Incredible Math Tasks!—Catalyst for Effective Formative Assessment

At the Baseball Shop

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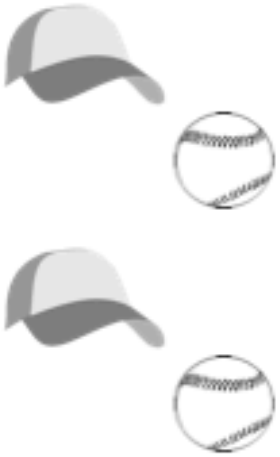		

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Incredible Math Tasks!—Catalyst for Effective Formative Assessment

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Howard County Public Schools

Jenny Novak

Delaware Department of Education

April 2017

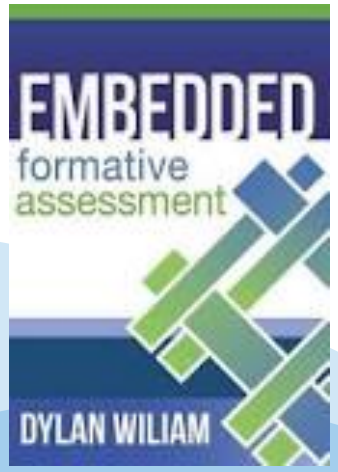
NCTM Annual Conference

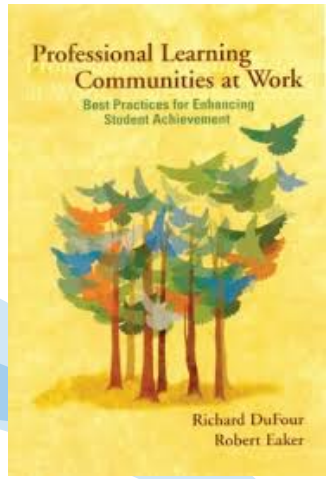
San Antonio, TX

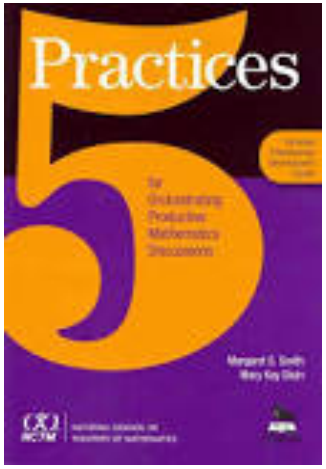
Today we will:

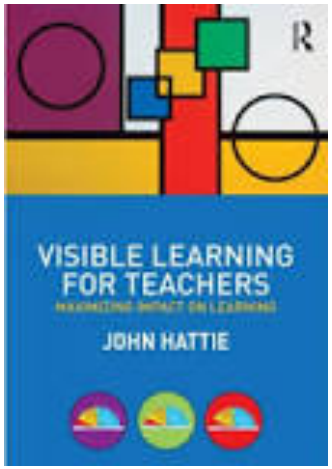
- Collaboratively engage in mathematical tasks designed to elicit and develop the Standards for Mathematical Practices. (DO MATH!)
- Examine how the strategic planning for and implementation of mathematical tasks during lessons serve as a catalyst for formative assessment processes.

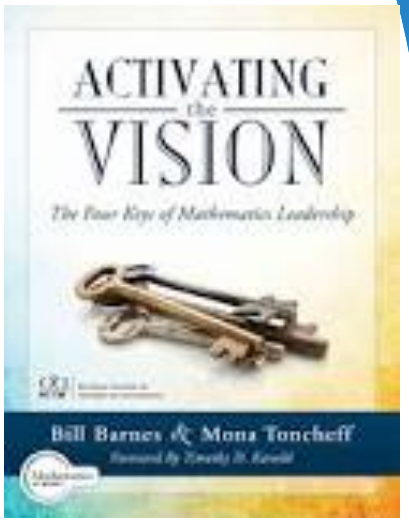
Thank you in advance to….

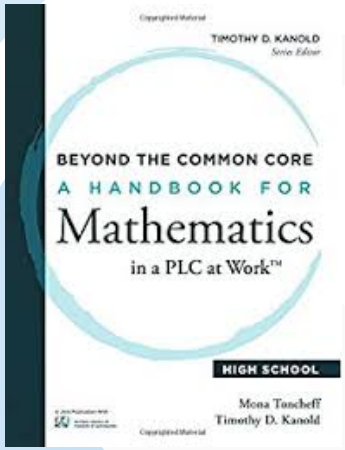












One Definition of Formative Assessment

An assessment that functions formatively…

> "to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better or better founded than the decisions that would have been made in the absence of that evidence."
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> (Black & Wiliam, 2009)

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Eliciting Evidence of Student Learning

	Where the learner is going	Where the learner is	How to get there
Teacher	Clarifying, sharing and understanding learning intentions	Engineering effective discussions, tasks, and activities that elicit evidence of learning	Providing feedback that moves learners forward
Peer		Activating students as learning resources for one another	
Learner		Activating students as owners of their own learning	

Wiliam, 2014 (Philadelphia)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Principles to Action: Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- **Implement tasks that promote reasoning and problem solving.**
- Use and connect mathematical representations.
- Facilitate meaningful discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- **Elicit and use evidence of student thinking.**

