Student Errors, Mistakes, and Wrong Answers, - Opportunity Rethink!

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NCTM San Antonio #367

How I Let Errors Live in My Classroom

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It's okay to rethink in my class!

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NCTM San Antonio #367

It's ~~okay~~ good to rethink in my class!

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NCTM San Antonio #367

What is a glide ratio?

A glide ratio is the ratio of the horizontal movement to the vertical drop of a glider – almost like slope but glide ratios have no negative to indicate the drop.

http://en.wikipedia.org/wiki/Otto_Lilienthal

Gliding

http://www.famousinventors.org/otto-lilienthal

Otto Lilienthal, the Glider King, German pioneer of aviation in mid-flight, circa 1895

http://en.wikipedia.org/wiki/Otto_Lilienthal

Glide Ratios

Two different gliders start from the same height.

Which glider goes farther: one with a glide ratio of 185:25 or one with a glide ratio of 155:20?

Explain your reasoning.

Recall: A glide ratio is the ratio of the distance traveled to altitude lost.

Please complete this task the way you think one of your students might approach it.

If you think of one way, try a different way a student might approach the task or share your solution with your friends next to you and compare strategies.

https://30daysofautism.wordpress.com/2012/03/22/the-wing-tales-from-an-aspie-childhood/

How did you decide?

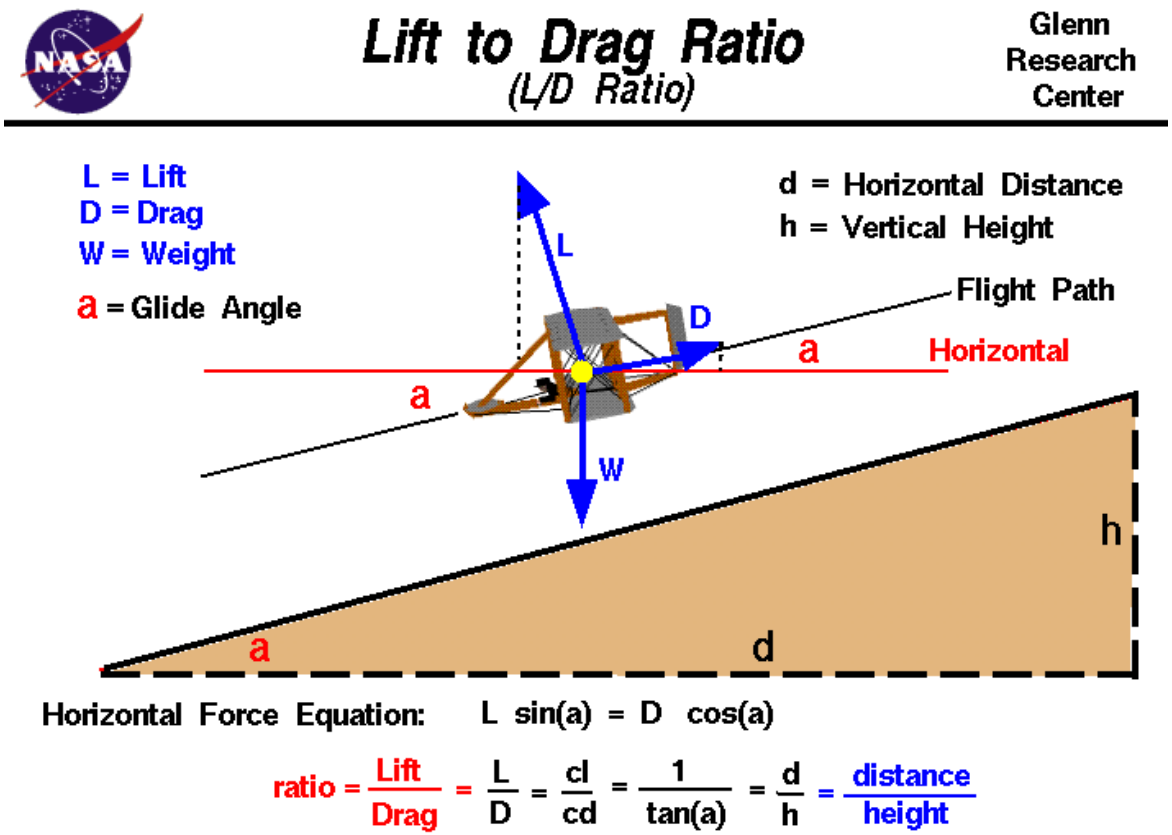
Which glider goes farther: one with a glide ratio of 185:25 or one with a glide ratio of 155:20?

