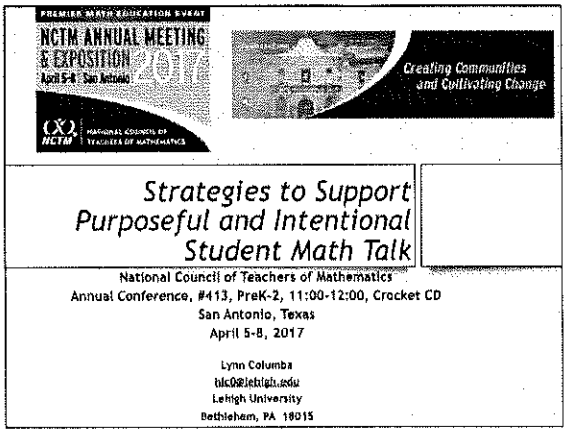


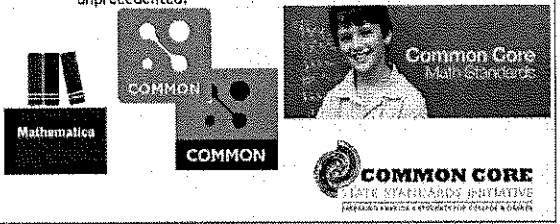
Why Mathematical Discourse?

- CCSSM
- Mathematical Practices
- Defining Mathematical Discourse
- Why should we talk?
- Creating a learning community

To think is to communicate
To communicate is to think

The Common Core State Standards in Mathematics (CCSSM)

- National Council Teachers of Mathematics
  - The opportunity to change the way math is taught is unprecedented.



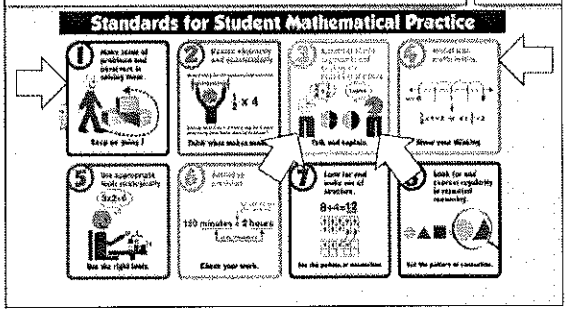
Many parents' perspective of the reform mathematics:

"It's like, Bill has three goldfish. He buys two more. How many dogs live in London?"

Comedian Louis C. K.
The Late Show with
David Letterman

MATH is FUN!

CCSSM's Mathematical Practices



Words for mathematical discourse

- Math-talk
- Dialogic
- Teacher-student interaction
- Student-student interaction
- Accountable talk
- Number talks
- Math conversations
- Classroom discussions
- Dialogue
- Math expressions
- Sharing strategies
- Think like mathematicians

Start the conversation

Definitions of mathematical discourse

- that they reveal their understanding of concepts. Discourse involves asking strategic questions that elicit from students both how a problem was solved and why a particular method was chosen. (Ministry of Education, New Zealand)
- Mathematical discourse occurs when teachers ask questions and students respond. (ASCD)
- Schwols & Dempsey (2012)--identify the following components: questioning, facilitation of conversation, appreciation for accuracy on reasoning and proof, and, collaborative exchange of ideas.
- Discourse-ways of representing, thinking talking, agreeing, and disagreeing; the way ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage as well as by the nature of the learning environment (NCTM, 1991).
- Whole classroom discussions in which students talk about mathematics in such a way that they reveal their understanding of concepts. Students also learn to engage in reasoning and debate (Maruire & Neill, 2006).