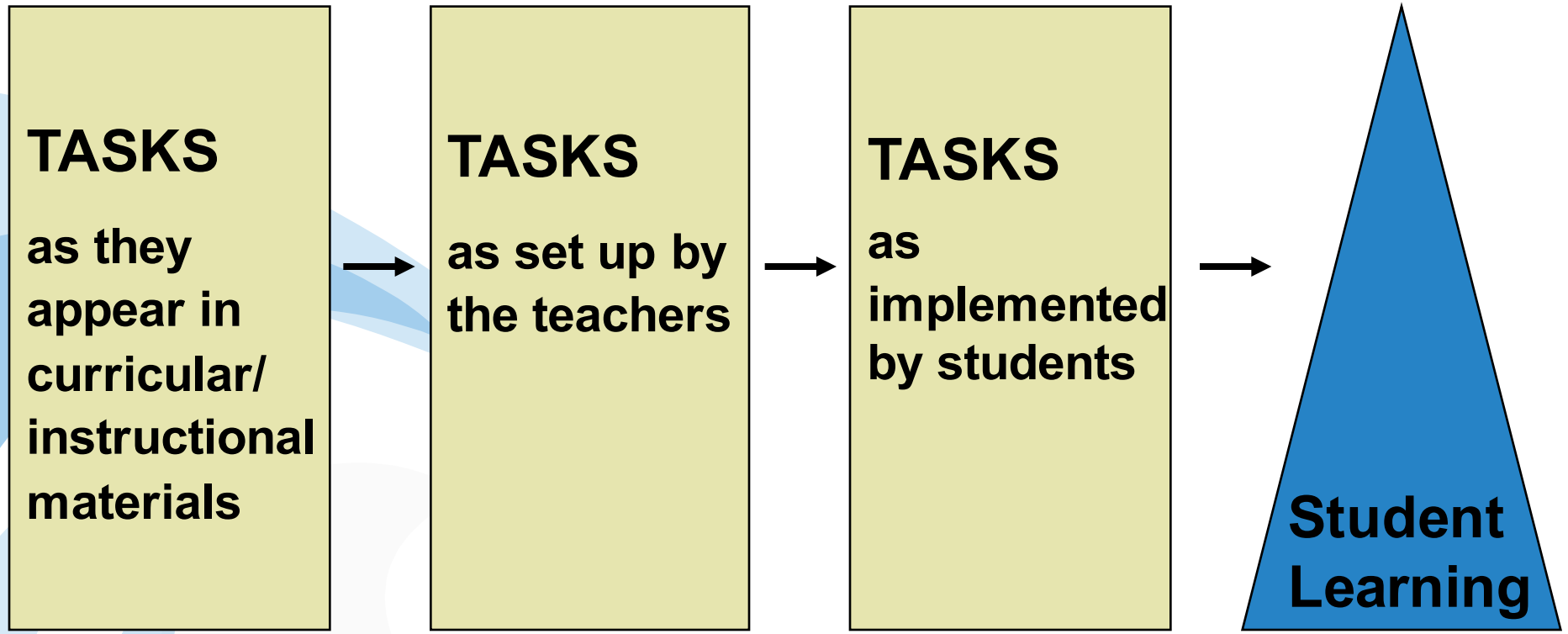
NCTM's Principles to Actions (2014)

What is a Worthwhile Math Task?

1. **The problem has important, useful mathematics embedded in it.**
2. **The problem requires higher-level thinking and problem solving.**
3. **The problem contributes to the conceptual development of students.**
4. **The problem creates an opportunity for the teacher to assess what his or her students are learning and where they are experiencing difficulty.**
5. **The problem can be approached by students in multiple ways using different solution strategies.**
6. **The problem has various solutions or allows different decisions or positions to be taken and defended.**
7. The problem encourages student engagement and discourse.
8. The problem connects to other important mathematical ideas.
9. The problem promotes the skillful use of mathematics.
10. The problem provides an opportunity to practice important skills.

http://www.nctm.org/news/content.aspx?id=25713

The Mathematical Tasks Framework



Stein, Smith, Henningsen, & Silver, 2000, p. 4

Revisiting the Opening Task

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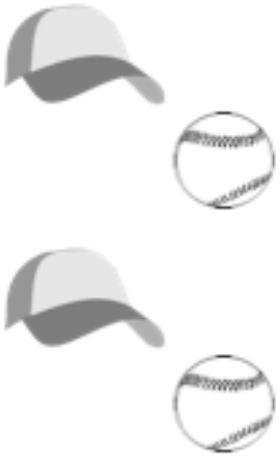		

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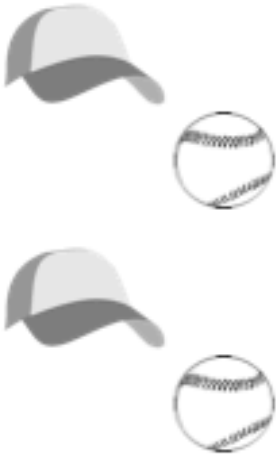		

2h+2b=46
1h+2b=31
1t+1b=33

If the baseball shop sells each item individually, what is the cost of each item?

h=cost of hat, b=cost of ball, t=cost of bat

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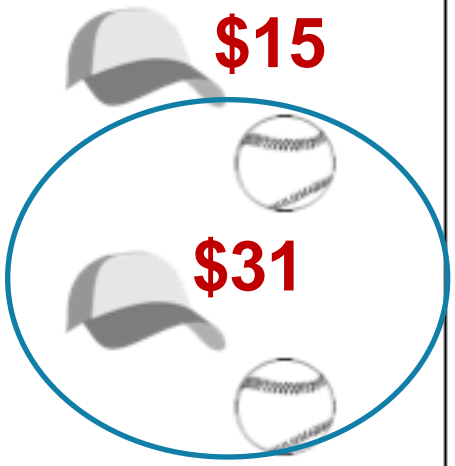		

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 	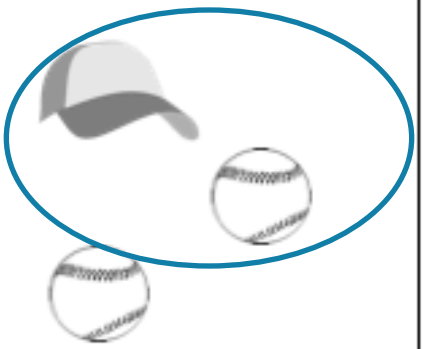	

