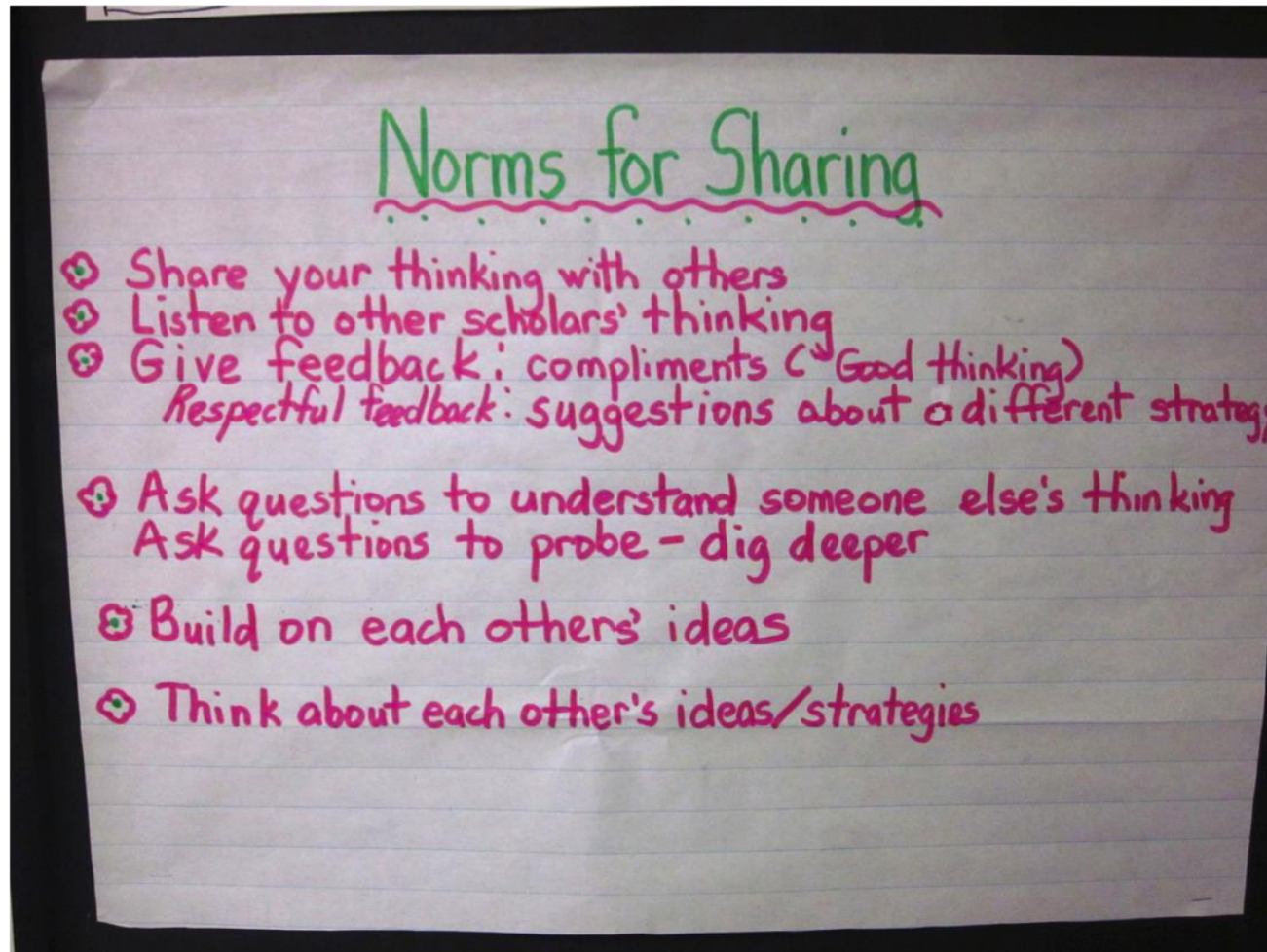
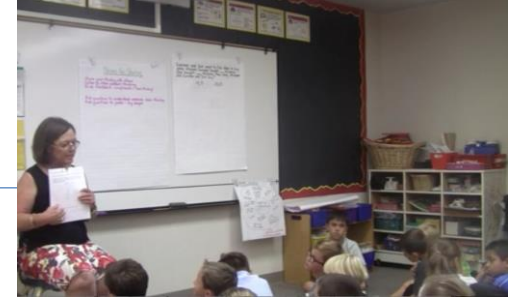
Mathematics Learning Environment

- Teach towards the understanding of powerful mathematics.
- View students as sense makers with valuable, important ideas.
- Nurture a mathematics community of learners.

