

A Whole New World

$$\begin{array}{c} \text{Number Talks} \\ + \\ \text{High School Content} \\ = \\ \text{Math Talks} \end{array}$$

A blue-outlined speech bubble with a tail pointing towards the bottom-left.

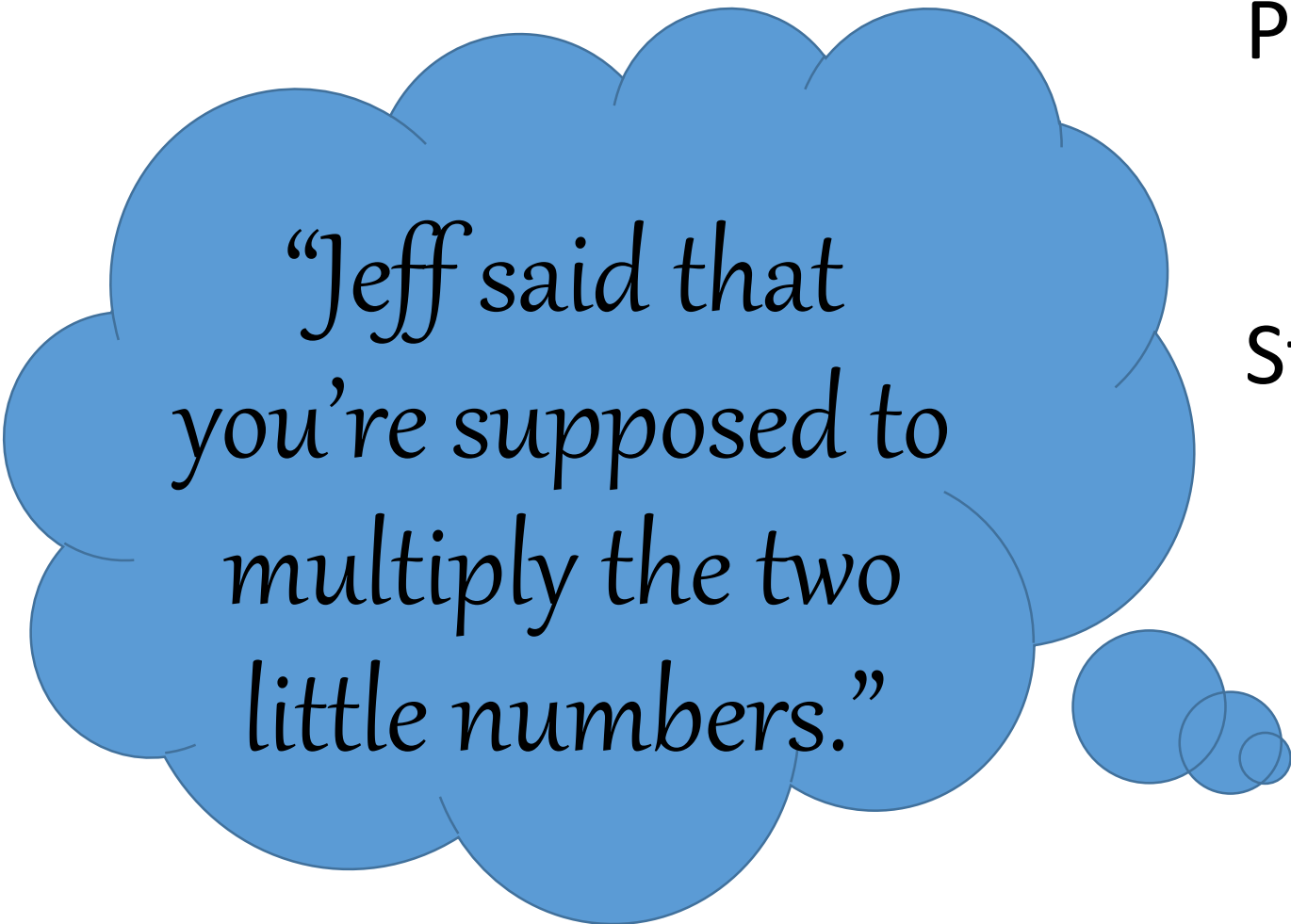
I scored all
three goals!

A blue-outlined speech bubble with a tail pointing towards the bottom-left.

I got the
right answer!

Overview

- Student performance vs Student thinking
- Background of our experiences: Math Talks with High School content
- Number Talks
- Adapting Number Talk principles to High School content
- Ideas for High School “Math Talks”
- Places to find Math Talk ideas



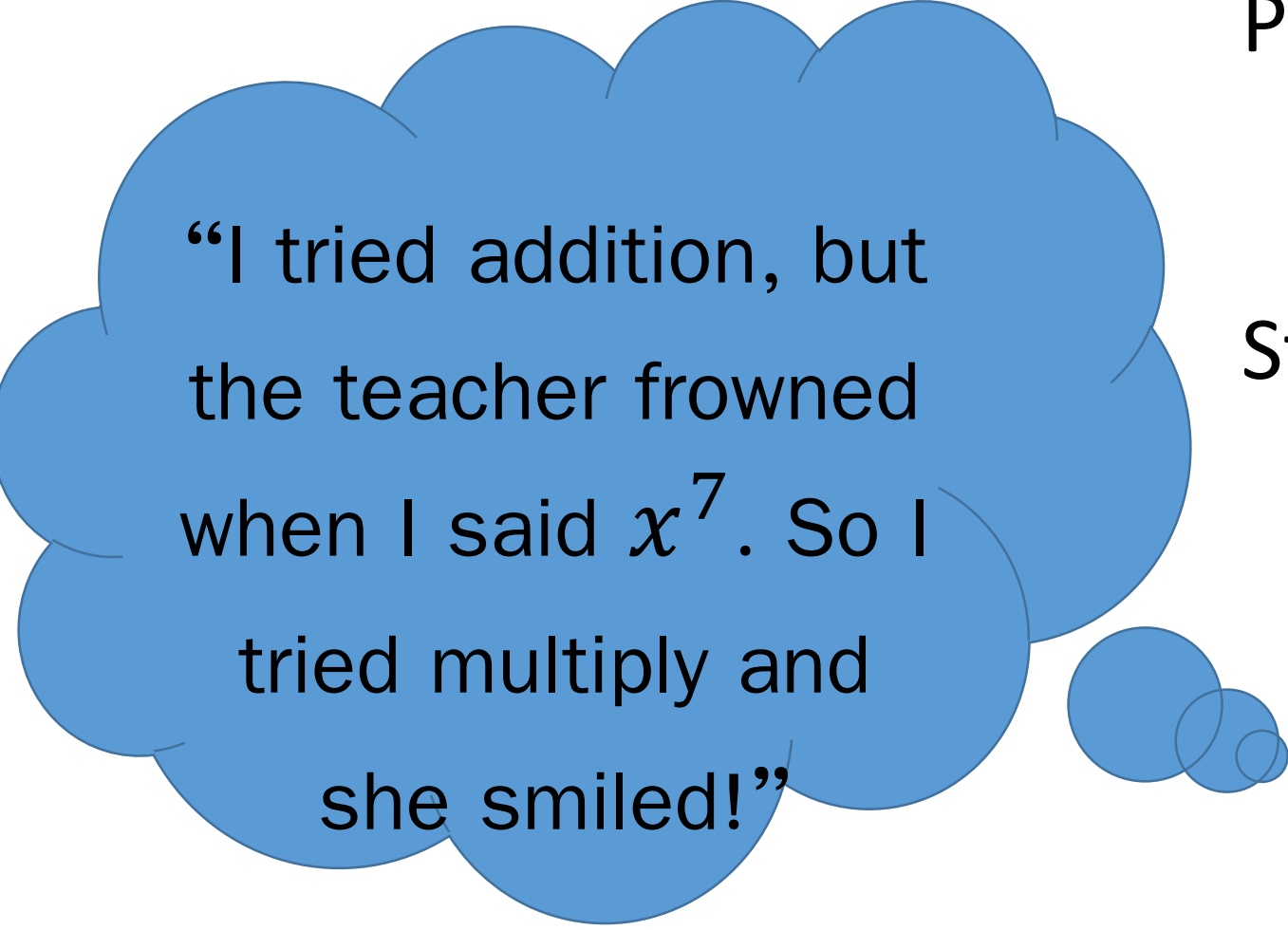
“Jeff said that
you’re supposed to
multiply the two
little numbers.”