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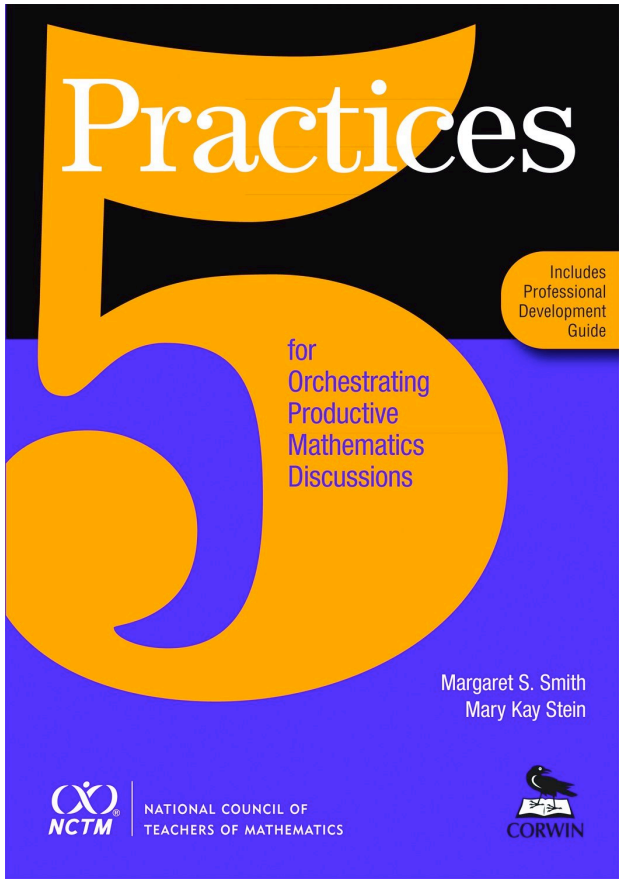
The 5 Practices for Orchestrating Productive Mathematics Discussions

- **Anticipating**
- **Monitoring**
- **Selecting**
- **Sequencing**
- **Connecting**



1. Anticipating

For implementation to be productive….

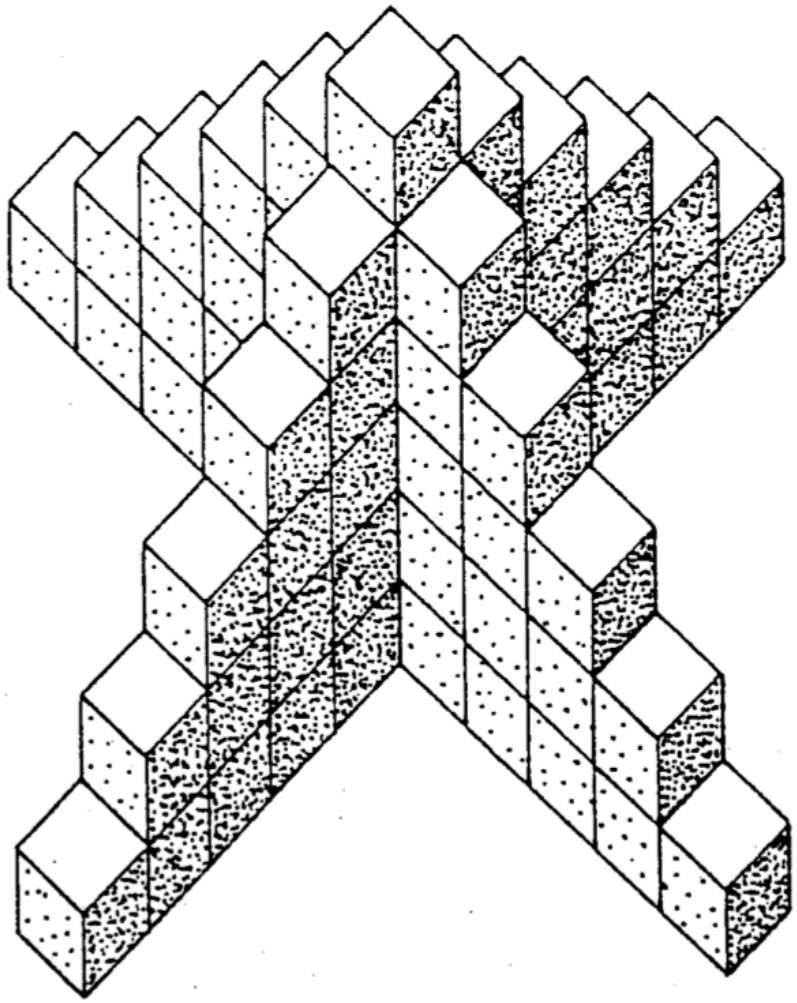
- How might students approach the task?
- How are these strategies (both correct and incorrect) related to the mathematical concepts, representations, procedures, and practices that you want the students to learn?

To effectively **Anticipate**, you should:

- Do the problem as many ways as you can (and with others if possible)
- Review any available responses to the task to determine which mistakes/misconceptions students might make?...and how they might be useful in developing understanding

Skeleton Tower Task

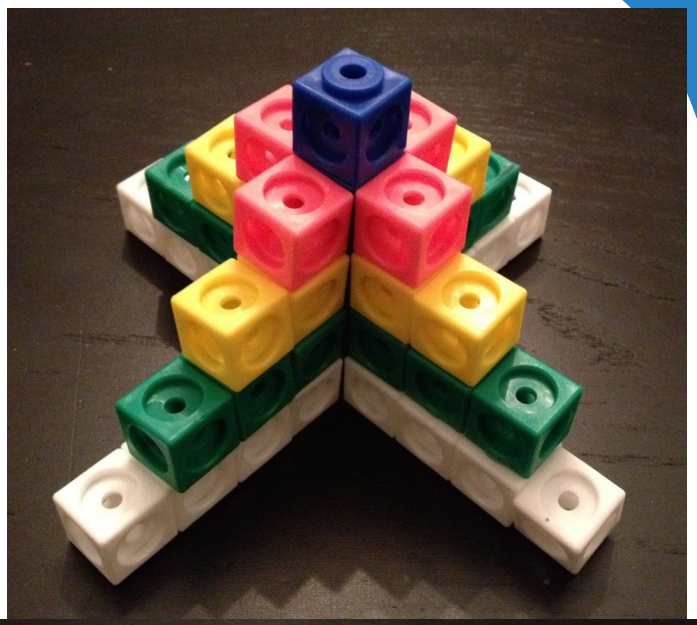
- How many cubes are needed to build this tower?
- How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you determined your answer.
- How would you calculate the number of cubes needed for a tower *n* cubes high? Describe the pattern for building the tower?