Explain your reasoning.

Recall: A glide ratio is the ratio of the distance traveled to altitude lost.

?

Glide Ratio vs Lift to Drag Ratio



https://www.grc.nasa.gov/www/K-12/airplane/ldrat.html

1848-1896 (August 10th)

https://vimeo.com/155960641

Outrageous Acts of Science

Kings of Carnage episode – Gary Connery

Franz Reichelt 1879-1912

https://en.wikipedia.org/wiki/Franz_Reichelt

http://www.vintag.es/2015/08/death-jump-story-of-franz-reichelt.html

https://britishpathe.wordpress.com/2012/02/04/franz-reichelt-day-february-4th-100th-anniversary-of-eiffel-tower-death-jump/

In gliding flight, the Glide Ratio ratio is equal to the L/D (Lift over Drag) (when flown at constant speed).

Flight article	Scenario	Glide ratio
Modern Sailplane	Gliding	40-60 (depending on span)
Hang glider	Gliding	15
Wingsuit	Gliding	2.5 (Gary C. 3.3)
Northern flying squirrel	Gliding	1.98

http://www.animalspot.net/northern-flying-squirrel.html

http://mkalty.org/wings-of-an-albatross/

Common Lift/Drag Ratios

Flight article	Scenario	Lift/Drag Ratio
Albatross	Flight	20
Boeing 747	Cruise	17
Helicopter	100 kts speed	4.5
House Sparrow	Flight	4

Standards for Mathematical Practice

- MP 3 – Construct viable arguments and critique the reasoning of others:
  - build a **logical progression** of statements
  - **justify** their conclusions & **communicate** them to others
  - make plausible **arguments** that take into account the **context**
  - decide whether reasoning of others **makes sense**
  - ask useful **questions to clarify or improve** understanding
- MP 6 – Attend to precision:
  - **communicate precisely with precision appropriate for the problem context**

TIMSS video – Hong Kong Identity Lesson

http://www.timssvideo.com/65

From beginning to 2 minutes 18 seconds

What happens in your class when a student makes a public error?

(an error that is clearly recognized as an error by other students)

A 1½ minute free write – what happens in your class?

This is a personal write so you can be honest with yourself.

Think about your classes.

What actually happens?

What do you say?

What do your students do?

Is there tension in the room? If so, how do you deal with those feelings?

What should/could happen in your class?

After a public error, what needs to happen next?

- Support – classroom culture accepted way of working
- Chance to fix the error (rethinking)
- Join the club (we all make errors, it is a natural part of thinking hard)
- Encouragement from teachers and students (we support each other in our thinking)
- Fix the error – correct math (logical and productive ways of thinking should be final understandings)
- Class (including the student) needs to understand the math

Absolute Correct Initial Thinking

~~Absolute Correct Initial Thinking~~

Errors are OPPORTUNITIES

- OPPORTUNITY to Rethink
- OPPORTUNITY to Discuss
- OPPRTUNITY to Ask:

What MAKES SENSE?

OPPORTUNITY to Rethink

We are all learning!

(Reduce the stigma attached to being wrong)

Mistakes give us the right to think again

Ilana Horn's Blog: Teaching Math Culture

July 14, 2014

- How hard it was for most students to answer the question: What do you think and why?
- Why students don't share.