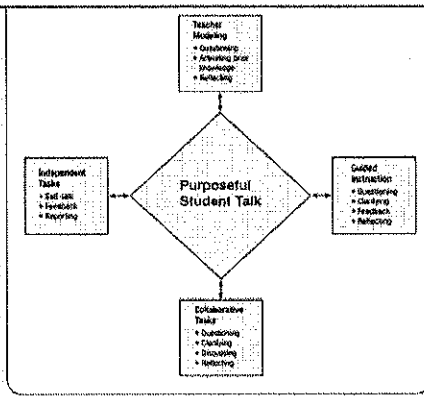
My working definitions of mathematical discourse

- Discourse, "math-talk," or orchestrates instruction in which young learners are challenged by asking them to clarify and justify their ideas and to express their mathematical thinking.
- Knowing what to do with students' ideas and teaching children how to in discussions by identifying a goal and then understand how to structure and facilitate the conversation to meet that goal (Kazemi & Hintz, 2014).
- An interactive and sustained discourse of a dialogic nature between teachers and students aligned to the content of the lesson that addresses learning issues (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008, p. 378)

Why Should We Talk?

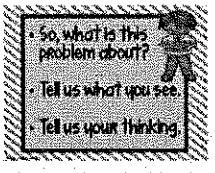
- and we get to help others learn." a 4th grader speaking (*TCM*, Wagganer, 2015)
- Vygotsky (1962) suggested that thinking develops into words in a number of phases, moving from imaging to inner speech to inner speaking to speech. Tracing this idea backward, speech—talk—is the representation of thinking.
- Language permits its users to pay attention to things, persons and events, even when the things and persons are absent and the events are not taking place. Language gives definition to our memories and, by translating experiences into symbols, converts the immediacy of craving or abhorrence, or hatred or love, into fixed principles of feeling and conduct. (Huxley, 1958p. 168)

Types of Talk (ASCD)



Creating a Math-Talk Learning Community

- A math-talk environment is a helping community where inquiry and meaning-making are nurtured, and everyone is a teacher and a learner.
- Differences Among Students
  - Linguistic
  - Proficiency in the home language
  - Number of languages spoken
  - Motivation
  - Poverty
  - Personality



Brief review of the research literature (1)

- (National Mathematics Advisory Panel so exploring instructional strategies to increase this practice is important (Boohen, Kolkman, & Kroesbergen, 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006).
- The language teachers and their students' use in the classroom affects conceptual development (Johnston, 2004).
- Standard of Mathematical Practice 3--Construct viable arguments and critique the thinking of others--guide students to explain their thinking and consider the mathematical thinking of others (NGA & CCSSO, 2010, p. 6)
- Recent studies in the professional development of teachers has focused on the importance of classroom discourse as it relates to teacher learning --a verbal window into the teacher's developing practice by documenting patterns of interactions between the teacher and students (Wood, 1995).

Discourse, Number Talks, Classroom Discussion, Dialogue...

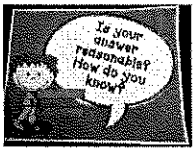
No matter what you call it, Math Talk" is essential.

Brief review of the research literature (2)

- In teacher education, we need to cultivate a practice that engages students in dialogic interactions. Changing the nature of classroom discourse likely requires confronting existing norms for doing mathematics. Preservice teachers are at a critical conjuncture at which teacher educators can assist in renegotiating the nature of classroom discourse (Blanton, Berenson, & Norwood, 2001).
- In a study of 35 kindergarten teachers suggests that teachers should be careful and selective with the amount of teacher talk they offer to young children (Boonen, Kolkman, & Kroesbergen, 2011).
- Analysis of two preservice teachers conceptions and practices of various forms of classroom communication: uni-directional, contributive, reflective, and instructive.

Strategies to Teach Mathematical Discourse

- Discuss why math talk is important.
- Teach students how to listen and respond.
- Introduce sentence stems.
- Contrast explanation versus justification.
- Give an example.



Productive Talk Moves

- **Revoicing:** Teacher repeats some of all of what the student has said. Students verify.
- **Repeating:** Asking students to restate someone else's reasoning.
- **Reasoning:** Asking students to apply their won reasoning to someone else's reasoning.
- **Adding on:** Promoting students for further participation.
- **Waiting:** Using wait time.