(Yeh, Ellis, & Hurtado, 2017)



A Peek Inside a Classroom: Co-Creating Classroom Norms



Co-Creating Classroom Norms

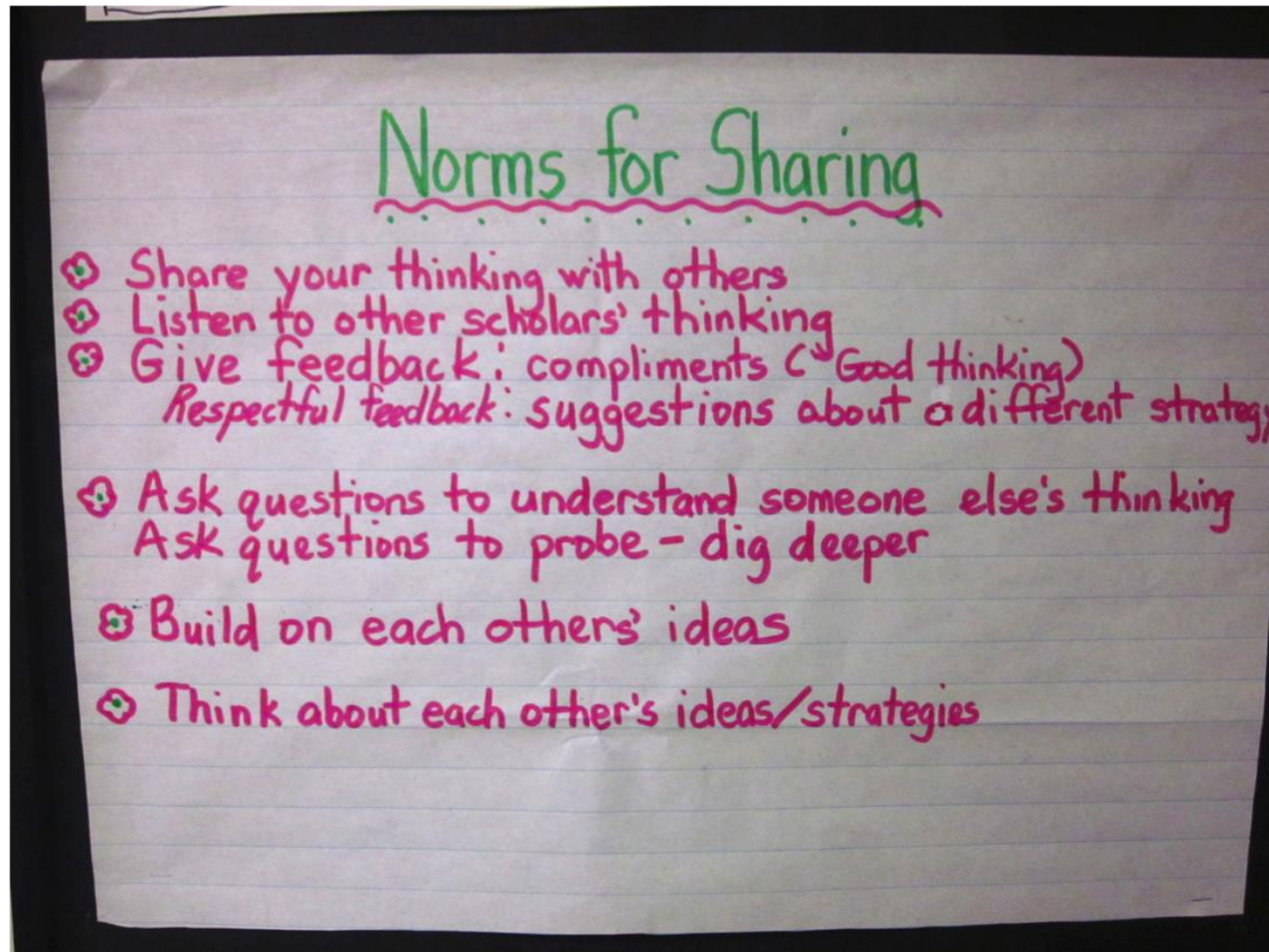
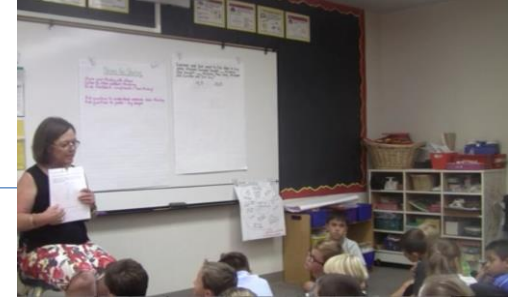
Posing the Question



Fishbowl Discussion



A Peek Inside a Classroom: Co-Creating Classroom Norms



Participation Structures to Support Math Discourse

Individual Analysis



STOP AND REFLECT

Before reading further, try to solve this problem:

Maya's mother is making a patchwork quilt. She wants to make a rectangular quilt that is 8 ft by 17 ft. If each square for the patchwork is 1 ft by 1 ft, how many squares are needed?