Problem:

$$(x^3)^4$$

Student Work:

$$x^{3 \times 4} = x^{12}$$



“I tried addition, but the teacher frowned when I said x^7 . So I tried multiply and she smiled!”

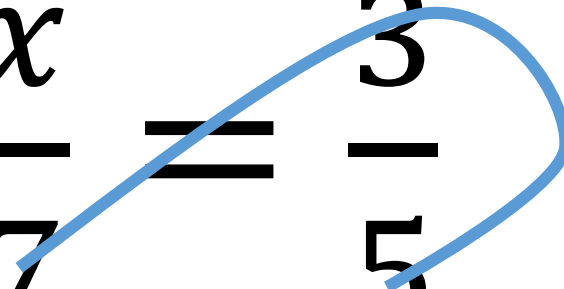
Problem:

$$(x^3)^4$$

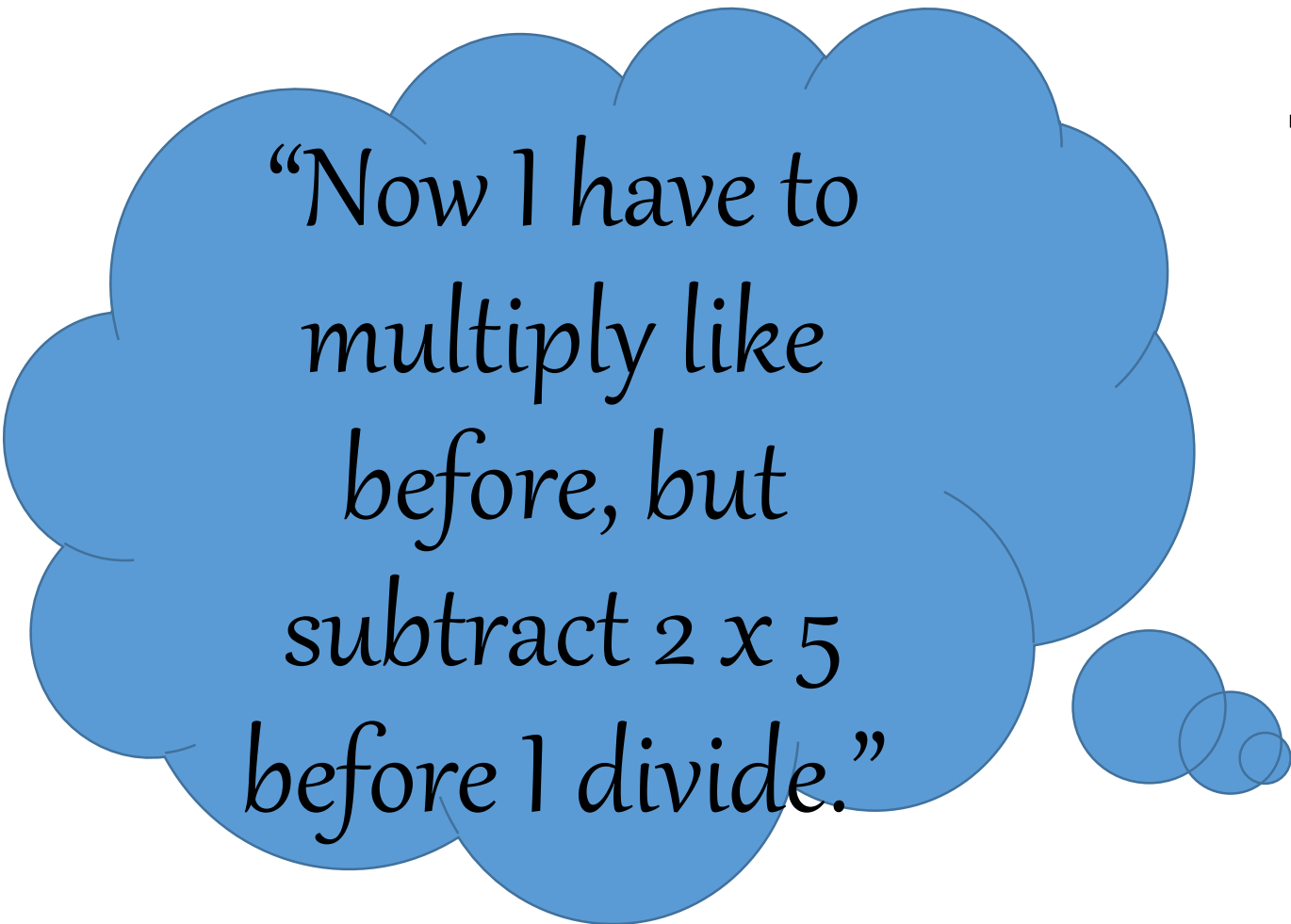
Student Work:

$$x^{3 \times 4} = x^{12}$$

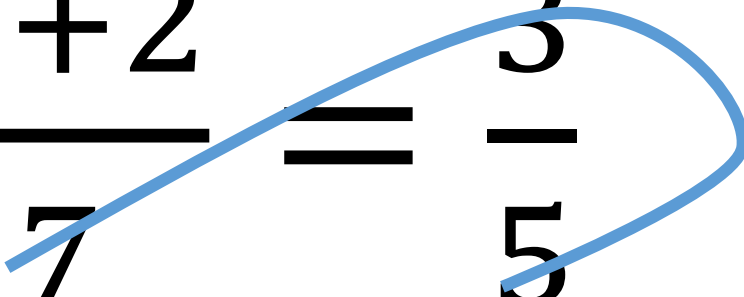
"I think this is where you multiply on the diagonal, then divide by the other number."