Classroom norms

Status and equity issues

http://mathforum.org/pcmi/hstp/sum2014/morning

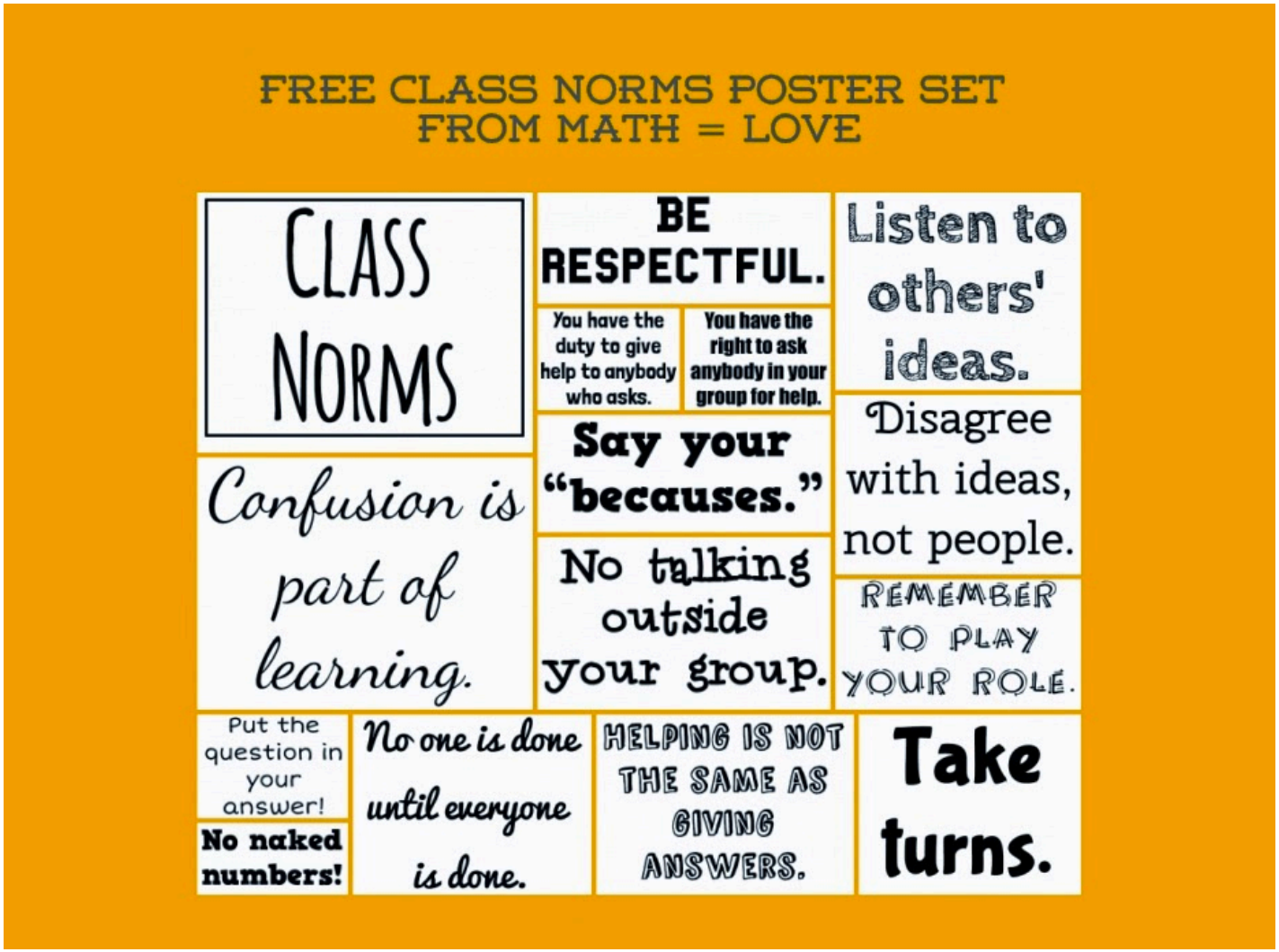
Ilana Horn's Blog:
https://teachingmathculture.wordpress.com

Classroom Norms are agreed upon ways of behaving

- Take turns
- Listen to others' ideas
- Disagree with ideas, not people
- Be respectful
- Helping is not the same as giving answers
- Confusion is part of learning
- Say your "becauses"

Classroom Norms help encourage rethinking

Sarah Hagan's blog – Classroom Norms



http://mathequalslove.blogspot.com/2014/07/class-group-work-norms-poster-set-free.html?m=1