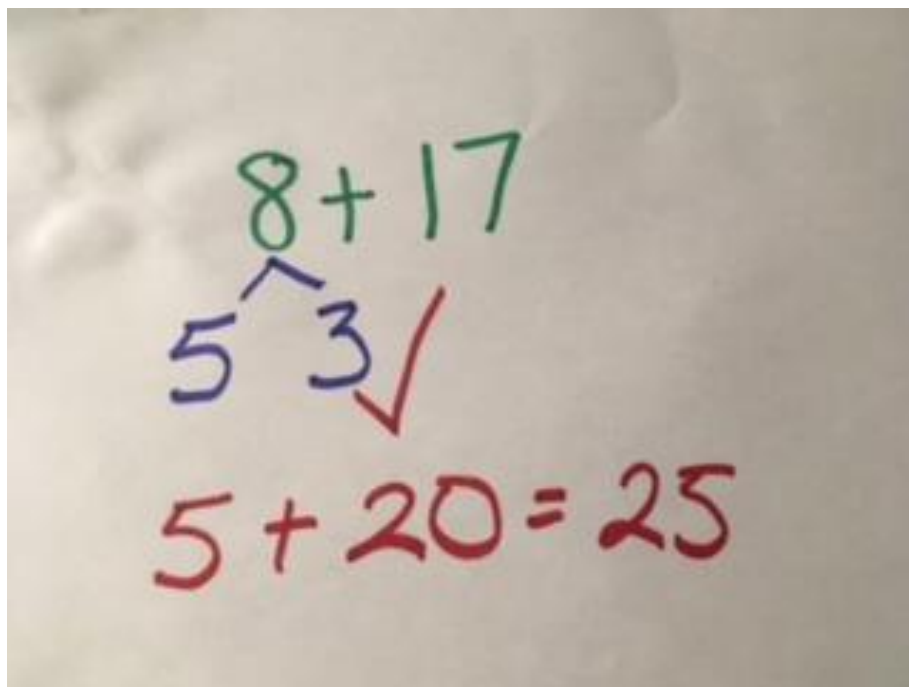
Maya finds the area of the quilt in the following way:

$$\begin{aligned} 8 \times 17 &= (5 + 3) \times 17 \\ &= 5 \times (3 + 17) \\ &= 5 \times 20 \end{aligned}$$

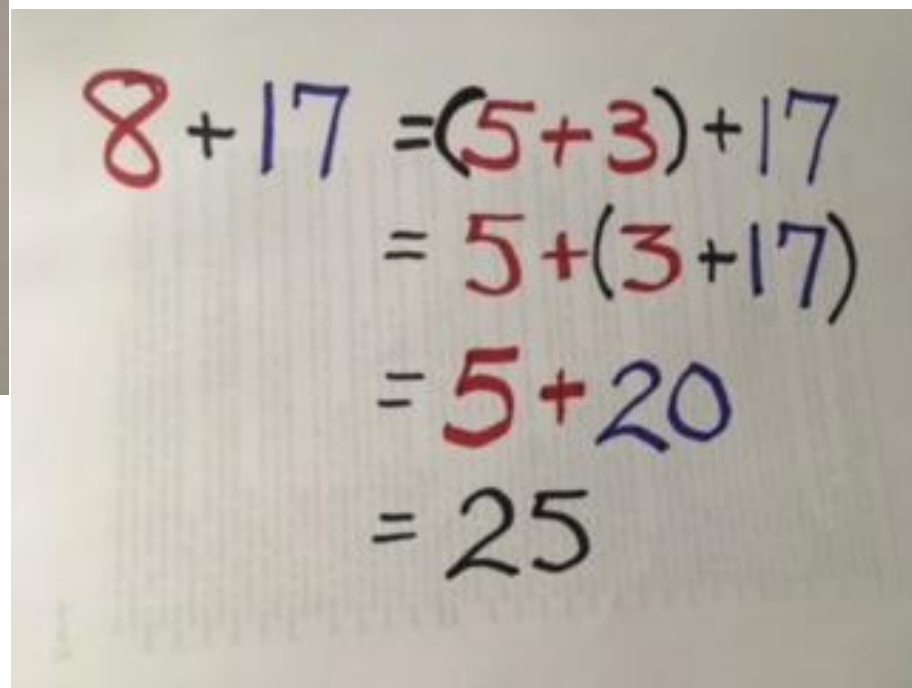
1. Do you agree with Maya's reasoning? Why or why not? Use pictures, numbers, and words to defend your thinking.
2. Find another way to calculate the area of the rectangular quilt. Explain why your method is correct.

Participation Structures to Support Math Discourse

Viewing Learning Along a Teaching and Learning Trajectory



Handwritten math showing a strategy for $8 + 17$. The number 8 is written in green, and 17 is written in green. Below 8, the number 5 is written in blue, and below 5, the number 3 is written in blue. A red checkmark is drawn next to the 3. Below this, the equation $5 + 20 = 25$ is written in red.