$$\frac{x}{7} = \frac{3}{5}$$


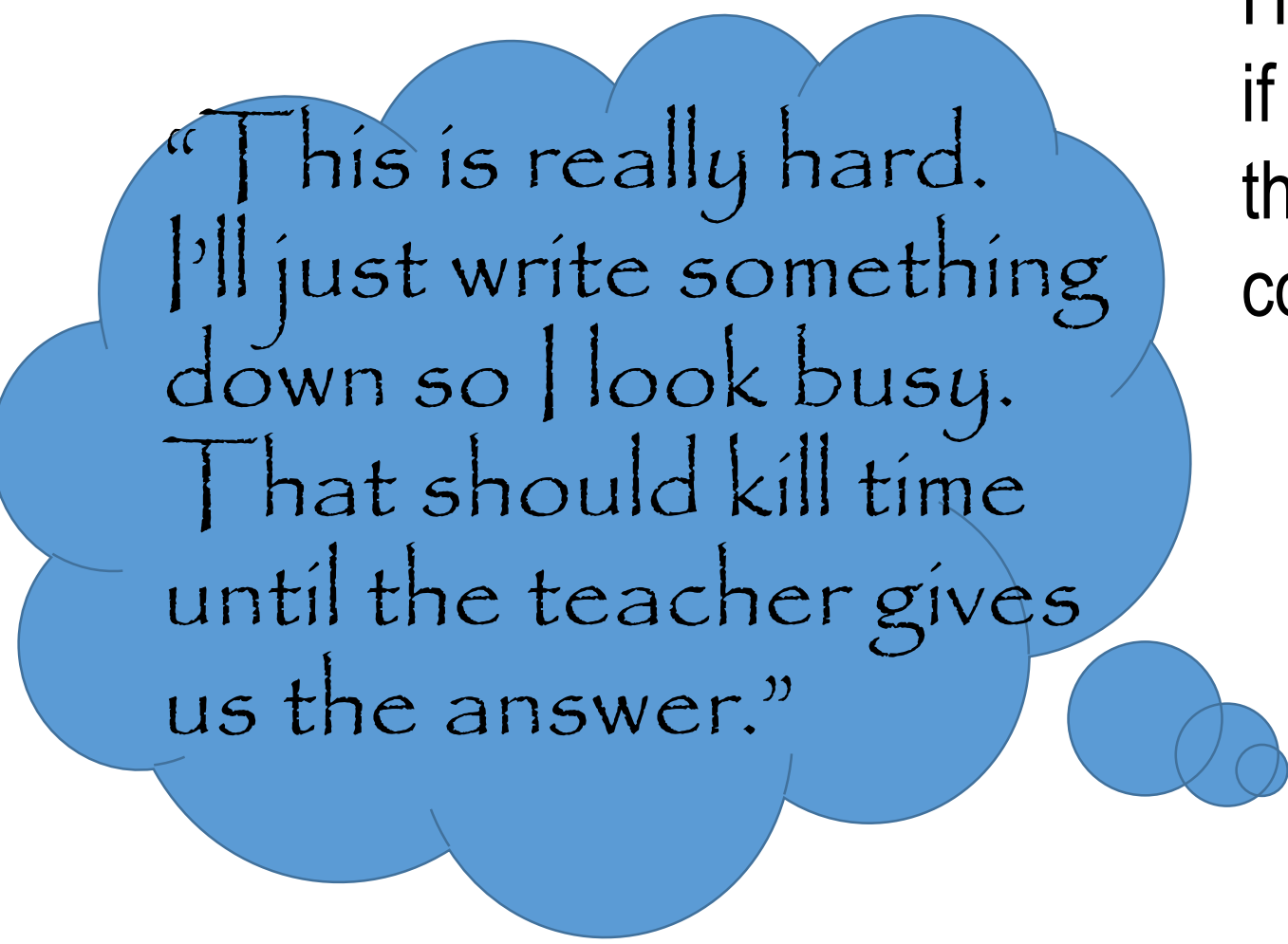
$$21 \div 5 = 4.2$$



“Now I have to
multiply like
before, but
subtract 2×5
before I divide.”

$$\frac{x+2}{7} = \frac{3}{5}$$


$$21 - 10 = 11$$
$$11 \div 5 = 2.2$$

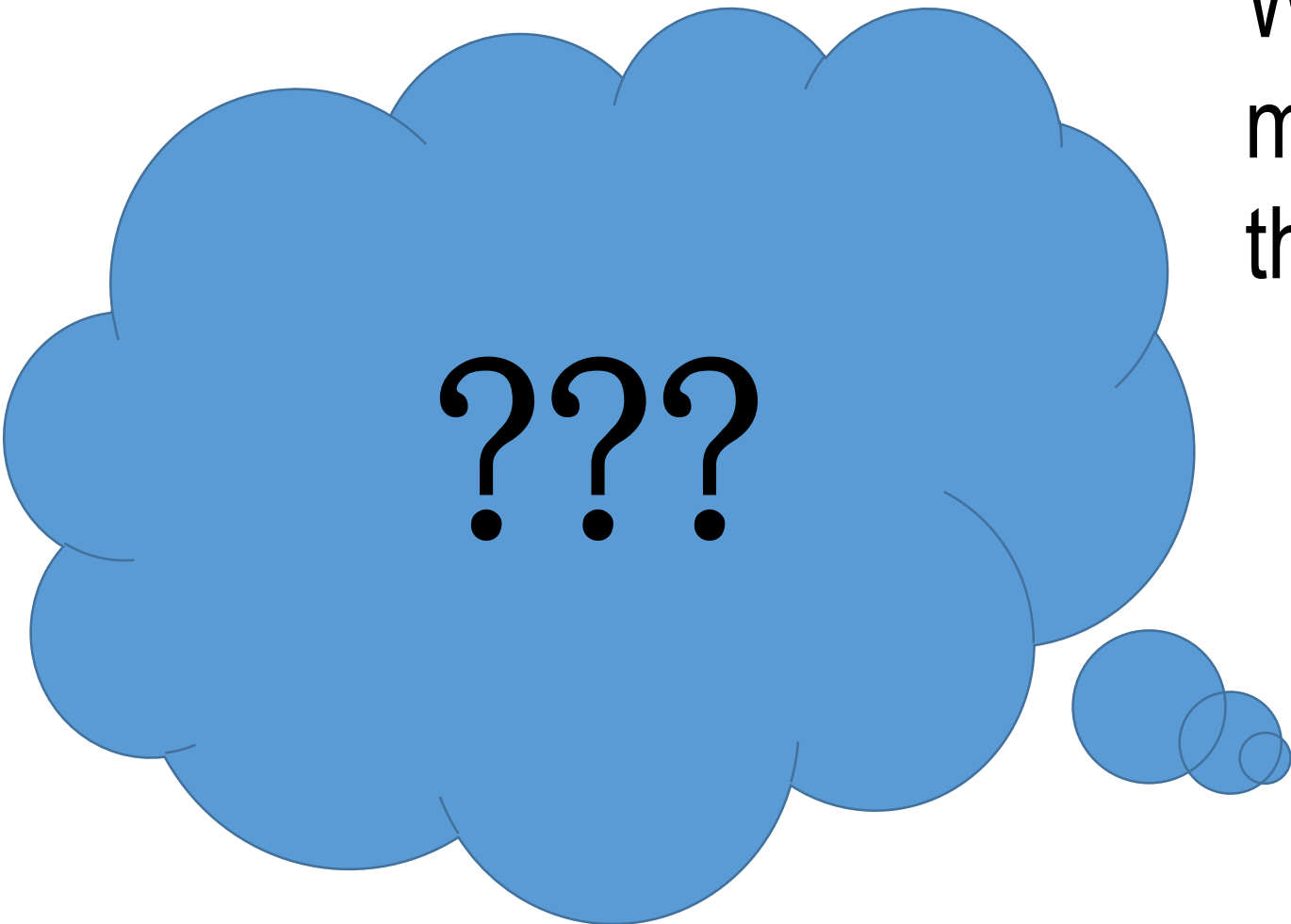


“This is really hard.
I’ll just write something
down so I look busy.
That should kill time
until the teacher gives
us the answer.”

How much money will Jeff have
if he invests \$1250 in account
that pays 4.5% interest
compounded monthly?

$$1250 \times 0.045 = \$56.25$$

$$56.25 +$$



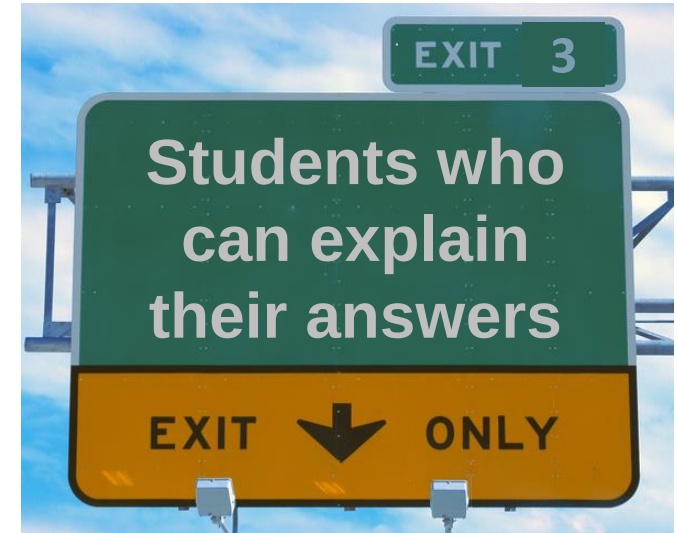
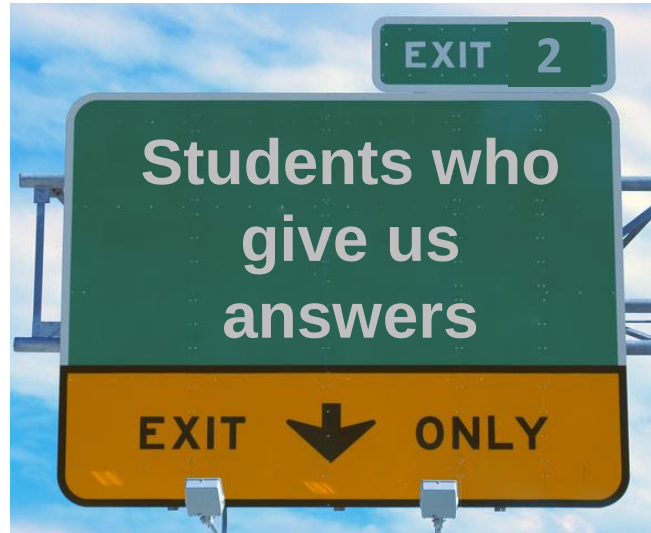
???

What other types of non-mathematical student thinking have you seen?

I Agree

CUS XR
Olwds RIT

What do we want? Where are we going?



