

What Does Domain Have to Do with It?

**NCTM Annual Meeting and Exposition
San Antonio, Texas
April 7, 2017**

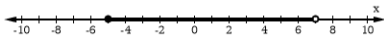
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FINDING DOMAIN

Warm Up

Using the given number lines, write the indicated inequalities and interval notation.

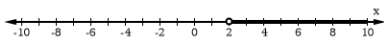
1.



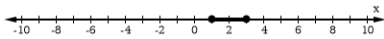
Inequality

Interval notation

2.



3.



4.



5.



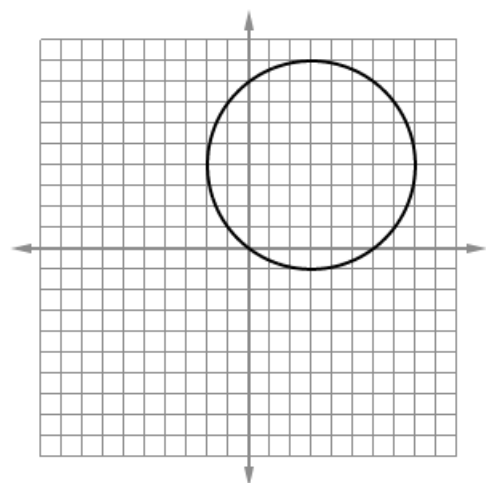
Finding Domain and Range

Examination of the graph of a relation can reveal the domain and range. Knowing how to read the graph is a key component to correctly identifying these important sets of real numbers. The process described and illustrated here will help to focus on the important aspects of this process.

Steps

1. Examine the graph and notice its shape.
2. With a ruler, draw vertical and horizontal lines to enclose the graph.
3. Notice the “ends” of the curve if they exist, and determine if there is a closed point, open point, or arrow.
4. Write the domain from the identified interval(s) on the x-axis.
5. Write the range from the identified interval(s) on the y-axis.

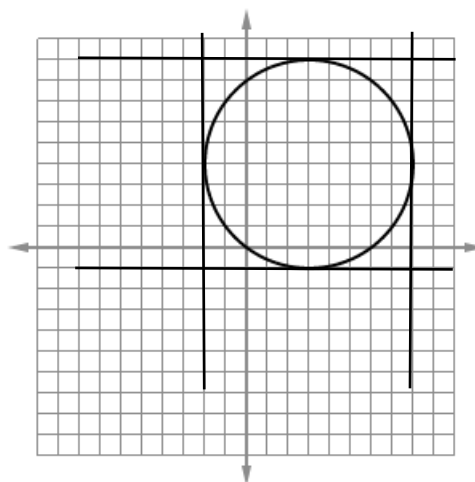
Example 1.



Step 1. We notice that the graph is a circle.

Step 2. With a ruler, we draw two vertical lines and two horizontal lines to enclose the graph. See the diagram below.

Step 3. We note that there are no “ends” on this graph.



Step 4. We see that the two vertical lines cross the x-axis at -2 and at 8. The domain is the interval $[-2, 8]$.

Step 5. Similarly, the two horizontal lines cross the y-axis at -1 and 9. The range is the interval $[-1, 9]$.

A Note About Notation: Square brackets $[$ or $]$ are used to indicate that the endpoint of an interval is included in the interval. Parentheses $($ or $)$ are used to indicate that the endpoint of the interval is not included. There are eight types of intervals which are shown below along with their interpretation in inequality notation.