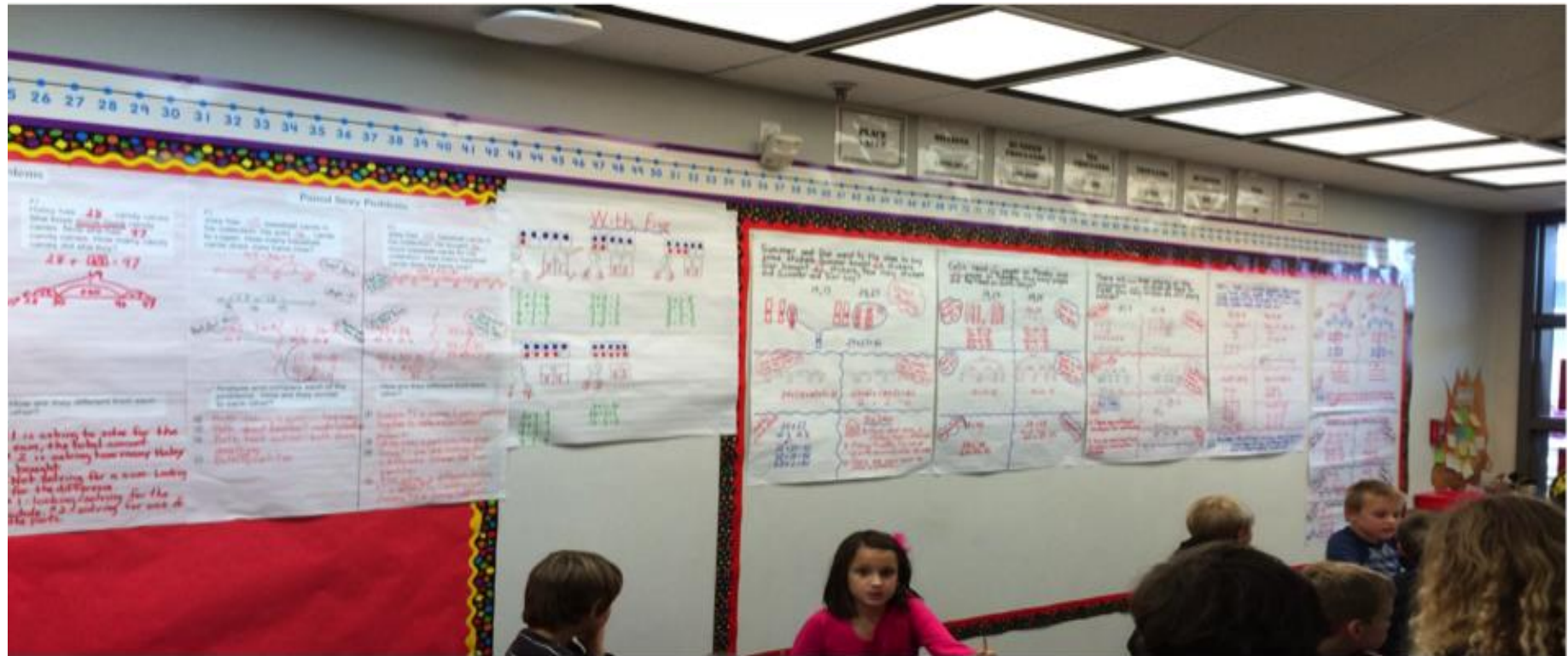


Handwritten math showing the associative property for $8 + 17$. The equation is written in red and blue ink:

$$\begin{aligned} 8 + 17 &= (5 + 3) + 17 \\ &= 5 + (3 + 17) \\ &= 5 + 20 \\ &= 25 \end{aligned}$$