Regional Math
Specialist



Rayna
Chatfield



Common questions/concerns:

Common Concerns

- I can't get my high school students to talk about the math.
- I'd like a more active learning environment, but when I try something different, the students **just sit there**.
- If I do get a student to say something, no one else seems to be listening.

Rayna's Main Goals

- Foster student sharing and discussing.
- Encourage students to learn to listen to others' thinking.
- Continue to address **high school content**.

Number Talks

Excellent for number sense,
not directly applicable to High School content

$$32.25 \times 8$$

Number Talk structure

- Mental math
- Student shares aloud, teacher records
- Teacher represents student thinking without evaluation
- Teacher represents student thinking without interjecting

Math Talk 5/4/16

Multiply 32.25×8

Erin

$$\begin{array}{r} 25 \times 8 = 200 \\ (25 \times 4 = 100) \downarrow \\ 200 \\ + 32 \\ \hline 232 \\ 34 \times 8 = 480 \end{array}$$

José

$$\begin{array}{l} 32.25 \times 2 = 64.50 \\ 64.50 \times 2 = 129 \\ 129 \times 2 = 258 \\ 2^3 = 8 \end{array}$$

$$\begin{array}{r} 25 \times 8 = 200 \\ \downarrow \\ 200 \\ 32 \times 8 = 256 \\ \hline 258 \end{array}$$

Math Talk 5/4/16

Multiply 32.25×8

Grant

$$32.25 \rightarrow 32 \quad 0.25$$

$$8 \times 0.25 \rightarrow 2$$

(quarters)

$$\begin{array}{ccccccc} 32 & + & 32 & + & 32 & + & 32 & + & 32 & + & 32 & + & 32 & + & 32 \\ \checkmark & & \checkmark & & \checkmark & & \checkmark & & \checkmark & & \checkmark & & \checkmark & & \checkmark \\ 64 & + & 64 & + & 64 & + & 64 & & & & & & & & \\ 258 & & & & & & & & & & & & & & \end{array}$$

Mia/Maryn

$$\begin{array}{r} 32.25 \\ \times 8 \\ \hline 258.00 \end{array}$$

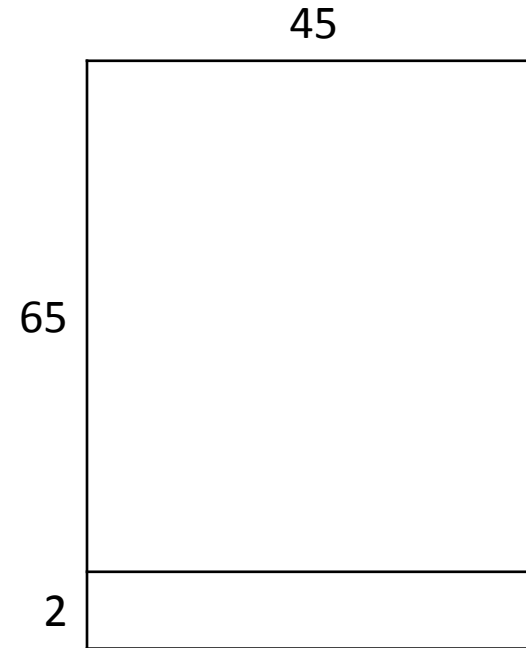
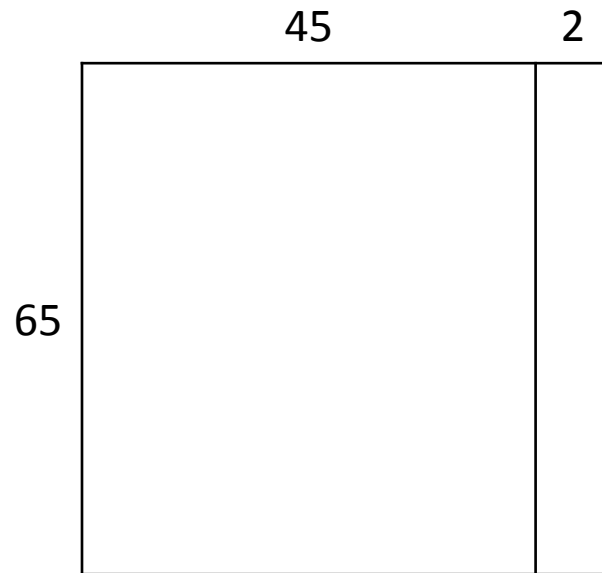
Isabella

$$\begin{array}{r} 32.25 \times 4 \\ (32 \times 4) \quad (0.25 \times 4) \\ \times 128 \\ \hline 256 \end{array} + \begin{array}{r} 1 \\ \times 2 \\ \hline 2 \end{array} = 258$$

$32 + \frac{1}{4}$ Stephen

$$\begin{array}{l} \sqrt[5]{32} = 2 \\ (2^5 + \frac{1}{4}) 2^3 \\ 2^8 + \frac{8}{4} \\ 256 + 2 = 258 \end{array}$$

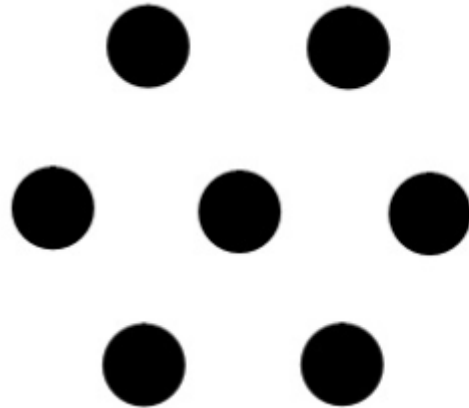
Which is greater, 65×47 or 67×45 ?



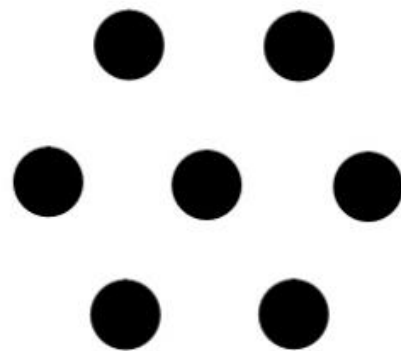
Dot Talks

A way to get the ball rolling...

Without counting, how many dots are in the picture?
Think of as many ways as you can to determine the
number of dots.



Without counting, how many dots are in the picture? Think of as many ways as you can to determine the number of dots.

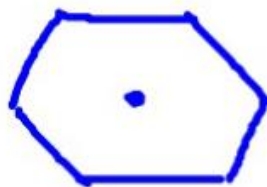


Jose



Erin
< . >

Kahner



Stetson



Carson



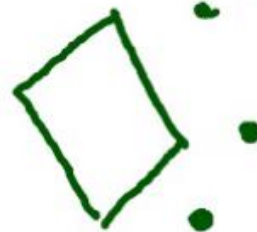
Kurtland



Carson



Gavin



Brenden



Which One Doesn't Belong?

A way to keep the ball rolling...

(Who knew Sesame Street was on to something so mathematical?)

Math Talk #2

9/16/16

Which of the following doesn't belong?

$x - 1$	$3(x + 2)$
x^2	$4 - 2y$

Math Talk #2

9/16/16

Which of the following doesn't belong?

Glen
 $3(x+2)$
has parentheses

Kai
 $4-2y$
variable is y not x

Alyssa
 x^2
variable has an
exponent

$x-1$	$3(x+2)$
x^2	$4-2y$

Elaina
 $x-1$
variable minus number

Kolin
 $3(x+2)$
not simplified

Glen
 $3(x+2)$
uses addition

Ms. Krevitzky
 x^2
doesn't have a constant

Math Talk 4/28/16

Which of these numbers doesn't belong in the group?

16	9
29	25

Math Talk 4/28/16

Which of these numbers doesn't belong in the group?