From *Classroom Discussion: Using Math Talk to Help Students Learn, 2004*

Creating Math Talk Communities *TCM*, Wagganer, 2015

After in-depth reading about how to foster classroom math talk, the author presented sentence stems, a tool introduced by Rawding and Wills (2012).

Sentence stems

I agree with ____ because . . .

This is what I think . . .

I have a different perspective because . . .

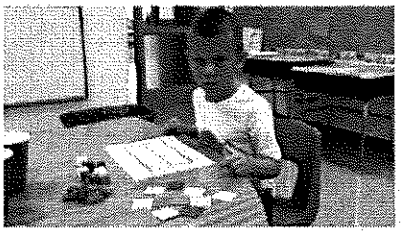
I made a connection with what ____ said . . .

When I thought about the question, I remembered . . .

I chose this method because . . .

Rawding & Wills, 2012

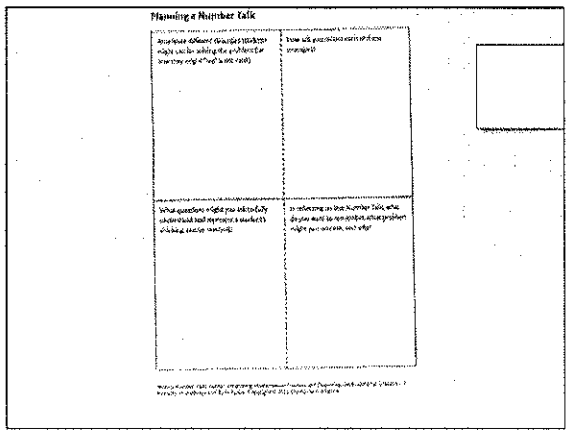
Video—Preservice Elementary Teacher-Kindergarten Student



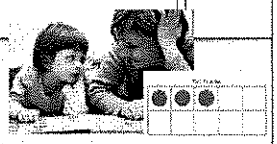
Planning a Number Talk

- Anticipate different strategies
- How will you record each of these strategies
- What question might you ask to fully understand and represent a student's thinking or method?
- Reflecting—what do you remember, what problem might you do next, why?

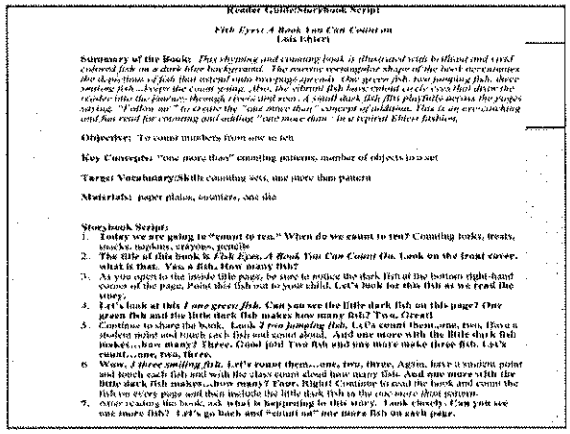
Deepening Understanding, Grades 4-10,



Picture Books as the Springboard-- A Reader's Guide



- Summary of the book
- Objectives
- Key Concepts
- Recommended Questions to Ask your Child(ren)—to encourage and engage in dialog
- Engaging the child(ren)-hands-on exploration to continue the dialog and enhance the concept
  - Example (handout)--*Fish Eyes*, Lois Ehlert



Reader Guide/Storybook Script

Fish Eyes: A Book You Can Count on
Lois Ehlert

Summary of the Book: *This rhyming and counting book is illustrated with brilliant and vivid colored fish on a dark blue background. The reverse rectangular shape of the book accentuates the depictions of fish that extend onto two page spreads. One green fish, two jumping fish, three smiling fish…Keep the count going. Also, the vibrant fish have critical circle eyes that draw the reader into the journey through rivers and seas. A small dark fish flits playfully across the pages saying, "Follow me" to create the "one more than" concept of addition. This is an eye-catching and fun read for counting and adding "one more than" in a typical Ehlert fashion.*