Participation Structures to Support Math Discourse

Viewing Learning Along a Teaching and Learning Trajectory

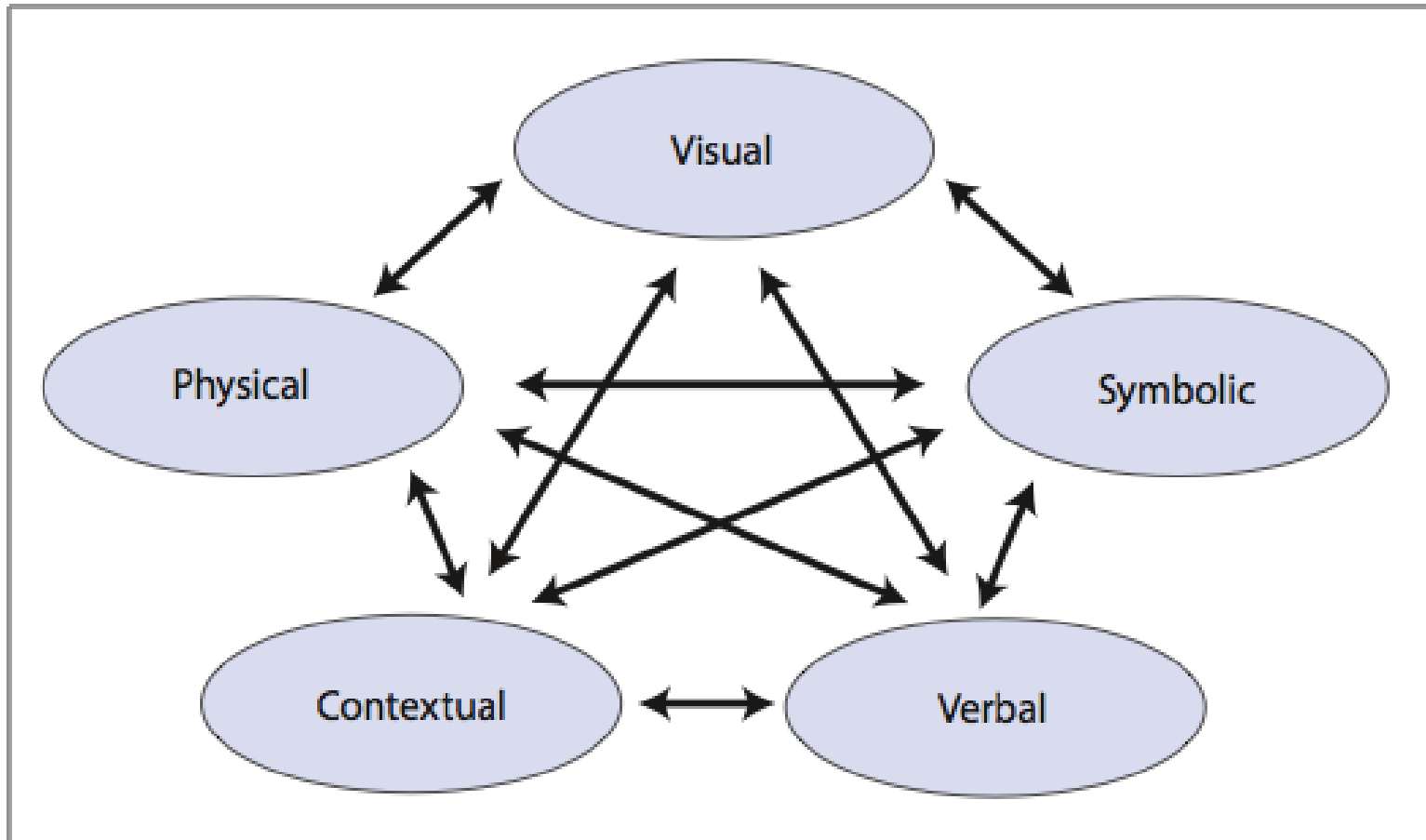


Participation Structures to Support Math Discourse



Individual Think Time

Connections Across Mathematical Representations



Representations are useful in all areas of mathematics because they help us develop, share, and preserve our mathematical thoughts. "[They] help to portray, clarify, or extend a mathematical idea by focusing on its essential features" (NCTM, 2000, p. 206).

Individual Engagement

- Encourage the use of multiple representations
- Allow students to code-switch when expressing ideas
- Use student-made dictionaries to link concepts and terms with drawings and examples
- Provide students daily opportunities to explain and justify in writing and speech
- Use math journals to record solutions and mathematical thinking.
- Spend time unpacking the problem (but allow students to engage in the beauty of the mathematical struggle)