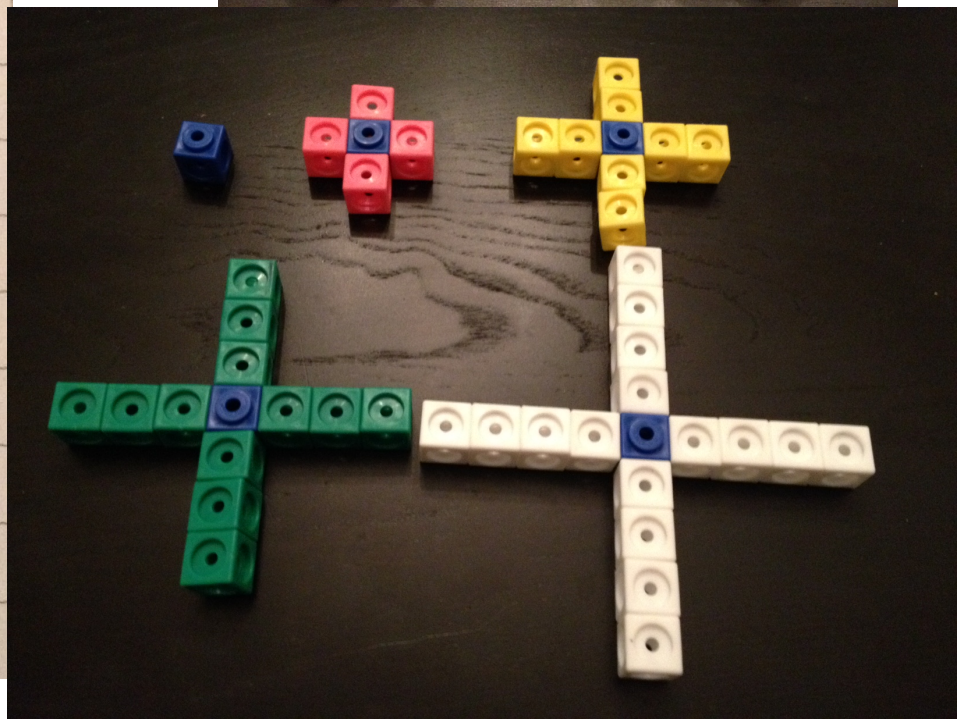


http://map.mathshell.org/materials/tasks.php?taskid=279&subpage=expert

Some Possible Strategies

Height of Tower	Number of Cubes	
1	1	1
2	6	1+4+1
3	15	6+8+1
4	28	15+12+1
5	45	28+16+1
6	66	45+20+1
7	91	66+24+1
8	120	91+28+1
9	153	120+32+1
10	190	153+36+1
11	231	190+40+1
12	276	231+44+1