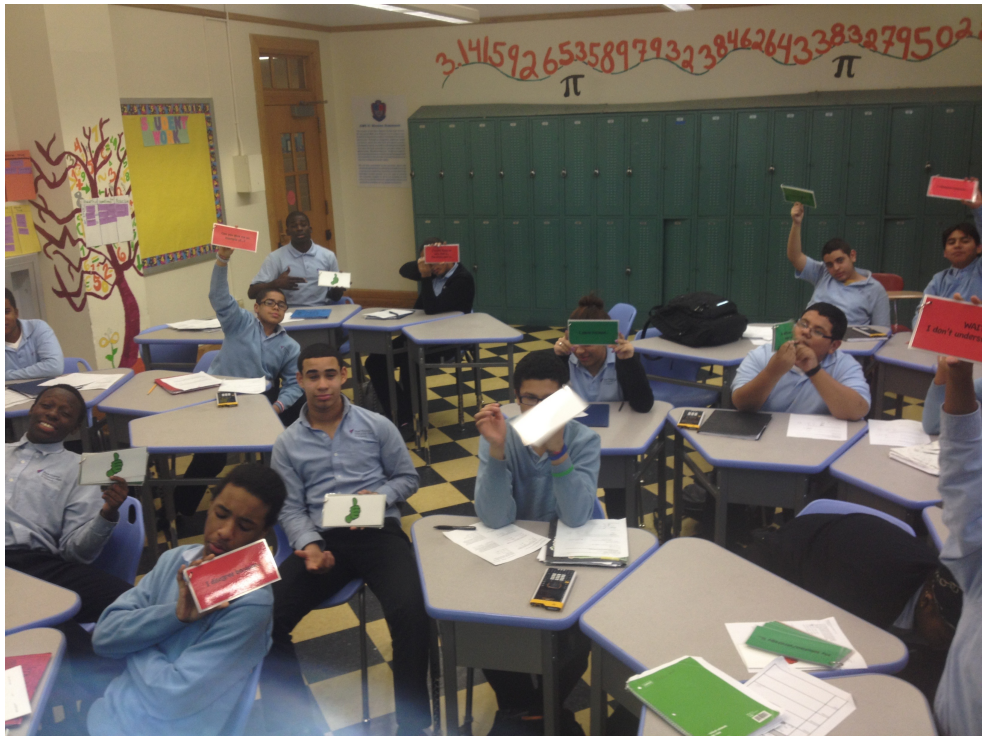
Accountable Talk – Jennifer Parker

Green Cards

I agree because …

I also notice …

I would like to add that …



Red Cards

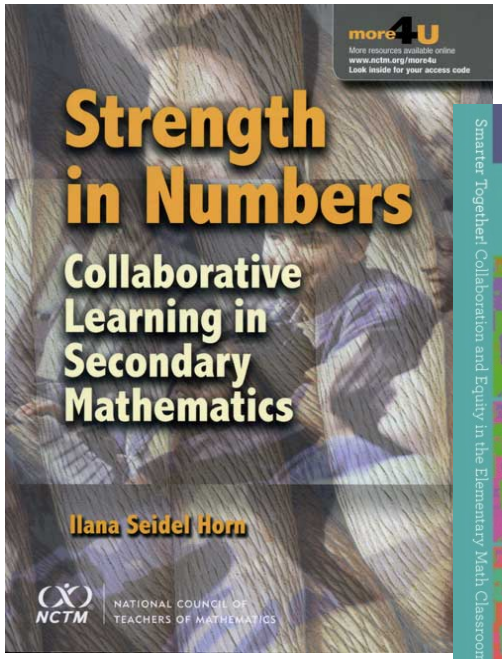
Can you say more about …

Can you give an example of …

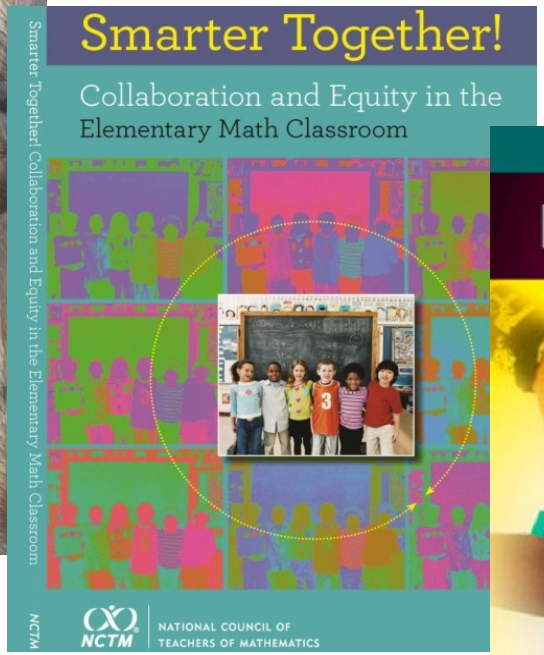
Wait! I don't understand …

What would happen if…?