16	9
29	25

Jackson
 $29 - 16 + 9 = 25$

Mia
29 - not a perfect square

Stephen
9 - isn't a factor
11600
and
only 1 digit

Conrad
16 - even

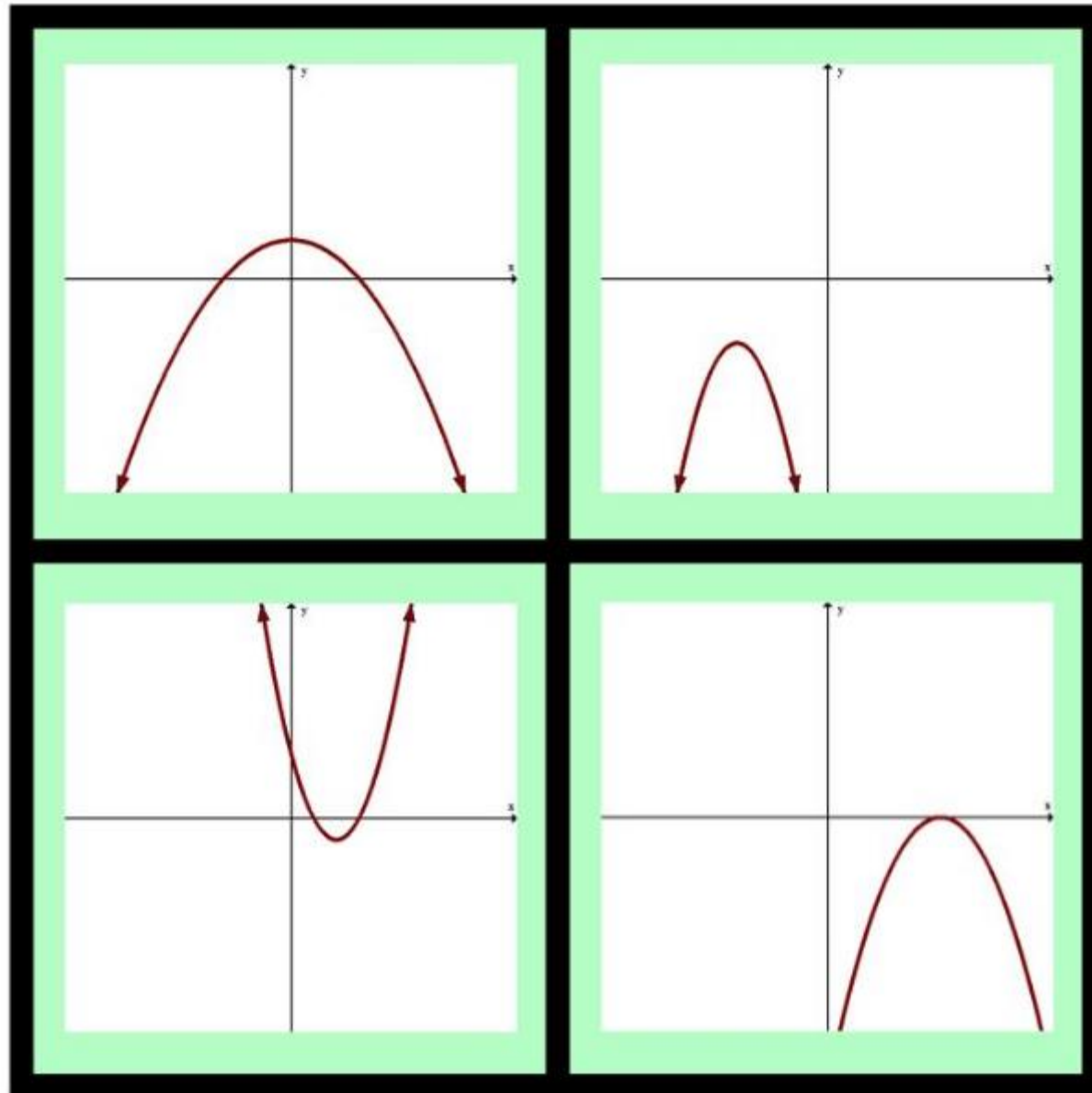
Isabella
25 - 2nd digit
isn't divisible
by 3

Grant
25 - multiply ~~or~~ 5

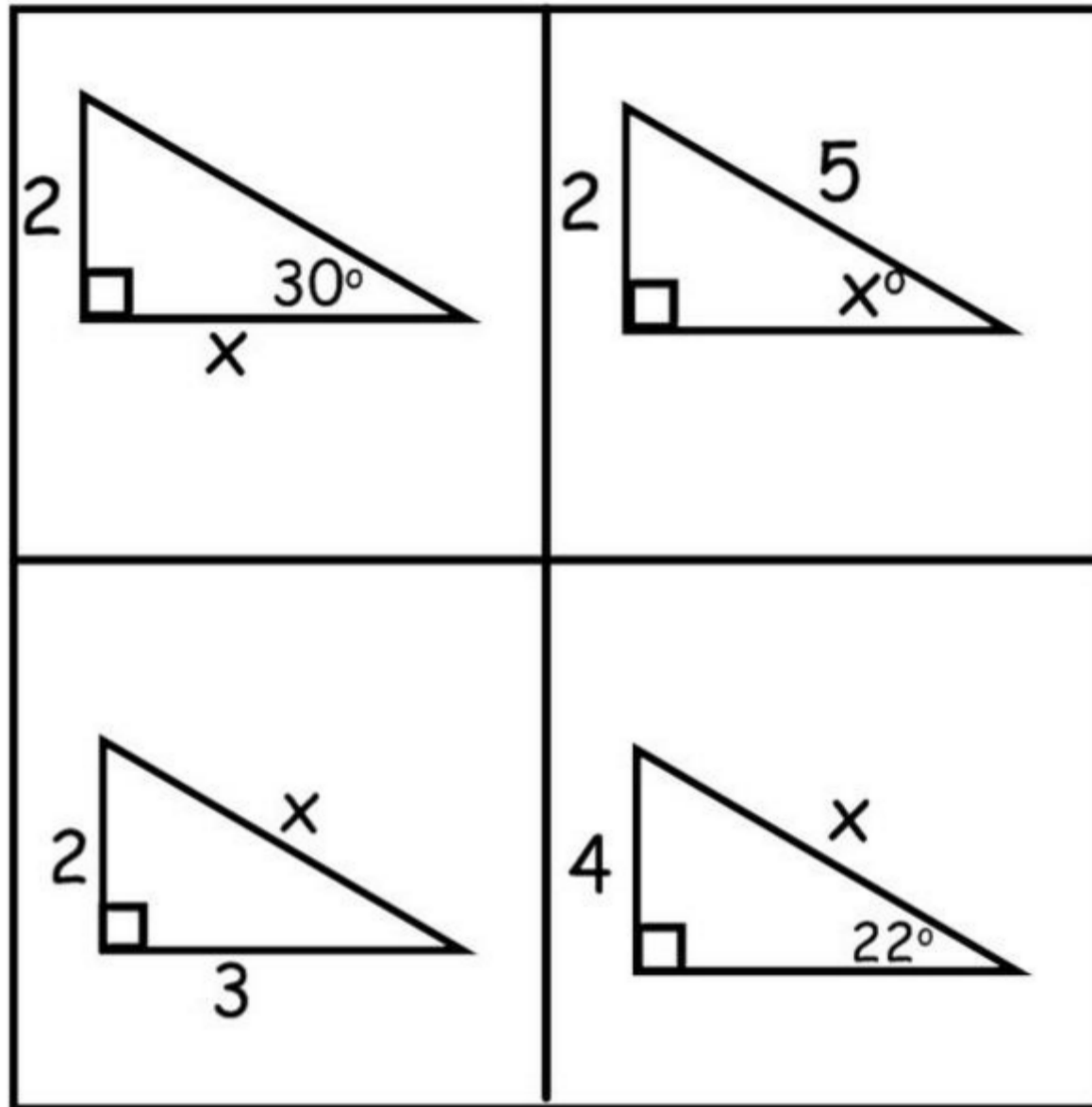
Stephen
25 - doesn't have
a loop (0)
circle
hole