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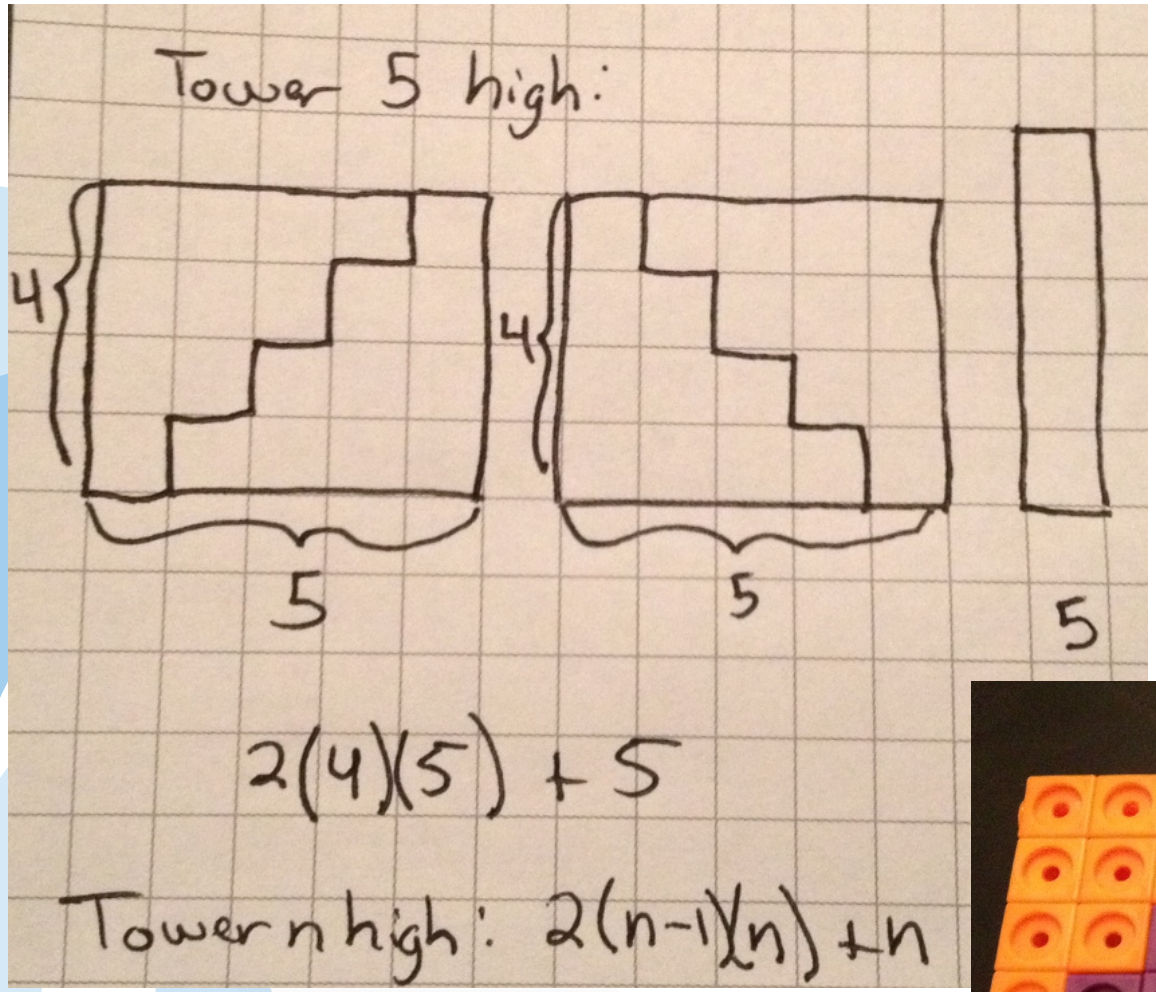
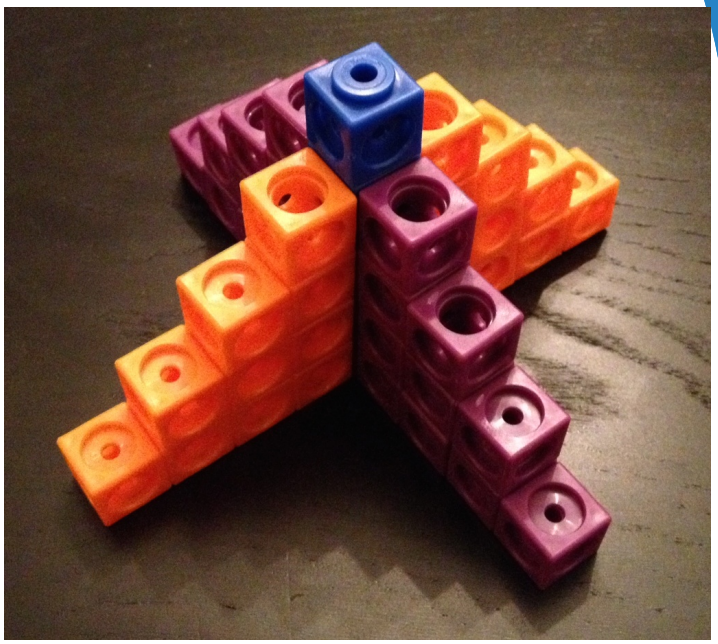
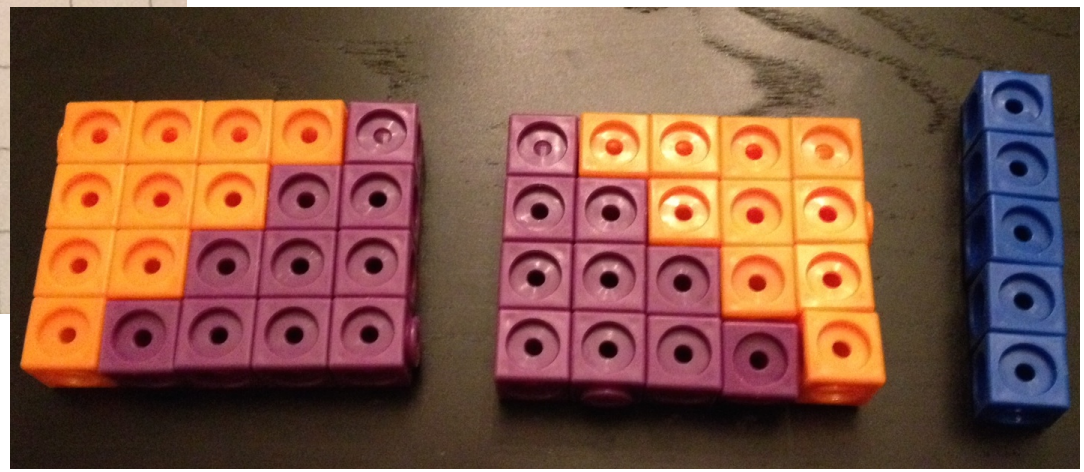






2. Monitoring

For implementation to be productive….

- How are students actually responding to the task?
- What are they saying, writing, or diagramming that might prove useful later in the lesson?
- Which of the anticipated solutions and/or misconceptions are present? Which might I have to introduce to extend thinking?

To effectively **Monitor**, you should:

- Pay close attention to students' mathematical thinking and solutions strategies as they work through the task
- Create a possible list of solution strategies before administering the task, then make note of who is using that approach

2. Monitoring

Strategy	Who and What	Order
Recursive Pattern		
Table of Values—Explicit		
Row by Row Analysis		
Analysis of “Wings”		
Gauss Formula		
Decompose/Products		
Summation		
Other		

3/4. Selecting & Sequencing

For implementation to be productive….