Objectives: To count numbers from one to ten

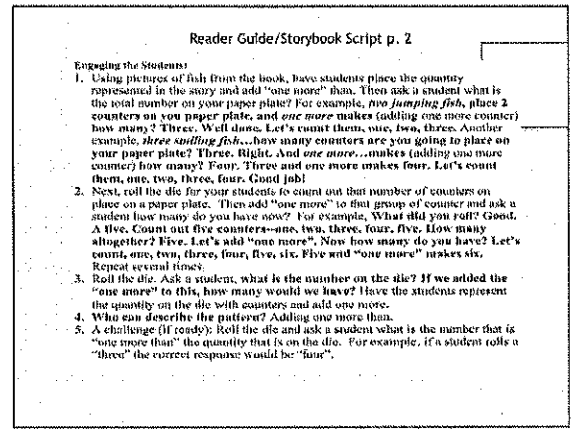
Key Concepts: "one more than" counting patterns, number of objects in a set

Target Vocabulary/Skills: counting sets, one more than pattern

Materials: paper plates, counters, one die

Storybook Script:

1. **Today we are going to "count to ten." When do we count to ten?** Counting books, treats, snacks, napkins, crayons, pencils
2. **The title of this book is *Fish Eyes: A Book You Can Count On.* Look on the front cover. what is that. Yes, a fish. How many fish?**
3. As you open to the inside title page, be sure to notice the dark fish at the bottom right-hand corner of the page. Point this fish out to your child. **Let's look for this fish as we read the story.**
4. **Let's look at this *1 one green fish.* Can you see the little dark fish on this page? One green fish and the little dark fish makes how many fish? Two. Great!**
5. **Continue to share the book. Look *2 two jumping fish.* Let's count them…one, two. Have a student point and touch each fish and count aloud. And one more with the little dark fish makes…how many? Three. Good job! Two fish and one more make three fish. Let's count…one, two, three.**
6. **Wow, *3 three smiling fish.* Let's count them…one, two, three. Again, have a student point and touch each fish and with the class count aloud how many fish. And one more with the little dark fish makes…how many? Four. Right! Continue to read the book and count the fish on every page and then include the little dark fish in the *one more than* pattern.**
7. **After reading the book, ask what is happening in this story. Look closely. Can you see one more fish? Let's go back and "count on" one more fish on each page.**



Reader Guide/Storybook Script p. 2

Engaging the Students:

1. **Using pictures of fish from the book, have students place the quantity represented in the story and add "one more" than. Then ask a student what is the total number on your paper plate? For example, *two jumping fish*, place 2 counters on you paper plate, and *one more* makes (adding one more counter) how many? Three. Well done. Let's count them, one, two, three. Another example, *three smiling fish*…how many counters are you going to place on your paper plate? Three. Right. And *one more*…makes (adding one more counter) how many? Four. Three and one more makes four. Let's count them, one, two, three, four. Good job!**
2. **Next, roll the die for your students to count out that number of counters on place on a paper plate. Then add "one more" to that group of counter and ask a student how many do you have now? For example, What did you roll? Good. A five. Count out five counters--one, two, three, four, five. How many altogether? Five. Let's add "one more". Now how many do you have? Let's count, one, two, three, four, five, six. Five and "one more" makes six.** Repeat several times.
3. **Roll the die. Ask a student, what is the number on the die? If we added the "one more" to this, how many would we have?** Have the students represent the quantity on the die with counters and add one more.
4. **Who can describe the pattern?** Adding one more than.
5. A challenge (if ready): Roll the die and ask a student what is the number that is "one more than" the quantity that is on the die. For example, if a student rolls a "three" the correct response would be "four".

Power of Questioning as a Way of Learning

All our knowledge results from questions, which is another way of saying that question-asking is our most important intellectual tool.

--Neil Postman

Teachers use questions to serve many purposes in a classroom. Cotton (1988)

- To develop interest and motivate students to become actively involved in lessons;
- To evaluate students' preparation and check on homework or seatwork completion;
- To develop critical thinking skills and inquiring attitudes;
- To review and summarize previous lessons;
- To nurture insights by exposing new relationships;
- To assess achievement of instructional goals and objectives; and
- To stimulate students to pursue knowledge on their own.