$[a, b]$ corresponds to $a \leq x \leq b$

(a, b) corresponds to $a < x < b$

$[a, \infty)$ corresponds to $x \geq a$

$(-\infty, b]$ corresponds to $x \leq b$

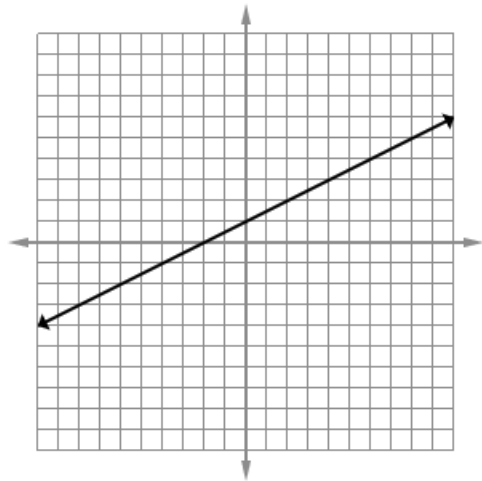
$[a, b)$ corresponds to $a \leq x < b$

$(a, b]$ corresponds to $a < x \leq b$

(a, ∞) corresponds to $x > a$

$(-\infty, b)$ corresponds to $x < b$

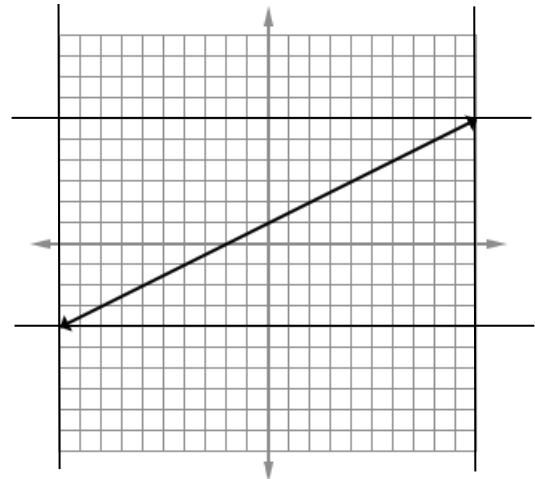
Example 2.



Step 1. The relation graphed here is a line.

Step 2. With a ruler, we draw two vertical lines and two horizontal lines to enclose the graph. See the diagram below.

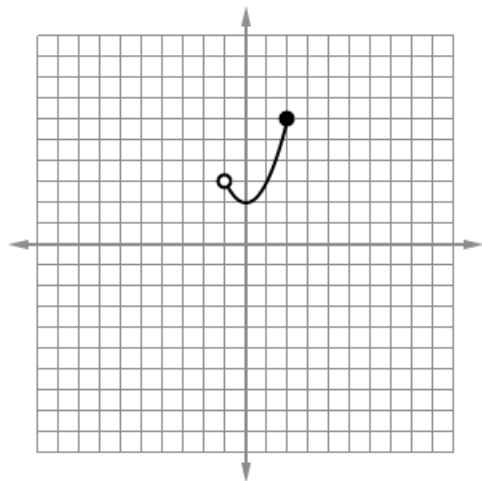
Step 3. We notice that the “ends” of the graph on the left and the right are marked arrows. We interpret this to mean that the graph continues to down as it goes left and the graph continues to go up as it goes right. It never stops.



Step 4. The vertical lines are at the arrow heads, so the graph continues infinitely in both directions. We say that the domain is “all real numbers” and designate with this with a bold **R** or by the interval $(-\infty, \infty)$.

Step 5. The horizontal lines are also at the arrow heads, so the graph continues infinitely in both directions. The range is also $(-\infty, \infty)$.

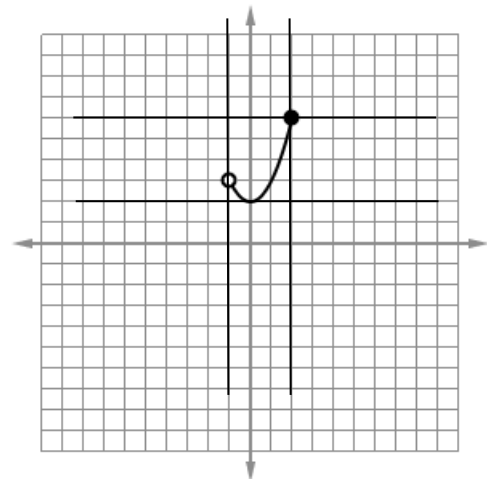
Example 3.



Step 1. The relation graphed here is a portion of a curve.

Step 2. With a ruler, we draw two vertical lines and two horizontal lines to enclose the graph. See the diagram below.

Step 3. We notice that the “ends” of the graph on the left and the right are marked an open point on the left and a closed point on the right. We interpret the open point to mean that the graph does not include this point. The closed point indicates that it is part of the graph.