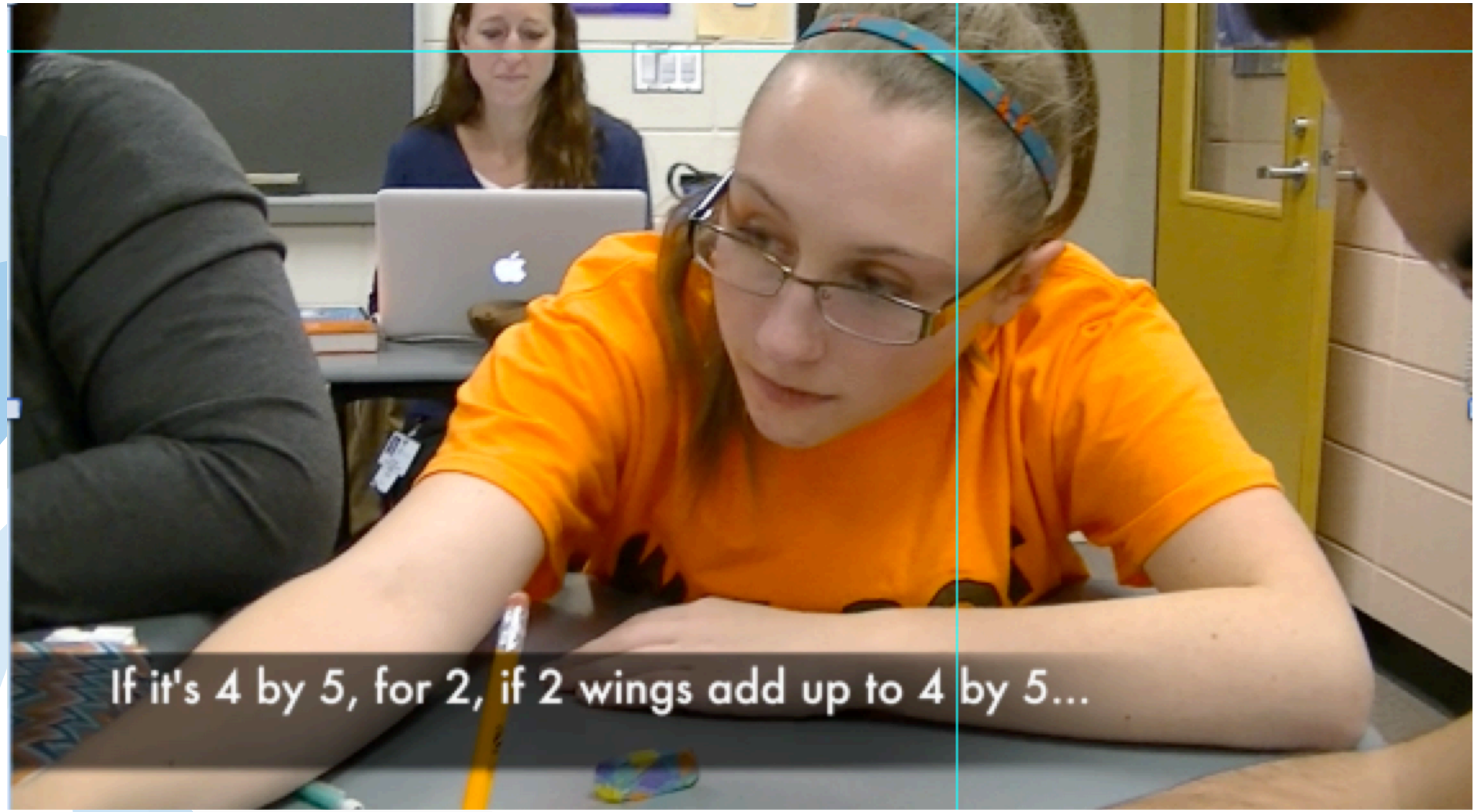
- Who should be selected to share their work?
- In what order will these strategies be select?

To effectively **Select & Sequence**, you should:

- Identify who should share and when.
- Determine the best way to make connections between strategies.
- Determine how you will notify students that they will be sharing.
- Determine the purpose for your sequence.

Selecting & Sequencing

Selecting & Sequencing



5. Connecting

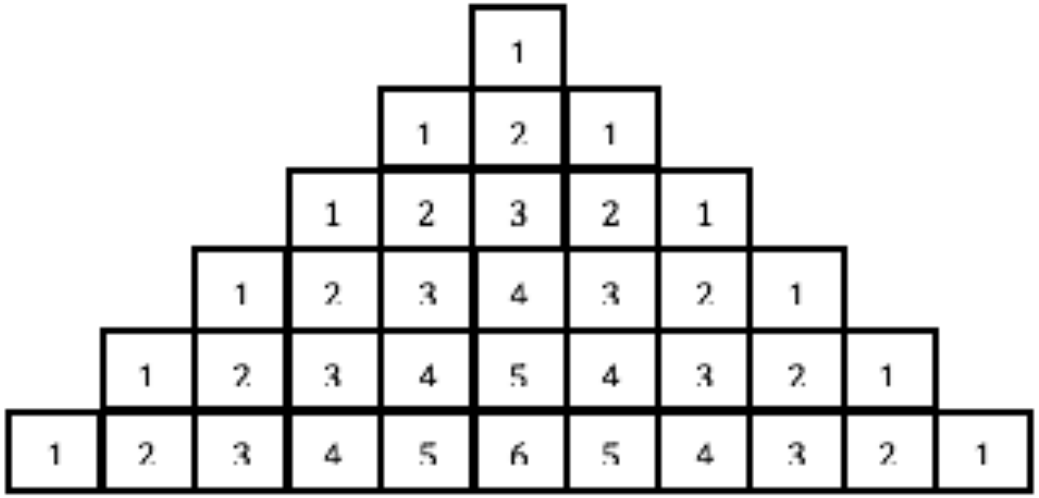
For mathematical discourse to be productive….

- What conclusions do you want students to draw between their solutions?
- What connections do you want students to make between their solutions and and the key mathematical ideas (i.e. instructional goal of task)?

To effectively **Connect**, you should:

- Plan follow-up questions to probe further or elicit connections.
- Provide adequate wait time and provide additional opportunities for students to discuss and/or revise strategies.