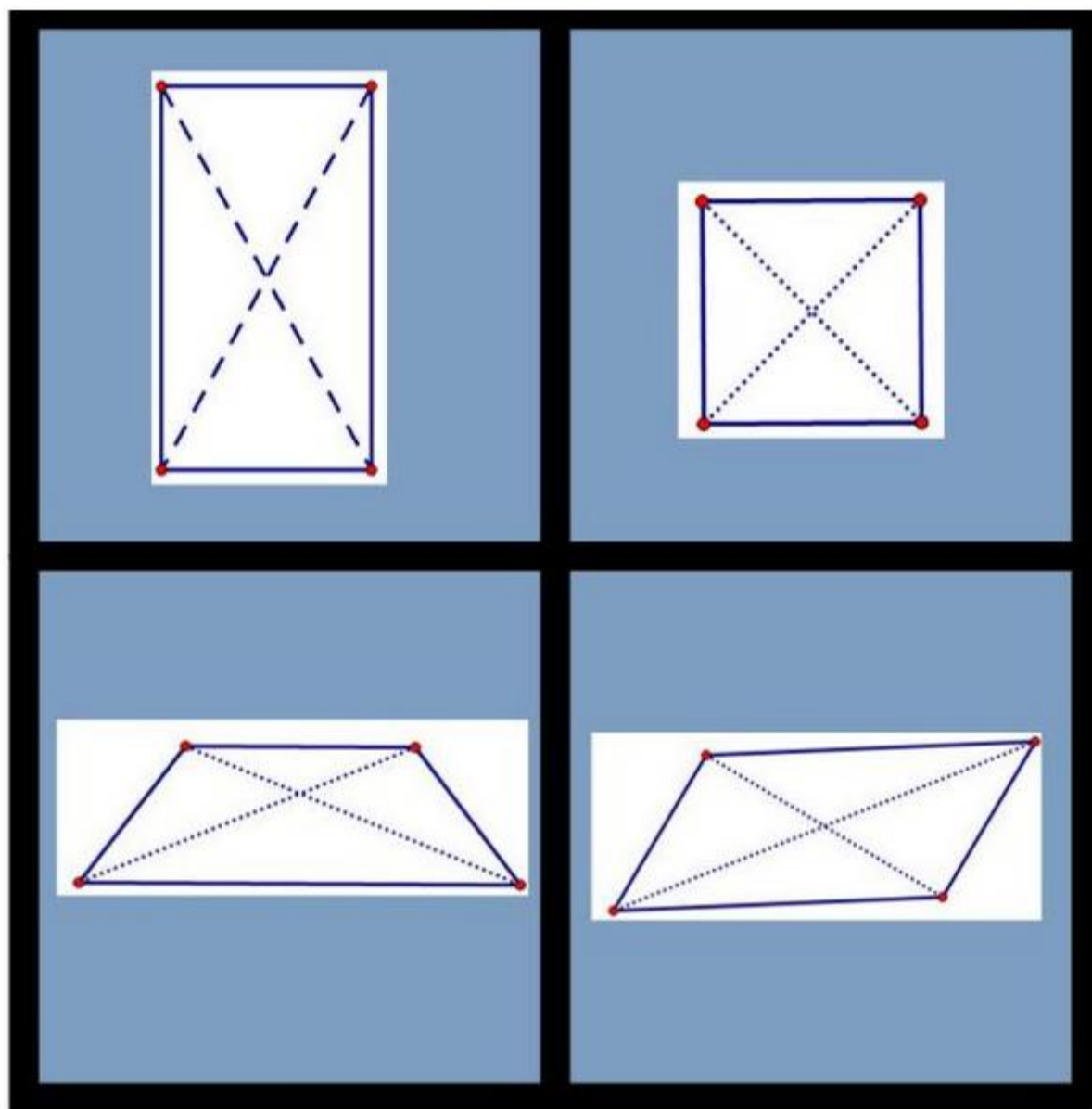
wodb.ca

Get ready to vote...



$3 - 4i$	$\sqrt{-81}$
$-2 + i$	-15





- “Math talks are an excellent way to allow time for students to vocalize their thoughts and practice giving reason and hearing reasons.”
- “I found that many students who are generally not engaged and participative in a traditional math lesson were very willing to share during the math talk time if facilitated well.”
- “I usually had to set a time limit and cut off responses because so many people were willing to share.”

Guiding Principles for Number Talks

1. All students have mathematical ideas worth listening to, and our job as teachers is to help students learn to develop and express these ideas clearly.

2. Through our questions, we seek to understand students' thinking.

3. We encourage students to explain their thinking conceptually rather than procedurally.

4. Mistakes provide opportunities to look at ideas that might not otherwise be considered.

5. While efficiency is a goal, we recognize that whether or not a strategy is efficient lies in the thinking and understanding of each individual learner.

6. We seek to create a learning environment where all students feel safe sharing their mathematical ideas.

7. One of our most important goals is to help students develop social and mathematical agency.

8. Mathematical understandings develop over time.

9. Confusion and struggle are natural, necessary, and even desirable parts of learning mathematics.

10. We value and encourage a diversity of ideas.

Making Number Talks Matter

Guiding Principles for Number Talks

- **All** students have mathematical **ideas** worth listening to, and our job as teachers is to help students learn to develop and express these ideas clearly.
- Through our questions, we seek to understand **students'** thinking.
- We encourage students to explain their thinking **conceptually** rather than procedurally.
- While efficiency is a goal, we recognize that whether or not a strategy is **efficient lies in the thinking** and understanding of each individual.
- We value and encourage a **diversity of ideas**.

Making Number Talks Matter

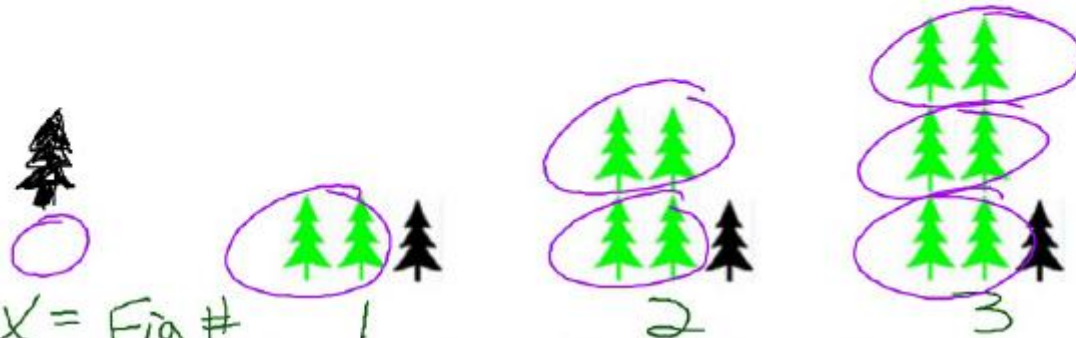
Pattern Talks

Number Talk principles, High School content!



- What do you notice about the pattern?
- How many trees would be in the 73rd picture?
- What would the next two pictures look like?
- Write an equation for the number of trees in the n th picture.

Math Talk 4/26/16



$x = \text{Fig \#}$
What do you know about the pattern?

Brianna
adds 2 green
to each figure
on the top

Ime
Linear
adding
up 2 each
time

Brenden
 $y = 2x + 1$

$$y = (x+1)^2$$

$$y = x^2 + 2x + 1$$

Recursive
 $\text{Next} = \text{Current} + 2 \text{ trees}$

Math Talk 4/26/16



7

x	y	1st diff	2nd diff
1	1	3	2
2	4	5	2
3	9	7	2
4	16		

Quadratic

Fig
What do you know about the pattern?

Joshua
Black + green trees
adds 2 green after
each figure

Isabella
linear
adding 2
Arithmetic

Math Talk 5/10/16

What does the 5th figure look like? How many small cubes are in it?



Figure 1



Figure 2

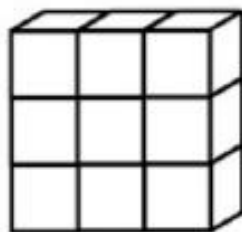


Figure 3

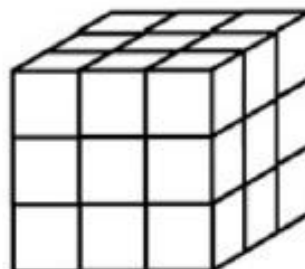


Figure 4

Math Talk 5/10/16

What does the 5th figure look like? How many small cubes are in it?