Participation Structures to Support Math Discourse



Individual Think Time

Participation Structures to Support Math Discourse

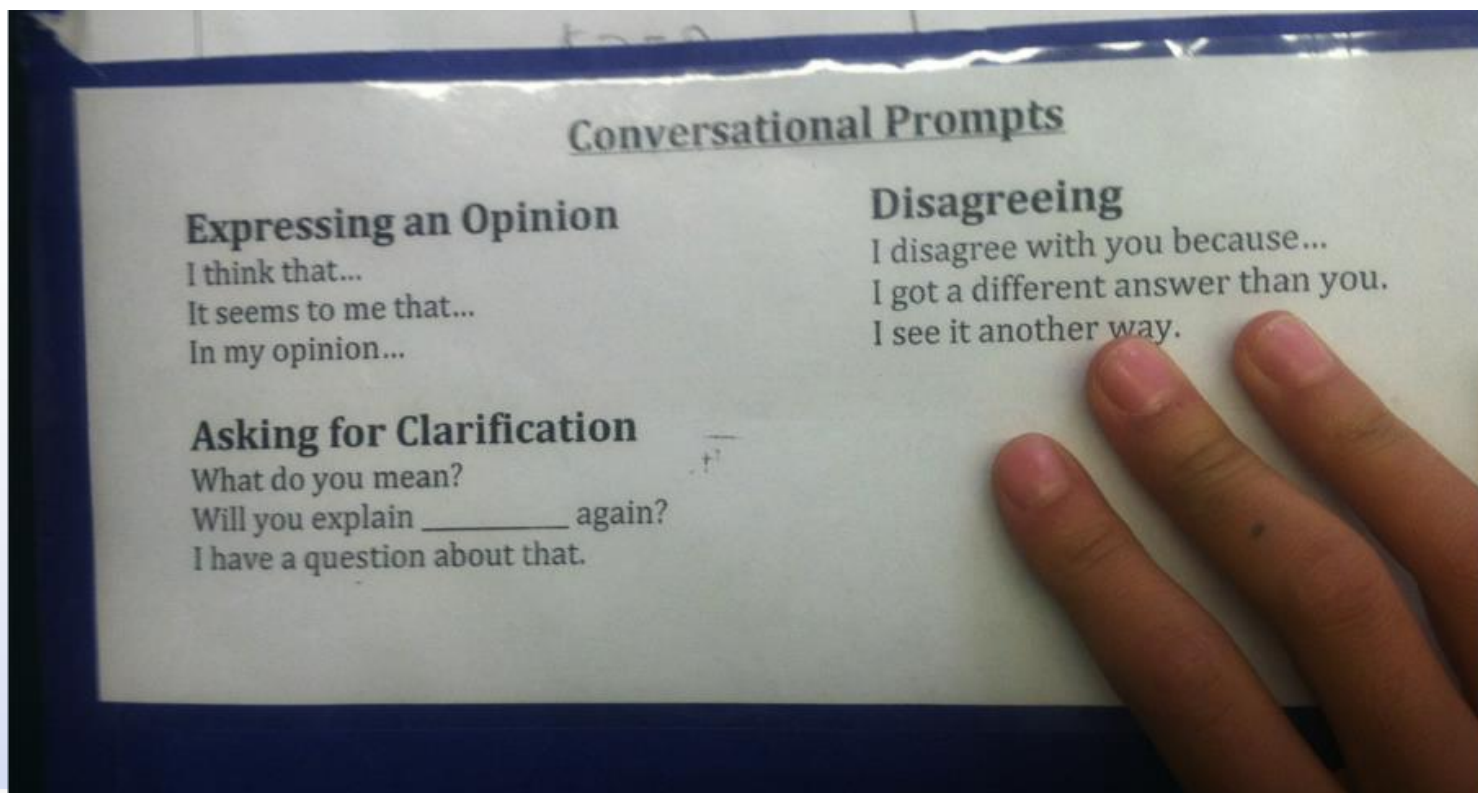


Individual Think Time



Focused Pair Share

Participation Structures to Support Math Discourse



Productive Group Interactions

- Value multiple representations and multiple perspectives (create the idea and not the person).
- Hold students accountable for individual and shared responsibility for learning.
- Establish flexible groups.



“Creating a culture of shared responsibility and equal participation requires strategic planning.”

Productive Group Interactions

→→→ step 1 Pokemon

$$\begin{array}{r} 6 \times 200 = 1,200 \\ 6 \times 40 = 240 \\ 6 \times 3 = 18 \\ \hline 1,458 \end{array}$$

$$\begin{array}{r} 3,982 \text{ - total cards} \\ - 1,458 \text{ - Pokemon} \\ \hline 1,544 \text{ - Yu-gi-oh} \end{array}$$

Yu-gi-oh divided into 4 equal stacks

$$1,544 \div 4 = 386$$

$$\frac{100}{100} \times \frac{100}{100}$$

$$\frac{50}{15} \times \frac{10}{10}$$

$$\begin{array}{r} 125 \\ 250 \\ 10 \\ \hline 386 \end{array}$$

$$\frac{80}{100} \times \frac{100}{100}$$

$$40 \div 4 = 10$$

$$1,000 \div 4 = 250$$

$$\frac{100}{100} \times \frac{50}{50}$$

$$\frac{100}{100} \times \frac{50}{50}$$

$$\frac{100}{100} \times \frac{50}{50}$$

$$\frac{100}{100} \times \frac{50}{50}$$

A = Robert stacked 386 Yu-gi-oh cards in each stack.

$$\begin{array}{r} 112 \\ 112 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 1,544 \\ + 1,458 \\ \hline 3,002 \end{array}$$

$$\begin{array}{r} 386 \times 12 \\ + 386 \times 4 \\ \hline 1,544 \end{array}$$

$$3 = 1,544$$

$$\begin{array}{r} \times 4 = 1,200 \\ \times 4 = 320 \\ \times 4 = 24 \end{array}$$

$$\begin{array}{r} 386 \times 772 \\ + 386 \times 772 \\ \hline 1,544 \end{array}$$

Robert decided to organize his Pokemon and Yu-gi-oh card collections. He put his Pokemon cards in 6 stacks with 243 cards in each stack. Then he put his Yu-gi-oh cards into 4 stacks. If altogether he had 3,002 cards, how many Yu-gi-oh cards did he put in each stack?

- Pokemon cards in 6 stacks with 243 cards each.
- 3,002 cards in all

add
divide
multiply
subtract

• each stack
• altogether

How many Yu-gi-oh cards did Robert place in each pile?

• = Sydney
• = Owen

Strategies for Whole Class Group Dialogue

- Student Talk Before the Share
- Anticipating
- Engage in Strategy Share Together
- Compare & Connect Class Discussion
- Finish the Thought and/or Error Analysis

There are ____ boxes of chocolates.
If there are ____ chocolates in each
box, how many chocolates are
there altogether?

(16, 9) (5, 9) (8, 9) (12, 9)

Principles Guiding Class Discussions

- Discussions should achieve a mathematical goal
- Different types of goals require leading discussions differently
- Students need to know what and how to share so their ideas are heard and are useful to others
- Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal
- Teachers must communicate that all students are sense makers and that their ideas are valued

Intentional Talk: How to Structure and Lead Productive Mathematical Discussions (Kazemi & Hintz, 2014)

Targeted Discussions: Compare & Connect

Goal: To compare similarities and differences among strategies

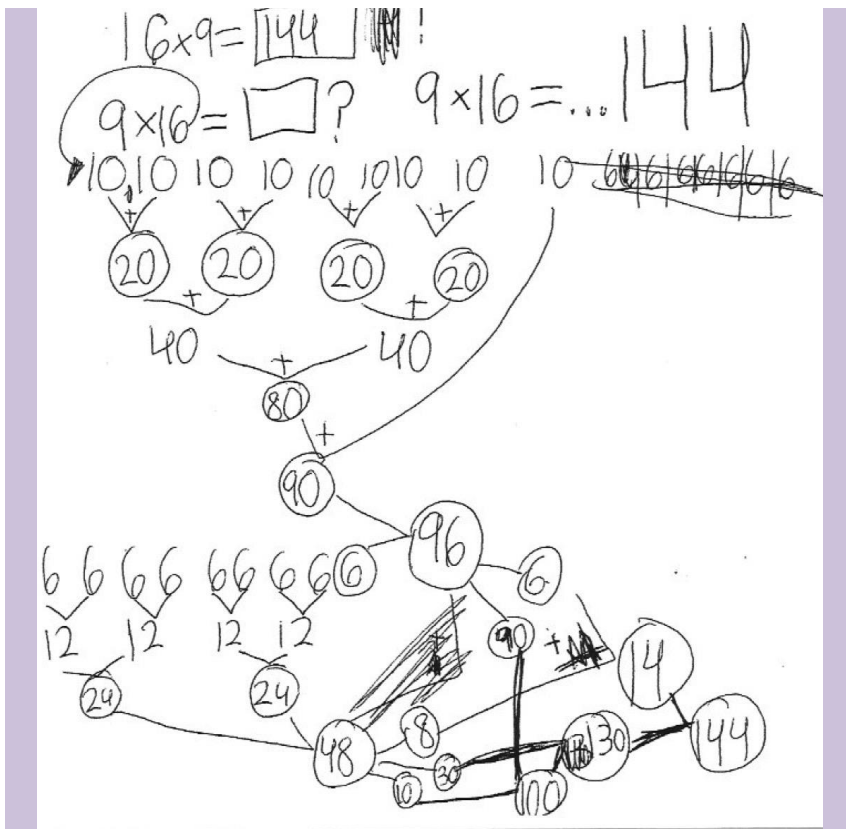
Considerations:

- What **strategies** might I see? How might I support student thinking as they engage in each others' strategies?
- What is a **key mathematical idea** I might want to highlight?
- What **connections** are important for students to notice?

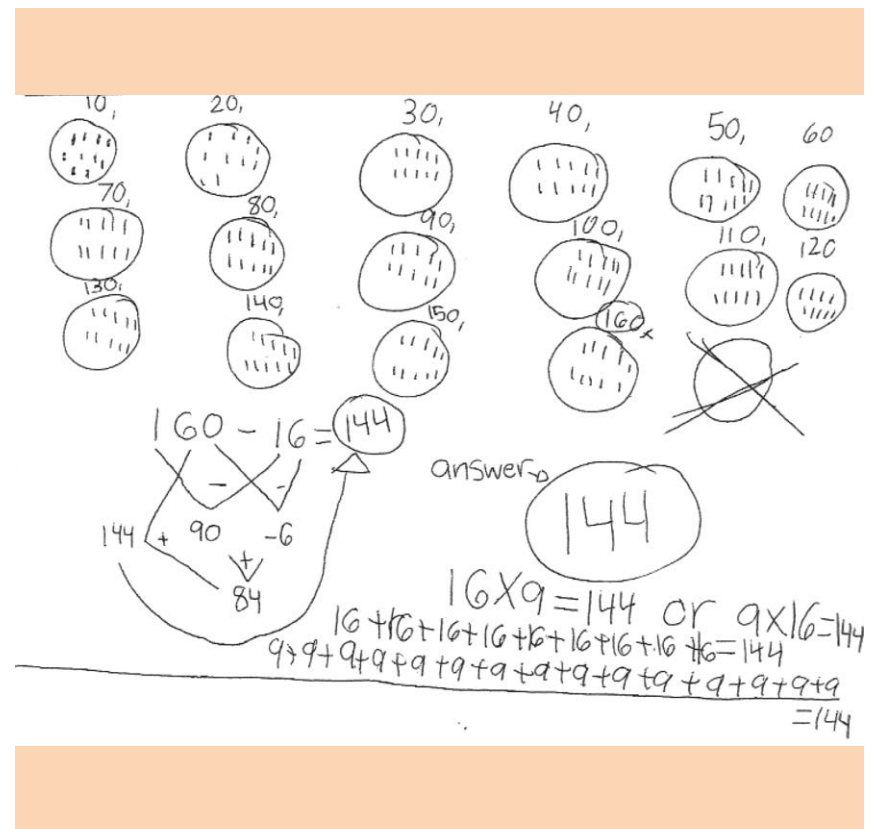
Intentional Talk: How to Structure and Lead Productive Mathematical Discussions (Kazemi & Hintz, 2014)

Compare & Connect

Ella



Chelsea



Finish the Thought and/or Error Analysis

How Many Ways Can You Make...

$$\frac{35}{10}$$

How Many Ways Can You Make $\frac{35}{10}$

Handwritten mathematical expressions showing various ways to make the fraction $\frac{35}{10}$:

- $(3 \times \frac{10}{10}) + \frac{5}{10}$
- $\frac{30}{10}$ (with a 3 above it)
- $\frac{11}{10} + \frac{11}{10} + \frac{11}{10} + \frac{2}{10}$
- $\frac{15}{10} + \frac{20}{10}$
- $\frac{20}{10} + \frac{8}{10} + \frac{7}{10}$
- $\frac{10}{10} + \frac{25}{10}$
- $1 + \frac{25}{10}$
- $(5 \times \frac{5}{10}) + \text{---}$
- A central circle containing $\frac{35}{10}$ with lines pointing to the various expressions.
- A boxed expression: $3\frac{1}{2} = \frac{35}{10}$ with "T/F" written above it.