Towering Numbers

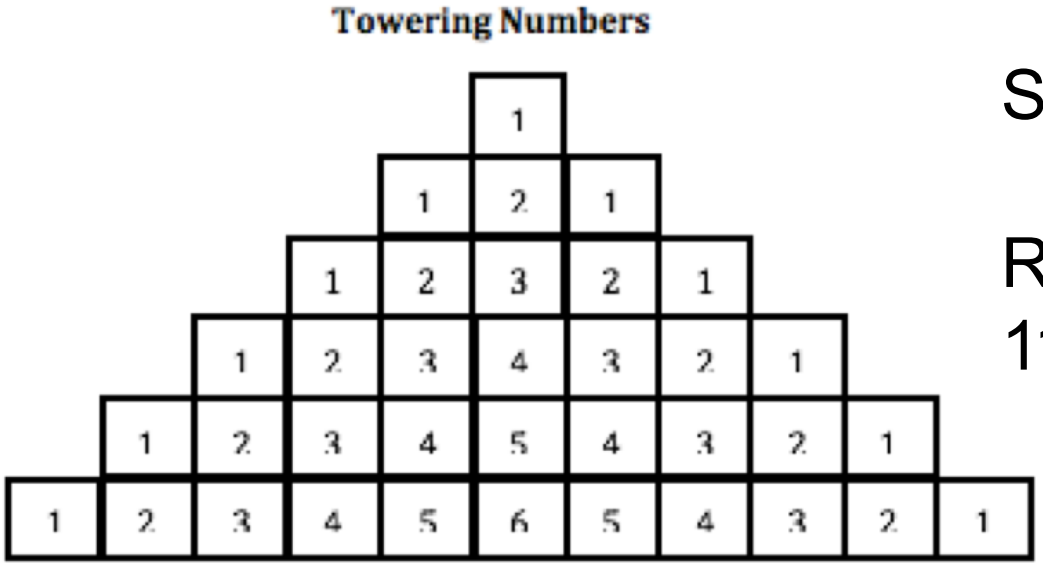


1. There are six rows in the tower pictured above. How many bricks would be in the seventh row?

2. Suppose you wanted to build a tower with 25 rows using the same design. Describe how you could figure out how many bricks you would need for the twenty-fifth row.

3. A very large tower was build using the same design. The longest row had 299 bricks in it. How many rows of bricks did the tower have?

An alternative approach



Starting Point:

Row 6 has
11 cubes

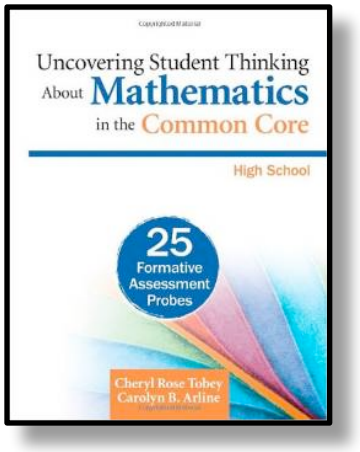
Row 6

Row 25 – Row 6 = 19 rows of adding 2 more for each row

2(19) + 11 = 49 cubes in row 25

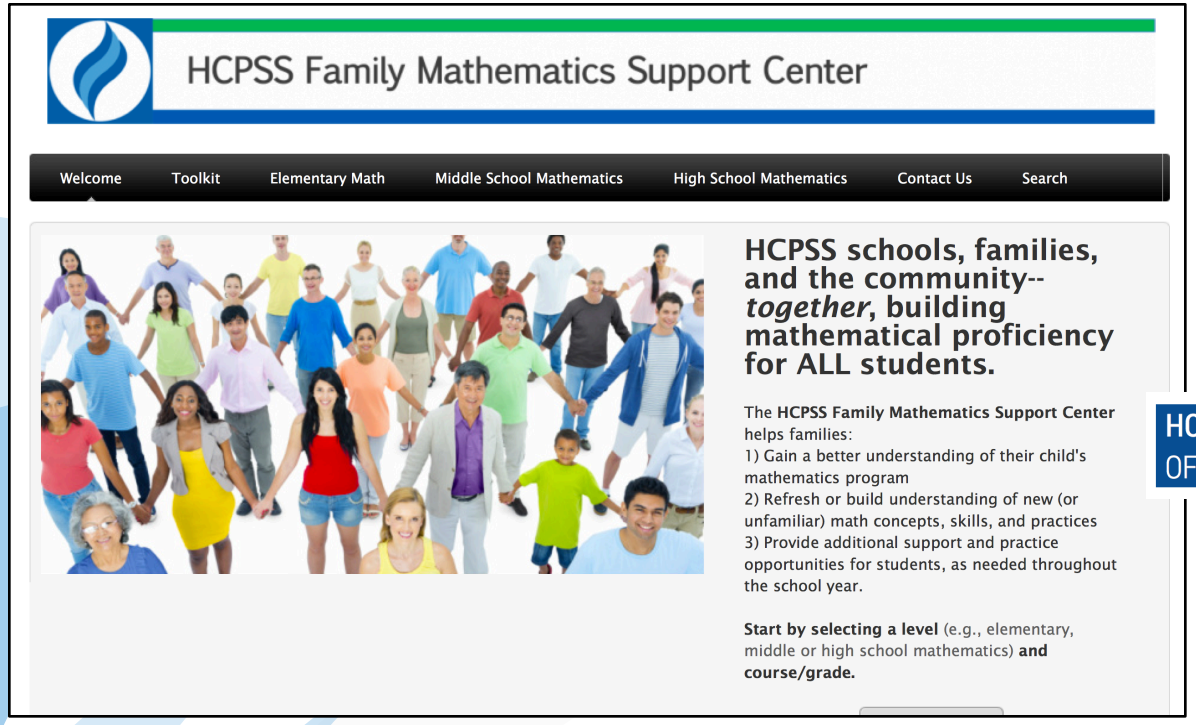
Other Formative Assessment "Moves"

- Maybe reference some authors best thinking here…
  - Tobey says,
    - Get more evidence
    - Provide further instruction
    - Provide formative feedback
    - Move on