Exponential
 $y = 3^{x-1}$
 $y = \frac{1}{3} \cdot 3^x$

Dante
 Multiply by 3
 every time

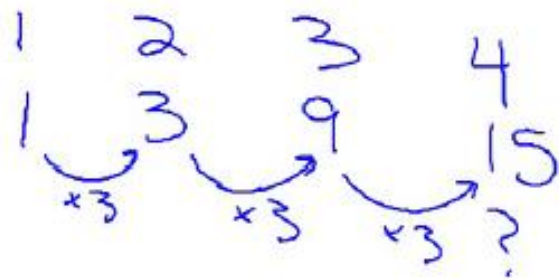


Figure 1



Figure 2

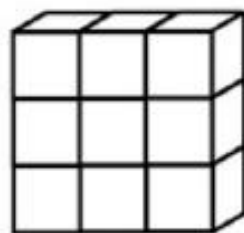


Figure 3

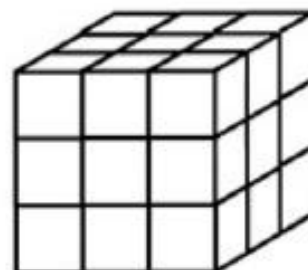
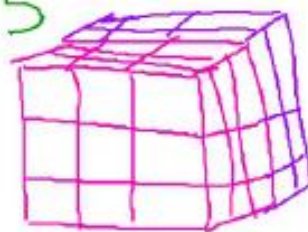


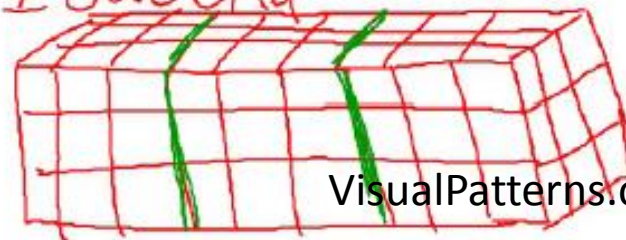
Figure 4

Joshua
 $3 \cdot 5 = 15$
 Fig 5



Aubrey
 Fig 5 has
 81 cubes

Isabella



VisualPatterns.org

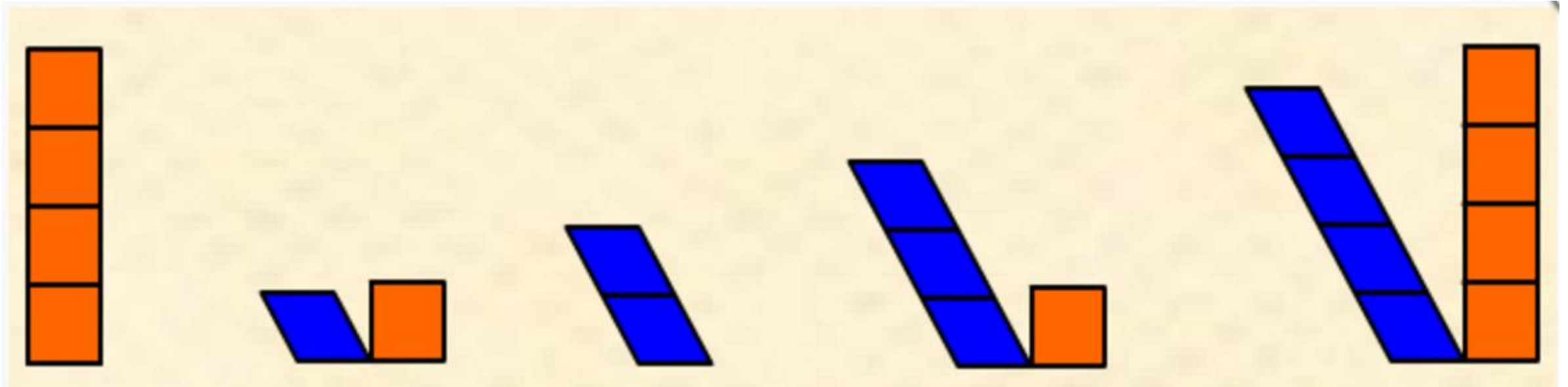
- “I found it helpful to ask for the next figure number or two as well as a figure number in the hundreds to try and get the students to generalize a rule/equation. I was surprised at how beneficial this was for my upper-classmen who are just more cognitively ready (and have had more ‘real-life’ experience to relate to) and were finally able to connect that whole $y=mx+b$ thing they vaguely remember to something more tangible.”



$$2(n - 1) + 1$$

$$n + (n - 1)$$

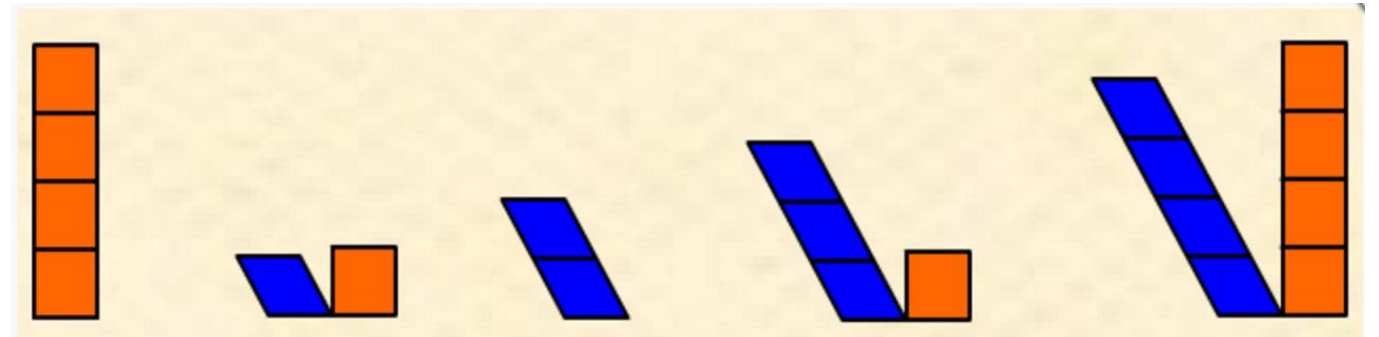
$$2n - 1$$



$$(n - 3)^2 + (n - 1)$$

$$n^2 + (n + 2)$$

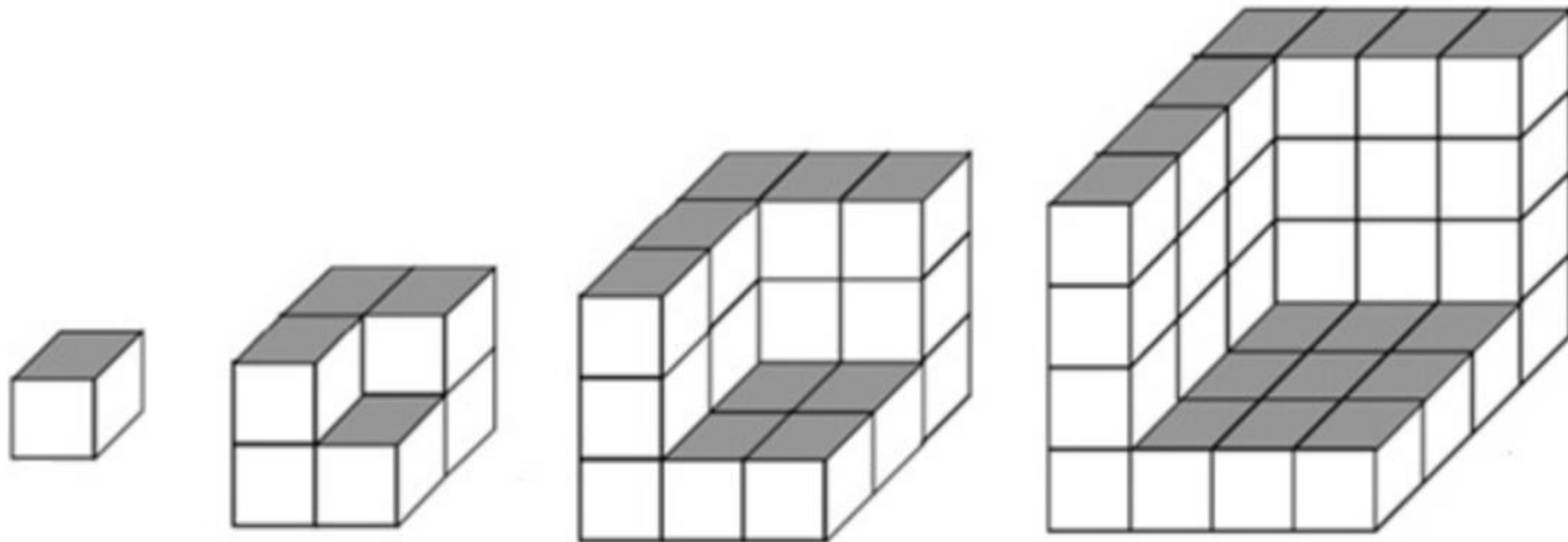
x	y
1	4
2	2
3	2
4	4
5	8



$$(n - 2)^2 + (n - 1)$$

$$n^2 + (n + 2)$$

- “In the upper division classes I tried to use the math talks that could apply in some way to what we were talking about that week.”
- “Students needs to learn how to communicate better mathematically and math talks are a good tool for that but they are more effective when they are centered around the curriculum (or maybe I am just more likely to do them when they are centered around the curriculum).”