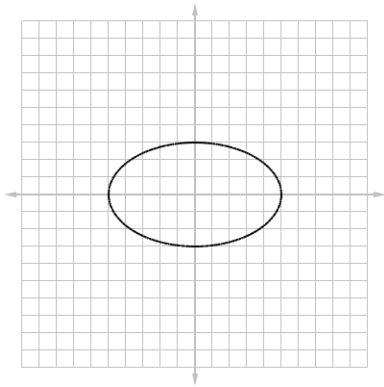
Step 4. The vertical lines are at -1 and 2, so the domain is $(-1, 2]$ since the point on the left is not included and the point on the right is included.

Step 5. The horizontal lines are at 2 and 6, so the range is $[2, 6]$ since the end point is included.

GRAPHICAL RELATIONS

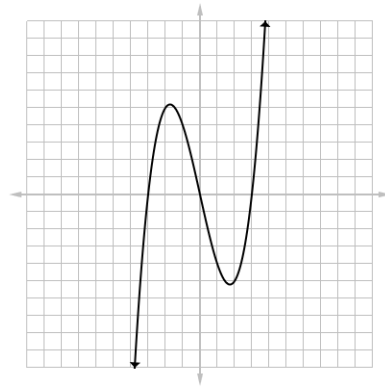
Determine the domain and range of each of the relations graphed below. Write the domain and range in interval notation.

1.



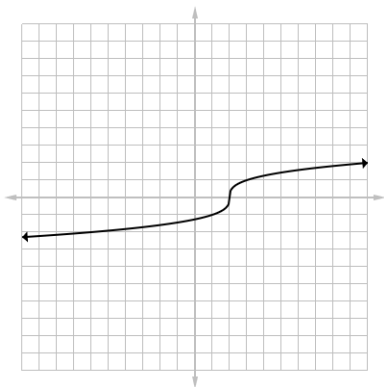
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2.



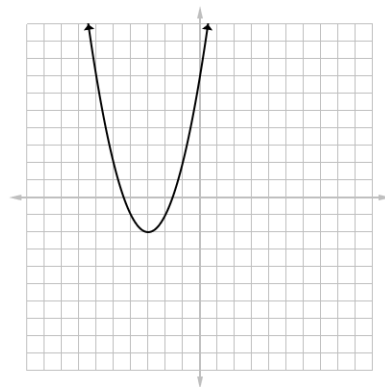
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3.



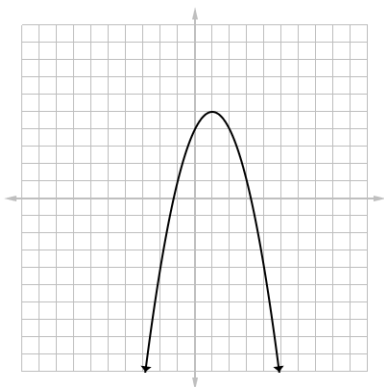
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4.



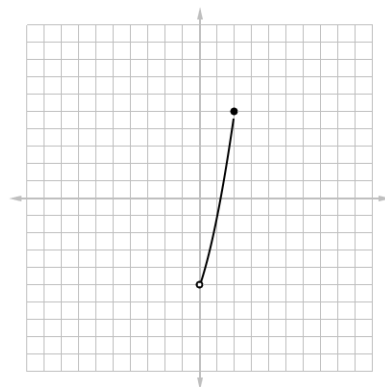
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5.



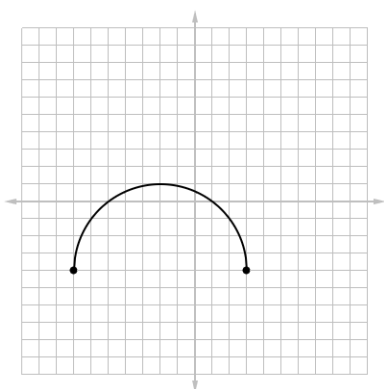
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6.



