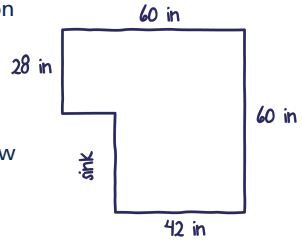
"Noticing and Wondering" as a Vehicle to Understanding the Problem

Annie Fetter, The Math Forum at NCTM

2017 NCTM Annual Meeting
Twitter: @MFAnnie, #NoticeWonder
http://mathforum.org/nctm/

Teresa's Tiles

Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.

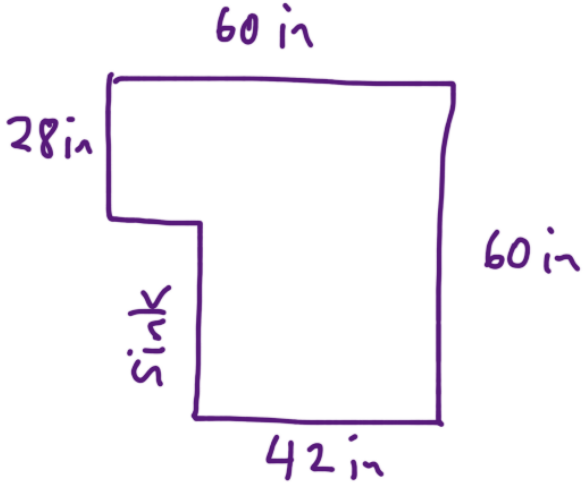


This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

Questions: How many tiles will she need to buy to cover her floor? How many tiles will she have to cut in order to cover the entire space?

Extra: What is the size, using whole numbers, of the largest square tile that could be used to tile the entire floor with no cut pieces?

Teresa's Tiles "Scenario"



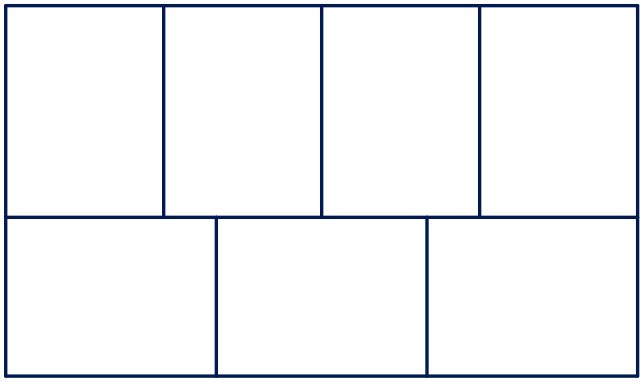
Teresa's Tiles, Student Work

Things that some "low-performing" 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces

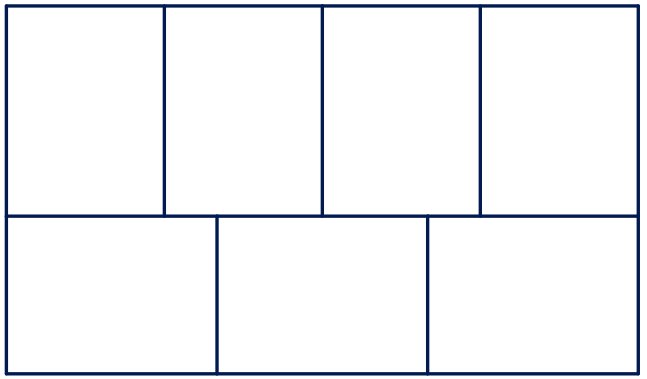
Congruent Rectangles Scenario I

The seven small rectangles in this picture are congruent.

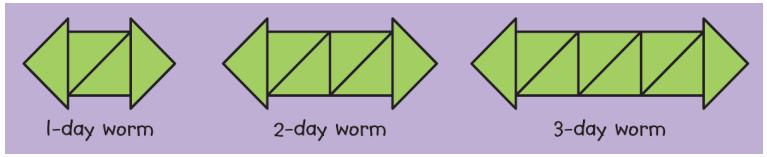


Congruent Rectangles Scenario II

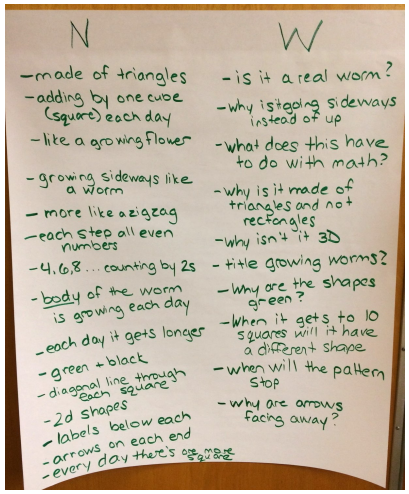
The seven rectangles in this picture are congruent.
The area of the large rectangle is 756 square centimeters.