- “I primarily used ‘pattern talks’ because it helped my students recognize the growth of linear, quadratic, and exponential functions (a standard I am supposed to directly address in my Algebra course).”
- “Students often saw the problems in a way that I had not even considered and I learned a few things from these pattern talks as well.”

Other Math Talk Ideas

The ball is rolling. Where can we take this?

Number Talk 4/25/16

Consider the equation $a \cdot b \cdot c \cdot d = 0$. What values of a , b , c , and d would make the equation true?

Number Talk 4/25/16

Consider the equation $a \cdot b \cdot c \cdot d = 0$. What values of a , b , c , and d would make the equation true?

Carson

$$0 \cdot 20 \cdot 20 \cdot 20 = 0$$

$$\begin{aligned} b &= 20 \\ c &= 20 \\ d &= 20 \end{aligned}$$

Brenden

$$1 \cdot 2 \cdot 3 \cdot 0 = 0$$

Shawn

$$1 \text{ of them} = 0$$

Math Talk 4/27/16

Find values for the blank that make the expression factorable.

$$x^2 - 8x + \underline{\hspace{2cm}}$$

Math Talk 4/27/16

Find values for the blank that make the expression factorable.

Brenden
-20
 $x^2 - 8x - 20$
 $(x-10)(x+2)$

Completing the square $x - 4$

x	x^2	$-4x$
-4	$-4x$	16

$$x^2 - 8x + \underline{\hspace{2cm}}$$

Carson
 $x^2 - 8x + 16$
 $(x-4)(x-4)$

$x^2 - 8x + 15$
 $(x-5)(x-3)$

$x^2 - 8x$
 $x(x-8)$
 $(x+0)(x-8)$

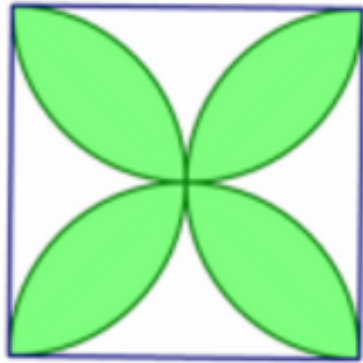
$0 + \underline{-8} = -8$

$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

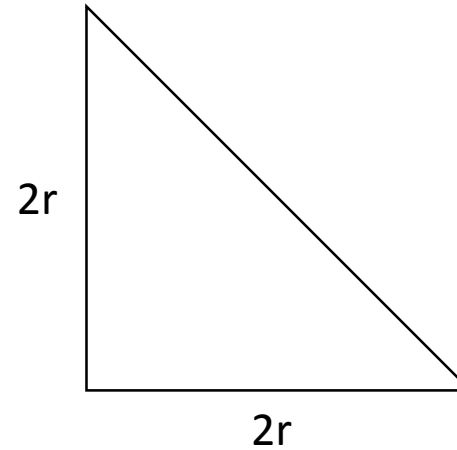
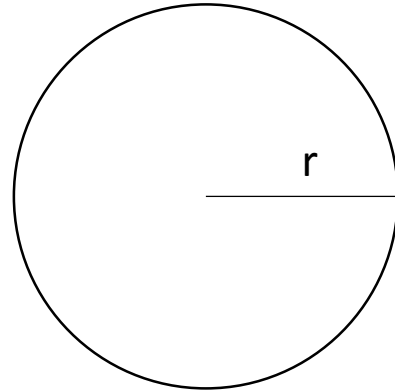
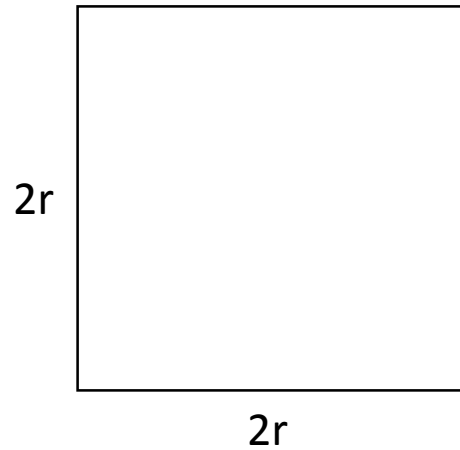
$(x-9)(x+1)$
 $x^2 - 8x - 9$

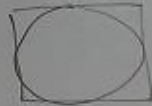
26. HOW WOULD YOU FIND THE AREA OF THE SHADED?

This 4-leaf clover was created by drawing 4 semicircles of radius equal to $\frac{1}{2}$ of side of square.

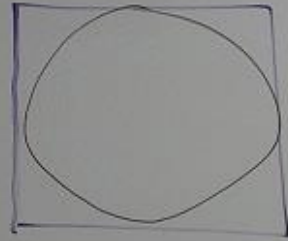


If the lengths double every second, which area grows the fastest?





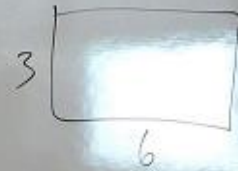
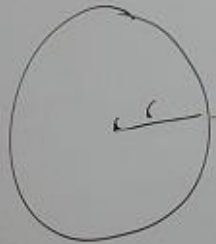
$2r$



Area will
get 4x
bigger.
 $\frac{dA}{dt} = 8r \frac{dr}{dt}$

$$A = 4r^2$$

$$A_1 = \pi r^2$$
$$A_2 = \pi (2r)^2$$
$$A_2 = 4\pi r^2$$



$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \left(\frac{dr}{dt} \right)$$

	1 sec		
	Before	After	change
Sq.	$4r^2$	$16r^2$	$12r^2 \text{ ?}^2/\text{sec}$
Circle	πr^2	$4\pi r^2$	$3\pi r^2 \text{ ?}^2/\text{sec}$
triangle	$2r^2$	$8r^2$	$6r^2 \text{ units}^2/\text{sec}$
rect.	18	72	54 $\text{units}^2/\text{sec}$

$A_1 = lw$
 $A_2 = (2l)(2w)$
 $A_2 = 4lw$

Math Talk Resources

Where do I find this stuff?

Math Talk Resources

- [VisualPatterns.org](https://www.visualpatterns.org)
- WODB.ca
 - [Shapes](#)
 - [Numbers](#)
 - [Graphs](#)
 - [Incomplete Sets](#)
- MathTalks.net
 - [Number Talks](#)
 - [Pattern Talks](#)
- *Making Number Talks Matter*, Cathy Humphreys and Ruth Parker

Common questions/concerns:

Common Concerns

- ☐ I can't get my high school students to talk about the math.
- ☐ I'd like a more active learning environment, but when I try something different, the students **just sit there**.
- ☐ If I do get a student to say something, no one else seems to be listening.

Rayna's Main Goals

- ☐ Foster student sharing and discussing.
- ☐ Encourage students to learn to listen to others' thinking.
- ☐ Continue to address **high school content**.

Challenges

- “I think that having math talks just for the sake of math talks works in some classes but not all. My kids seemed to get tired of them. I think it would be a good idea to have math talks for a few weeks to get used to them and get used to sharing and listening and then after that build them in to your lessons.”
- “Making sure you let students have a say – that their ideas are important – wait time – all that takes practice.”

Successes

- “These visual patterns are very easy to find online so they were perfect for a preparation procrastinator like myself.”
- “Math talks are also good for giving students some insight into proofs because when you do math talks you are looking for reasons behind doing what you are doing.”
- “My biggest challenge eventually became trying to get every student to be silent and listen to a full student response before chiming in and/or ‘helping’ that student explain their thinking.”

Common questions/concerns:

Common Concerns

- ☒ I can't get my high school students to talk about the math.
- ☒ I'd like a more active learning environment, but when I try something different, the students **just sit there**.
- ☐ If I do get a student to say something, no one else seems to be listening.

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- ☐ Continue to address **high school content**.

Please, Experiment & Contribute!

Jason Libberton

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