Revisit Bloom's Taxonomy of the Cognitive Domain (1956)

- Probing questions are a teaching/assessment strategy that provides insight into the mental process students' use by engaging them in conversation about mathematics.
- The goal of the *question* is to deepen students' understanding of the mathematics concepts.
- Preparing these *effective questions* are an integral component of designing quality mathematics lesson plans for the classroom.

An Effective Question

The ones for which students actively compose a response and thereby become engaged in the learning process.

Bloom's Taxonomy Revised (2002)

Category	Description	Related Cognitive Processes
Remember (formerly Knowledge)	Retrieving relevant knowledge from long-term memory	Recognizing Recalling
Understand (formerly Comprehension)	Determining the meaning of instructional messages, including oral, written, and graphic communication	Interpreting Exemplifying Classifying Summarizing Inferring Comparing Explaining
Apply	Carrying out or using a procedure in a given situation	Executing Implementing
Analyze	Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.	Differentiating Organizing Attributing
Evaluate	Making judgments based on criteria and standards.	Checking Critiquing
Create (formerly Synthesis)	Putting elements together to form a novel, coherent whole or make an original product.	Generating Planning Producing

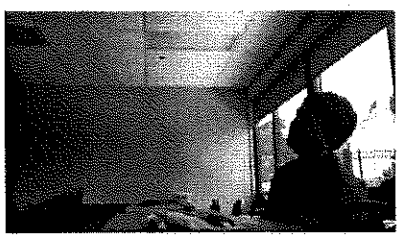
Levels	Definition	Sample Question
Knowledge	Recall of facts	What is 3 + 2?
Comprehension	Understanding, transforming info	Can you use cubes/counters to create a combination of 3 + 2?
Application	Using information to solve new problems	Can you show that 3 + 2 is the same as 2 + 3?

Levels	Definition	Sample Question
Analysis	Breaking down the idea into component parts	Explain the "Turnaround Property?" (Commutative Property)
Synthesis	Combine parts to form new solutions to a problem	Write the four members fact family for 3 + 2 = 5 (Addition/Subtraction Fact families); What are other ways to arrive at the answer of 5?
Evaluation	Suggesting well-reasoned decisions	What is the best way to arrive at the answer of 5? Justify your answer.

Hunkins (1995)

"In a classroom atmosphere conducive to good questions and questioning, students realize a shared responsibility for their learning."

Video Preservice Special Education Teacher-Elementary Student



Socratic Questions

Question Type	Sample Question
Questions of Clarification	*Why do you say that?*
Questions That Probe Purpose	*What is the purpose of ...?*
Questions That Probe Assumptions	*How can we verify that assumption?*
Questions That Probe Information, Reasons, Evidence, and Causes	*Can you give an example? Can you tell me why it happened?*
Questions about Viewpoints or Perspectives	*What is another way to look at it?*
Questions That Probe Implications and Consequences	*What do you think is an example of that assumption?*
Questions about Questions	*What was the point of this question?*
Questions That Probe Concepts	*Why or how is this idea important?*
Questions That Probe Inferences and Interpretations	*How did you reach that conclusion?*

Wait-time---the "miracle pause"—beyond 3 seconds

2005)

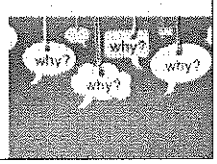
- achievement;
- Improvements in student retention, as measured by delayed tests;
- Increases in the number of higher cognitive responses generated by students;
- Increases in the length of student responses;
- Increases in the number of unsolicited responses;
- Decreases in students failing to respond;
- Increases in the amount and quality of evidence students offer to support their inferences;
- Increases in contributions by students who do not participate much when wait-time is under three seconds;
- Expands the variety of responses offered by students;
- Decreases in student interruptions;
- Increases in student-student interactions; and,
- Increases in the number of questions posed by students.