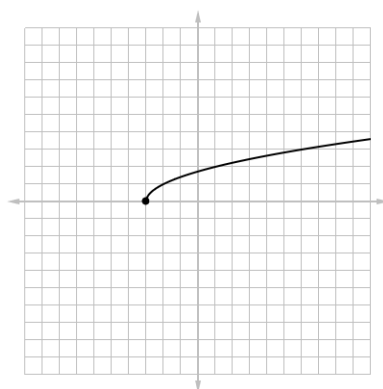
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7.



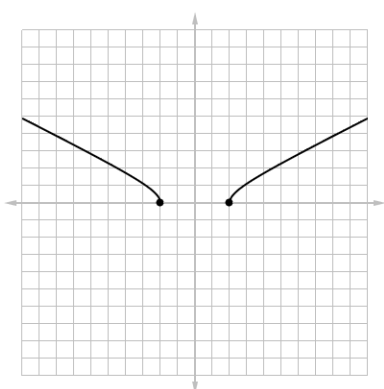
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8.



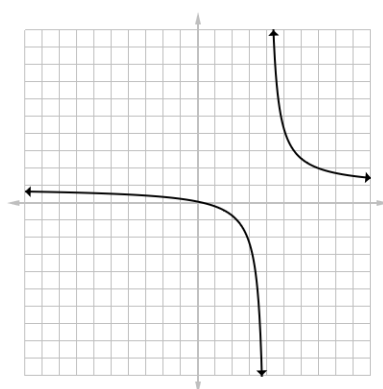
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9.



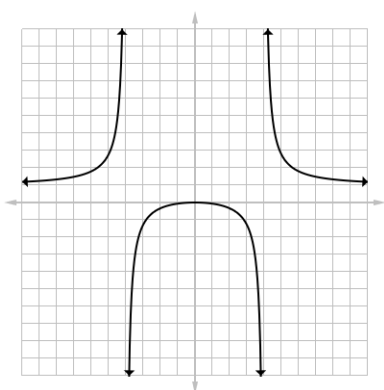
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10.



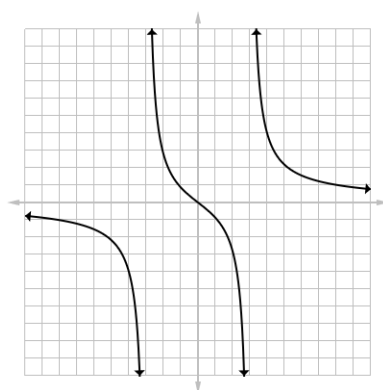
Domain: _____ Range: _____

11.



Domain: _____ Range: _____

12.



Domain: _____ Range: _____

Function Domain

A. Find the domain of each of the functions. Write the domain in interval notation.

1. $y = 3x - 4$ 2. $f(x) = 3x^2 + 6x - 4$ 3. $g(x) = \frac{3x-2}{4}$

Domain: _____

Domain: _____

Domain: _____

4. $h(x) = x^3 - 1$ 5. $p(x) = \frac{1}{2}x^2 + \frac{3}{5}x^4$ 6. $q(x) = x(2x-3)(x+4)$

Domain: _____

Domain: _____

Domain: _____

B. Find the domain of each of the square root functions. Write the domain in interval notation.

7. $f(x) = \sqrt{x-2}$ 8. $g(x) = \sqrt{x+7}$ 9. $h(x) = \sqrt{5x-4}$

Domain: _____

Domain: _____

Domain: _____

10. $p(x) = \sqrt{x^2+1}$ 11. $q(x) = \sqrt{\frac{1}{x}}$ 12. $t(x) = \sqrt{x^2-4}$

Domain: _____

Domain: _____

Domain: _____

C. Find the domain of each of the rational functions. Write the domain in interval notation.

13. $f(x) = \frac{x+1}{x-2}$ 14. $g(x) = \frac{x-5}{3x+4}$ 15. $h(x) = \frac{-7}{x(x+3)}$

Domain: _____

Domain: _____

Domain: _____

16. $p(x) = \frac{x^2+3x+2}{x^2-4}$ 17. $q(x) = \frac{3x+5}{x^2+9}$ 18. $t(x) = -3 - \frac{2x}{x+1}$

Domain: _____

Domain: _____

Domain: _____