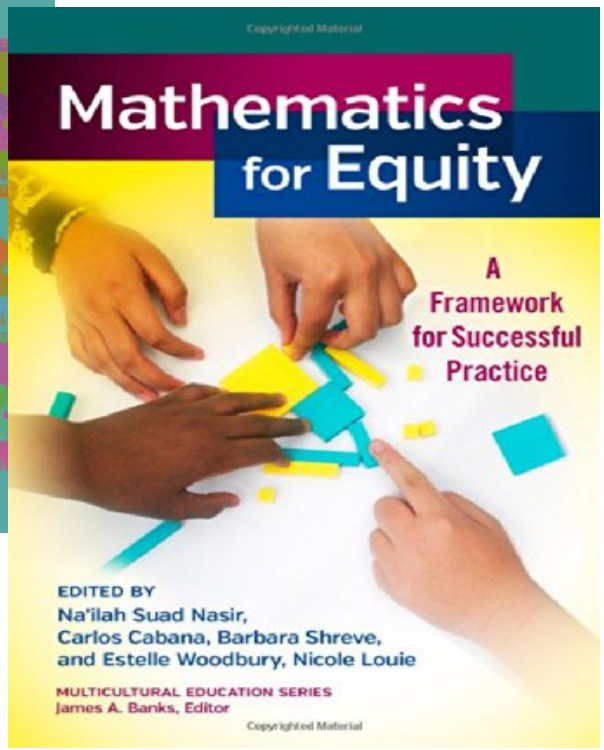
Status & Equity Issues



http://www.nctm.org/catalog/product.aspx?ID=13791



http://www.nctm.org/catalog/product.aspx?id=13785



http://www.nctm.org/catalog/product.aspx?id=14786

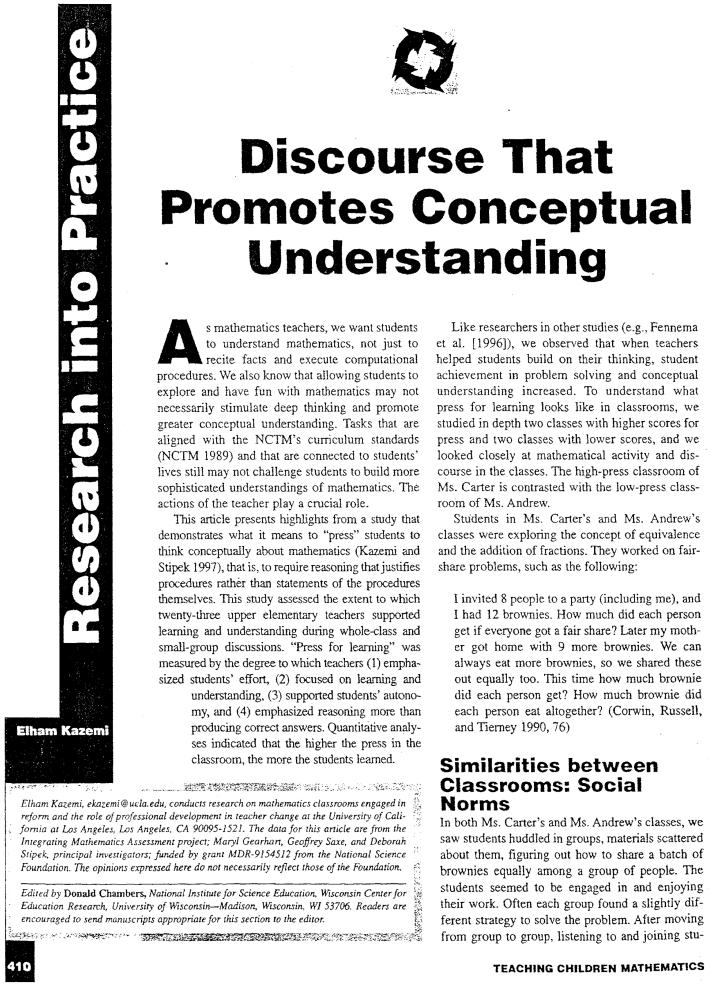
Journal for Research in Mathematics Education
July 1996