Three Steps to Encourage Interaction

1. Ask "what" questions. Point to an item in a book and say, "what's this?"
2. Expand on what the child says. Keep expansions short and simple. Add, "Yes, you're right! That's a square."
3. Ask open-ended questions. "Tell me about." "What if?" "What do you think?"

The value of "*Why*?"

- Answers to "Why?" questions require reflective thinking and diminish guesses or responses based on rote memorization.
  - Why did that happen?
  - Why didn't that happen?
  - Will that always happen?
  - What are the next steps?
  - What will happen next?
  - Can you illustrate your answer?
  - Is there another way to solve it?
  - Have you exhausted all the possibilities?



Dialogic Reading Technique-PEER

- **P**rompts the child to say something about the book/problem situation/manipulatives
- **E**valuates the child's response
- **E**xpands the child's response by rephrasing and adding information to it, and
- **R**epeats the prompt to make sure the child has learned from the expansion.

Teacher-Student Interaction Example--PEER

- Looking at the page of a book that has a picture of a red triangle on it. The teacher says, "What is this?" while pointing to the triangle. The child says, *triangle*, and the teacher follows with "That's right **(evaluation)**; it's a red triangle **(expansion)**; look at that red triangle" **(repetition)**

Questioning for Teachers and Students

- Elicit student thinking
  - Tell us what you see, Tell us your thinking
- Support student thinking
  - What did you mean when you said, Show us on your drawing what you mean
- Extend student thinking
- Increase participation of other students in the conversation
- Probe specific math topics

How to prompt children-CROWD

- **Completion prompts—fill** in the blank, provides child with information on the structure of language
- **Recall prompts—**Can you tell me? Explain steps,
- **Open-ended prompts—Tell** me what is happening, increase expressive fluency
- **Wh- prompts**—what, where, when, why, how, develop new vocabulary
- **Distancing prompts**—relate pictures or words outside the classroom, connect to the real world

Rubrics for Implementing Discourse for Teachers

- www.clemson.edu/iim
- Session 5 - How Can Professional Development Enable Teachers to Improve Student Achievement? Corwin Press Secondary Lenses on Learning: Team Leadership for Mathematics in Middle and High Schools Copyright 2009
- Silicon Valley Mathematics Initiative, The Mathematics Teaching Rubric--Teacher's Role, Student's Role, Tools for Enhancing Discourse

V. Discourse Factors

Construct Measured		Pre-Inquiry (Level 1)	Developing Inquiry (2)	Proficient Inquiry (3)	Exemplary Inquiry (4)
D1.	Questioning Level	Questioning rarely challenged students above the remembering level	Questioning rarely challenged students above the understanding level	Questioning challenged students up to application or analysis levels	Questioning challenged students at various levels, including at the analysis level or higher; level was varied to scaffold learning.
D2.	Complexity of Questions	Questions focused on one correct answer; typically short answer responses	Questions focused mostly on one correct answer; some open response opportunities	Questions challenged students to explain, reason, and/or justify.	Questions required students to explain, reason, and/or justify. Students were expected to critique others' responses.
D3.	Questioning Ecology	Teacher lectured or engaged students in oral questioning that did not lead to discussion.	Teacher occasionally attempted to engage students in discussions or investigations but was not successful.	Teacher successfully engaged students in open-ended questions, discussions, and/or investigations.	Teacher consistently and effectively engaged students in open-ended questions, discussions, investigations, and/or reflections.
D4.	Communication Pattern	Communication was controlled and directed by teacher and followed a didactic pattern.	Communication was typically controlled and directed by teacher with occasional input from other students; mostly didactic pattern.	Communication was often conversational with some student questions guiding the discussion.	Communication was consistently conversational with student questions often guiding the discussion.
D5.	Classroom Interactions	Teacher accepted answers, correcting when necessary, but rarely followed-up with further probing.	Teacher or another student occasionally followed-up student response with further low-level probe.	Teacher or another student often followed-up response with engaging probe that required student to justify reasoning or evidence.	Teacher consistently and effectively facilitated rich classroom dialogue where evidence, assumptions, and reasoning were challenged by teacher or other students.