How Many Ways Can You Make $\frac{35}{10}$

Handwritten mathematical expressions showing various ways to make $\frac{35}{10}$:

- $(3 \times \frac{10}{10}) + \frac{5}{10}$
- $\frac{30}{10}$
3
- $\frac{11}{10} + \frac{11}{10} + \frac{11}{10} + \frac{2}{10}$
- $\frac{15}{10} + \frac{20}{10}$
- $\frac{20}{10} + \frac{8}{10} + \frac{7}{10}$
- $\frac{10}{10} + \frac{25}{10}$
- $1 + \frac{25}{10}$
- $3\frac{1}{2} = \frac{35}{10}$ (labeled T/F)
- $(5 \times \frac{5}{10}) + \underline{\hspace{2cm}}$ (highlighted in yellow)

Levels of Classroom Discourse

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
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What Discourse Strategies Might You Use?



STOP AND REFLECT

Before reading further, try to solve this problem:

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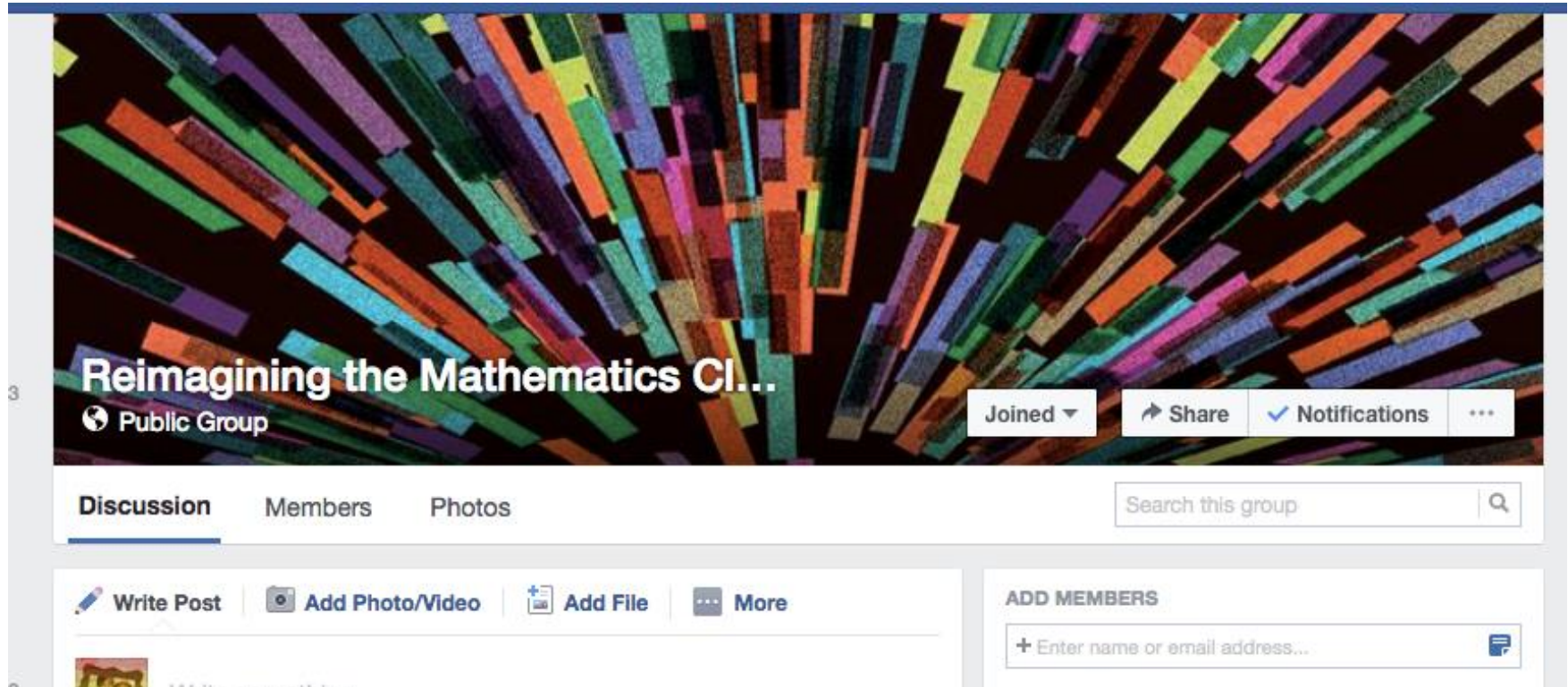
Maya finds the area of the quilt in the following way:

$$\begin{aligned} 8 \times 17 &= (5 + 3) \times 17 \\ &= 5 \times (3 + 17) \\ &= 5 \times 20 \end{aligned}$$

1. Do you agree with Maya's reasoning? Why or why not? Use pictures, numbers, and words to defend your thinking.
2. Find another way to calculate the area of the rectangular quilt. Explain why your method is correct.

Learning Together

Collaborative Learning Opportunities



Join us on: **Reimagining the Mathematics Classroom on Facebook!**

Thank you!



Questions



Join us on Saturday, April 8th for
Juicy Tasks to Nourish Students' Mathematical Reasoning
9:30AM - 10:30AM
Convention Center, Room 221

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References

- Kazemi, E., & Hintz, A. (2014). *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions*. Portland, Maine: Stenhouse Publishers, 2014.
- National Council of Teachers of Mathematics. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. 2014.
- Yeh, C, Ellis, M, & Hurtado, C. (2017). *Reimagining the Mathematics Classroom: Creating and Sustaining Productive Learning Environments, K-6*. National Council of Teachers of Mathematics. Reston, VA.