SOCIOMATHEMATICAL NORMS, ARGUMENTATION, AND AUTONOMY IN MATHEMATICS

ERNA YACKEL, *Purdue University*
PAUL COBB, *Vanderbilt University*

This paper sets forth a way of interpreting mathematics classrooms that aims to account for how students develop mathematical beliefs and values and, consequently, how they become intellectually autonomous in mathematics. To do so, we advance the notion of *sociomathematical* norms, that is, normative aspects of mathematical discussions that are specific to students' mathematical activity. The explication of sociomathematical norms extends our previous work on general classroom social norms that sustain inquiry-based discussion and argumentation. Episodes from a second-grade classroom where mathematics instruction generally followed an inquiry tradition are used to clarify the processes by which sociomathematical norms are interactively constituted and to illustrate how these norms regulate mathematical argumentation and influence learning opportunities for both the students and the teacher. In doing so, we both clarify how students develop a mathematical disposition and account for students' development of increas-

Discourse That Promotes Conceptual Understanding by Elham Kazemi, Mathematics Teacher - March 1998



Research into Practice

Discourse That Promotes Conceptual Understanding

Elham Kazemi

As mathematics teachers, we want students to understand mathematics, not just to recite facts and execute computational procedures. We also know that allowing students to explore and have fun with mathematics may not necessarily stimulate deep thinking and promote greater conceptual understanding. Tasks that are aligned with the NCTM's curriculum standards (NCTM 1989) and that are connected to students' lives still may not challenge students to build more sophisticated understandings of mathematics. The actions of the teacher play a crucial role.

This article presents highlights from a study that demonstrates what it means to "press" students to think conceptually about mathematics (Kazemi and Stipek 1997), that is, to require reasoning that justifies procedures rather than statements of the procedures themselves. This study assessed the extent to which twenty-three upper elementary teachers supported learning and understanding during whole-class and small-group discussions. "Press for learning" was measured by the degree to which teachers (1) emphasized students' effort, (2) focused on learning and understanding, (3) supported students' autonomy, and (4) emphasized reasoning more than producing correct answers. Quantitative analyses indicated that the higher the press in the classroom, the more the students learned.

Elham Kazemi, ekazemi@ucla.edu, conducts research on mathematics classrooms engaged in reform and the role of professional development in teacher change at the University of California at Los Angeles, Los Angeles, CA 90095-1521. *The data for this article are from the Integrating Mathematics Assessment project; Maryl Gearhart, Geoffrey Saxe, and Deborah Stipek, principal investigators; funded by grant MDR-9154512 from the National Science Foundation. The opinions expressed here do not necessarily reflect those of the Foundation.*

Edited by **Donald Chambers,** *National Institute for Science Education, Wisconsin Center for Education Research, University of Wisconsin—Madison, Wisconsin, WI 53706. Readers are encouraged to send manuscripts appropriate for this section to the editor.*

Like researchers in other studies (e.g., Fennema et al. [1996]), we observed that when teachers helped students build on their thinking, student achievement in problem solving and conceptual understanding increased. To understand what press for learning looks like in classrooms, we studied in depth two classes with higher scores for press and two classes with lower scores, and we looked closely at mathematical activity and discourse in the classes. The high-press classroom of Ms. Carter is contrasted with the low-press classroom of Ms. Andrew.

Students in Ms. Carter's and Ms. Andrew's classes were exploring the concept of equivalence and the addition of fractions. They worked on fair-share problems, such as the following:

> I invited 8 people to a party (including me), and I had 12 brownies. How much did each person get if everyone got a fair share? Later my mother got home with 9 more brownies. We can always eat more brownies, so we shared these out equally too. This time how much brownie did each person get? How much brownie did each person eat altogether? (Corwin, Russell, and Tierney 1990, 76)

Similarities between Classrooms: Social Norms

In both Ms. Carter's and Ms. Andrew's classes, we saw students huddled in groups, materials scattered about them, figuring out how to share a batch of brownies equally among a group of people. The students seemed to be engaged in and enjoying their work. Often each group found a slightly different strategy to solve the problem. After moving from group to group, listening to and joining stu-

Sociomathematical Norms

- Mathematical Justification
- Errors are an Opportunity to Rethink
- Mathematical Connections
- Individual Accountability in Collaborative Work

Lyons Pride Points

Lyons Pride Points

I sat by someone who didn't have anyone sitting by them.

I was honest in schooling saying the correct amount of time I did.

I got my drivers license.

I asked a girl on a date and it was waaaaaay fun!

I had my first kiss. Much divertido Oh My!!!

I made an error on the new math we've learned, but got the correct answer when we did another practice problem I learned from my past mistake.

I made a mistake on graphing. I graphed $x - 2$ as $(-2, 0)$ and then I re-thought it and re-graphed it at $(2, 0)$.

I was having a hard time with my math last night and I thought what I was doing wrong and fixed it.