INQUIRY IN MOTION
www.clemson.edu/iim

[p.1] *Handout 5A* Math Talk Learning Community Rubric

Describing Levels and Components of a Math-Talk Learning Community

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Overview of Shift over Levels 0-3: The classroom community grows to support students acting in central or leading roles and shifts from a focus on answers to a focus on mathematical thinking.

A. Questioning	B. Explaining mathematical thinking	C. Source of mathematical ideas	D. Responsibility for learning
Shift from teacher as questioner to students and teacher as questioners.	Students increasingly explain and articulate their math ideas.	Shift from teacher as the source of all math ideas to students' ideas also influencing direction of lesson.	Students increasingly take responsibility for learning and evaluation of others and self. Math sense becomes the criterion for evaluation.

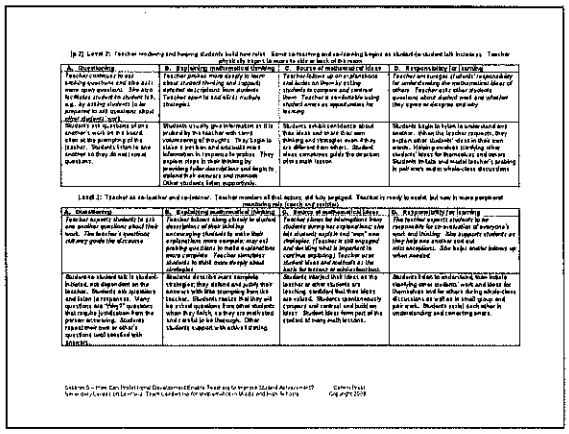
Level 0: Traditional teacher-directed classroom with brief answer responses from students.

A. Questioning	B. Explaining mathematical thinking	C. Source of mathematical ideas	D. Responsibility for learning
Teacher is the only questioner. Short frequent questions function to keep students listening and paying attention to the teacher.	*No or minimal teacher elicitation of student's thinking, strategies, or explanations; teacher expects answer-focused responses. Teacher may tell answers.*	*Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math.*	*Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students' answers by verifying the correct answer or showing the correct answer method.*
Students give short answers and respond to the teacher only. No student-to-student math talk.	No student thinking or strategy-focused explanation of work. Only answers are given.	Students respond to math presented by the teacher. They do not offer their own math ideas.	Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves.

Level 1: Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community.

A. Questioning	B. Explaining mathematical thinking	C. Source of mathematical ideas	D. Responsibility for learning
Teacher questions begin to focus on student thinking and focus less on answers. Teacher begins to ask follow-up questions about student methods and answers. Teacher is still the only questioner.	*Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself.*	*Teacher is still the main source of ideas, though she elicits some student ideas. Teacher does some probing to access student ideas.*	*Teacher begins to set up structures to facilitate students listening to and helping other students. The teacher alone gives feedback.*
As a student answers a question, other students listen passively or wait for their turn.	Students give information about their math thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide brief descriptions of their thinking.	Some student ideas are raised in discussions, but are not explored.	Students become more engaged by repeating what other students say or by helping another student at the teacher's request. This helping mostly involves students showing how they solved a problem.

Session 5 - How Can Professional Development Enable Teachers to Improve Student Achievement? Corwin Press
Secondary Lenses on Learning: Team Leadership for Mathematics in Middle and High Schools Copyright 2009



Final Thoughts on Math-Talk

- Goal—a nurturing, meaning-making math talk community
- Everyone is a teacher and a learner
- Enhance everyone's mathematical understanding, competence, and confidence

Thank you for your attention and participation. Wishing you good teaching and a terrific NCTM conference.

ready, set, math talk!

Math Talk is Critical to Teaching and Learning

Why Math Talk?

Explain your... Reasoning

Could you say more about...

I have another opinion...

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