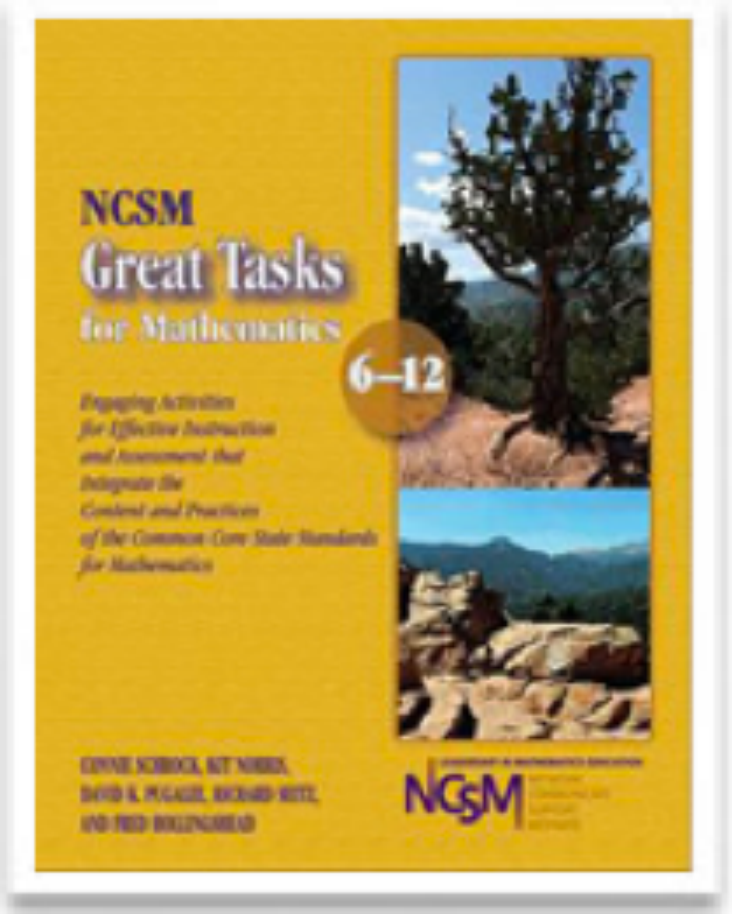


  - Fennell, Kobett, and Wray say,
    - Observations, Interviews, Show Me, Hinge Questions, Exit Tasks

HCPSS Open-Sourced Resources



Where else can you find tasks?



NCTM ILLUMINATIONS
Resources for Teaching Math

Illustrative Mathematics

MARS Mathematics Assessment Resource Service

ANNENBERG LEARNER

Thank you!

For more information please contact us.

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Jenny Novak
jennifer.novak@doe.k12.de.us
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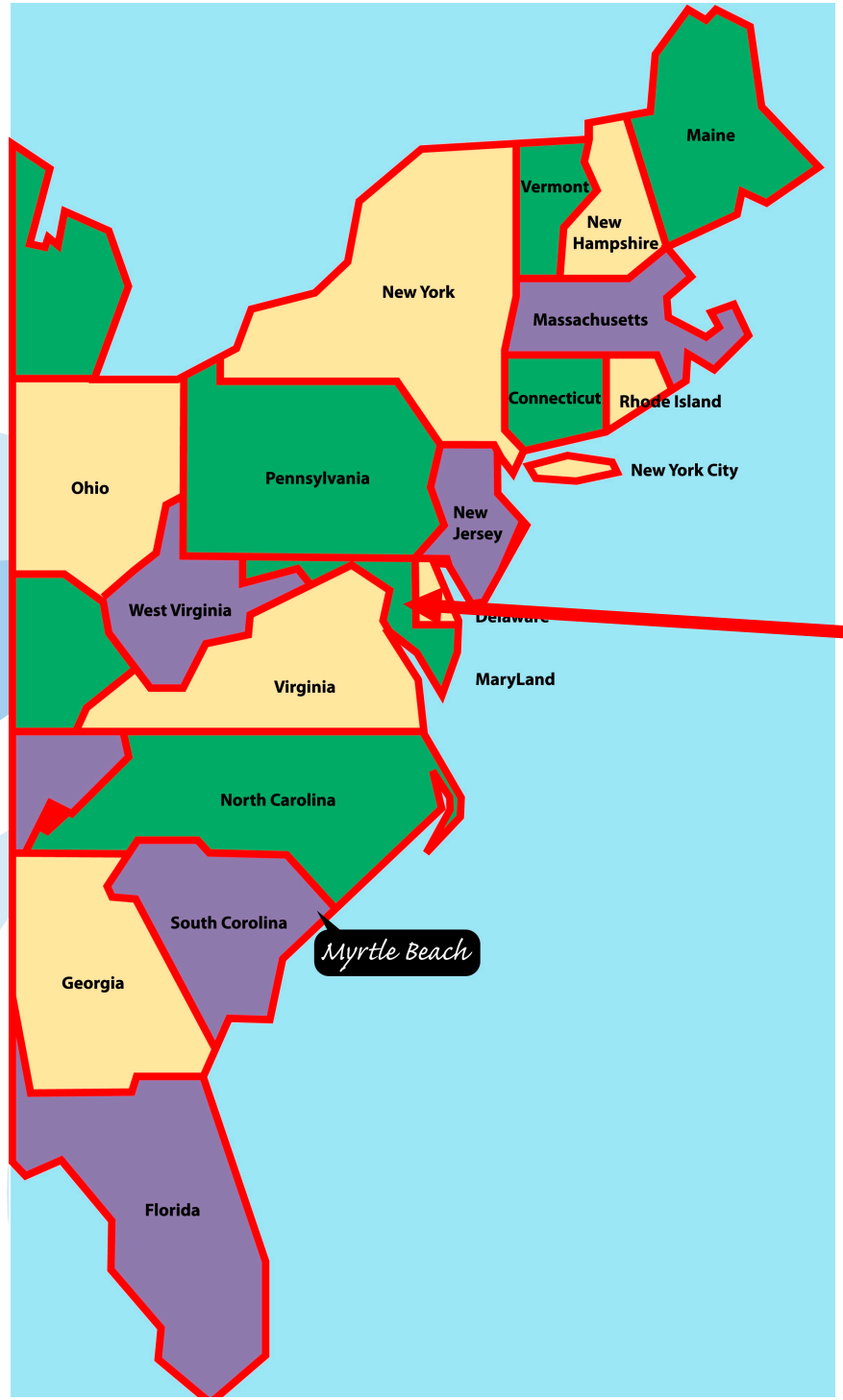
Welcome!



WE ARE FROM HERE!