I think I was getting the homework wrong and was really confused. So I came to class and I'm going to rethink it.

The mistakes I made showed me what I did wrong so I was able to fix them and then understand.

Doing test corrections I am able to see the mistakes I've made (even little ones) and it helps me to understand what I need to work harder on – I've learned it's the same in life!
When I don't get something I just have to keep working to get it!

OPPORTUNITY to Discuss

- Address common misconceptions
- Refine student thinking
- Prompt metacognition
- Engage students in developing conjectures & hypotheses

How long would a shelf need to be that would hold 100,000,000 Doraemon (anime) books?

What does the teacher say?

Student conversation?

What do you notice?

http://cartoonesia.com/wp-content/uploads/2014/07/doraemon_new_wallpaper_free_download_.jpg

How long would a shelf need to be that would hold 100,000,000 Doraemon (anime) books?

Email valyons@gmail.com for info about this video

How long would a shelf need to be that would hold 100,000,000 Doraemon (anime) books?

- Did you notice the teacher say listen to your **friend's** explanation?
- Did you notice the students conversing student to student?
- Which is the correct answer? Could you tell from the teacher?
- This is a classroom where differences of approaches, not all correct, are allowed to live in the classroom.

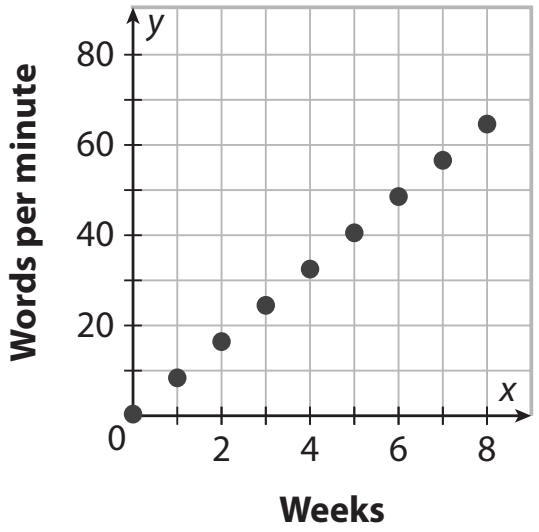
OPPORTUNITY to Ask

What MAKES SENSE?

Find the Error

The graph shown was given to represent this problem. Find the error(s) in the graph and then create a correct graph to represent the problem.

Jamie took an 8-week keyboarding class. At the end of each week, she took a test to find the number of words she could type per minute and found out she improved the same amount each week. Before Jamie started the class, she could type 25 words per minute, and by the end of week 8 she could type 65 words per minute.



Bring errors into discussions

“My Favorite No”
- a protocol for discussing errors -

https://www.teachingchannel.org/videos/class-warm-up-routine

https://www.teachingchannel.org/videos/class-warm-up-routine

Protocols for discussing errors

- Create safety – anonymity (if possible)
- Looked for the Good!
- Validate the student effort
- Praised the success – noted that this time no one made a previously often made error
- Teacher has knowledge
- Emphasis is on positive
- No tension – we can all learn!
- Student can think – I missed it but look at all the good things I did
- Students correct the mistake
- Students must convince teacher with their reasons for their thinking and the correctness of their answers

MAA Teaching Tidbits blog Suggestions

1. Create a safe space for incorrect answers (I'm so glad you raised that point)
2. Keep a poker face (ask the/a student to justify the reasoning, don't give away the answer)
3. Focus on the reasoning (encourage students to share, no negative reactions)
4. Distinguish between types of errors (typo? Sometimes it is important just to correct and move on)
5. Identify correct aspects of a solution (recognize the good work or "That would be the correct answer if [xxx], but actually we are thinking about [yyy]"

Be intentional about why you ask questions and how you solicit answers. Do you want to elicit discussion? The purpose of the question should dictate the way in which you handle responses, correct or incorrect.

Rachel Levy – Harvey Mudd College, September 20, 2016, http://maateachingtidbits.blogspot.de/

NCTM Research Brief
What Are Some Strategies For Facilitating Productive Classroom Discussions?

Exploring incorrect solutions can serve as a springboard for discussion.

Errors can serve to focus discussion and engage students in figuring out why an idea does or does not make sense. This creates opportunities to

- Address common misconceptions
- Refine thinking
- Prompt metacognition
- Engage in developing conjectures & hypotheses

http://www.nctm.org/Research-and-Advocacy/research-brief-and-clips/Strategies-for-Discussion/

NCTM Research Brief
What Are Some Strategies For Facilitating Productive Classroom Discussions?

Interestingly, in **collaborative discussions**, we found it rare for something to explicitly be identified as "wrong." Rather, students' ideas were treated as **"works in progress."**

The focus of the teacher's guidance was not to help students "correct" something that was wrong. Rather, it was to help the student and the class extend the idea that had been presented and continue to develop a viable solution collaboratively.

COURSE CORRECTION

Staples and Colonis *Making the Most of Mathematical Discussion* MT, Nov 2007

Principles to Action:
Ensuring Mathematical Success For All

Support productive struggle in learning mathematics Teacher and student actions	
What are teachers doing?	What are students doing?
Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.	Persevering in solving problems …

Principles to Action, p. 52

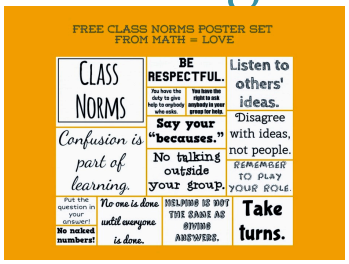
Errors are OPPORTUNITIES!

Research Brief: What Are Some Strategies For Facilitating Productive Classroom Discussions?

- OPPORTUNITY to Rethink

  Reduce Absolute Correct Initial Thinking

  We are all learning!



- OPPORTUNITY to Discuss
  - Address common misconceptions
  - Refine student thinking
  - Prompt metacognition
  - Engage students in developing conjectures & hypotheses

- OPPRTUNITY to Ask: What MAKES SENSE?

Mathematical Teaching Practices

- Implement tasks that promote reasoning and problem solving.
- Facilitate meaningful mathematical discourse.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Thank you for coming!

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http://www.nctm.org/PrinciplestoActions/

Resources

- Otto Lilienthal's First Film https://vimeo.com/155960641
- Outrageous Acts of Science, Kings of Carnage episode, Gary Connery wingsuit flying http://turbotunehd.com/vid/WZZ5Bg7F0DQ
- Glide Ratio: Adapted from Looking at an angle. (2003). Mathematics in Context Project. Directed by Tom Romberg & Jan deLange. Chicago IL: Encyclopedia Britannica.
- NASA Glenn Research Center, A Beginner's Guide to Aerodynamics, Aircraft Forces, Lift to Drag Ratio, https://www.grc.nasa.gov/www/K-12/airplane/ldrat.html
- Wikipedia *Lift-to-drag Ratio*, https://en.wikipedia.org/wiki/Lift-to-drag_ratio #Glide_ratio
- Common Core State Standards for Mathematics, Standards for Mathematical Practice pp. 6-7 http://www.corestandards.org/Math/
- TIMSSVIDEO, Hong Kong Mathematics Lessons, HK4 Identity: http:www.timssvideo.com/videos/mathematics/Hong%20Kong

Resources

- Ilana Horn's blog: http://teachingmathculture.wordpress.com/2014/07/14/what- do-you-think-and-why/
- Classroom Norms poster, Sarah Hagan's blog: http://mathequalslove.blogspot.com/2014/07/class-group-work-norms-poster-set-free.html?m=1
- Accountable Talk Cards, Jennifer Parker, July 13, 2015 *http://mathforum.org/pcmi/hstp/sum2015/morning/shorts.html*
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- Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). Smarter together! collaboration and equity in the elementary math classroom. Reston, VA: The National Council of Teachers of Mathematics.

Resources

- Nasir, N., Cabana, C., Shreve, B., Woodbury, E., & Louie, N. (Eds) (2014). Mathematics for equity: a framework for successful practice (multicultural education). New York, NY: Teachers College Press
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Resources

- "My Favorite No" with Leah Alcala: https://www.teachingchannel.org/videos/class-warm-up-routine
- Houghton Mifflin Harcourt Integrated Mathematics I, Module 1 – Quantitative Reasoning p. 25
- MAA Teaching Tidbits blog, http://maateachingtidbits.blogspot.de/2016/09/5-ways-to-respond-when-students-offer.html?m=1
- NCTM Research Brief What Are Some Strategies For Facilitating Productive Classroom Discussions? http://www.nctm.org/Research-and-Advocacy/research-brief-and-clips/Strategies-for-Discussion/
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