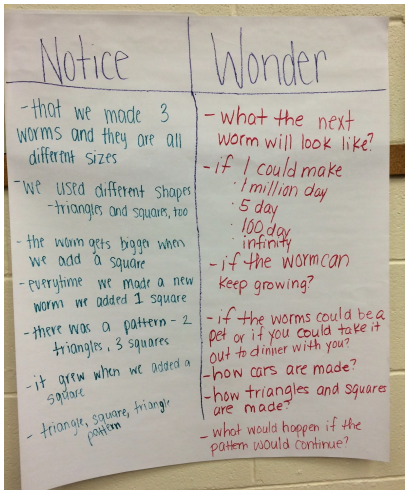


Growing Worms Scenario

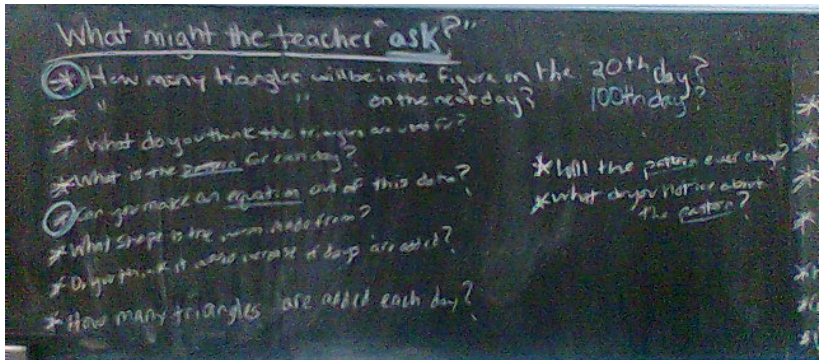


Growing Worms Student NW





What Might the Teacher Ask?



Variations on "Notice and Wonder"

What Did You Hear?
What Did You See?

CCSS Mathematical Practice 1

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

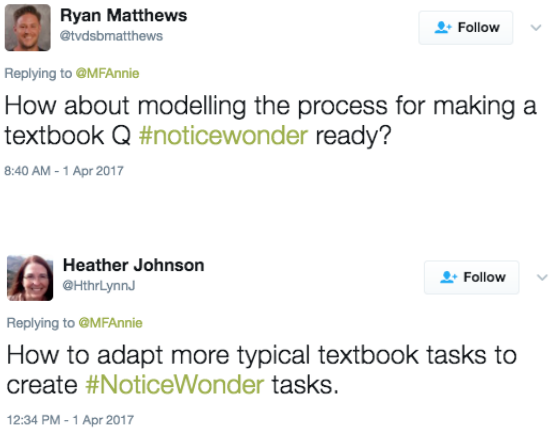
They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

I Asked Twitter for Advice

Annie Fetter
@MFAnnie

Talking #NoticeWonder at #NCSM17 + #NCTMannual. What's something thing you'd want me to be sure to tell/show/share with folks? #mtbos

#NoticeWonder with Textbooks



#NoticeWonder with Textbooks

The Math Forum
PEOPLE LEARNING MATH TOGETHER
Annie Fetter
@MFAnnie
#NoticeWonder

#NoticeWonder with Textbooks

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

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PEOPLE LEARNING MATH TOGETHER
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#NoticeWonder

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Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

Mr. Gavin has a ladder that is 100 centimeters tall.
Ms. Cornell has a ladder that is 2 meters tall.

The Math Forum
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#NoticeWonder

#NoticeWonder with Textbooks

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

Mr. Gavin has a ladder that is 100 centimeters tall.
Ms. Cornell has a ladder that is 2 meters tall.

To make a stained glass window, Robert used 16 pieces of glass. Seven of the pieces were red.

#NoticeWonder with Textbooks

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PEOPLE LEARNING MATH TOGETHER
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#NoticeWonder

#NoticeWonder with Textbooks

Mike had 3 puzzles.
Now he has 5 puzzles.

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#NoticeWonder

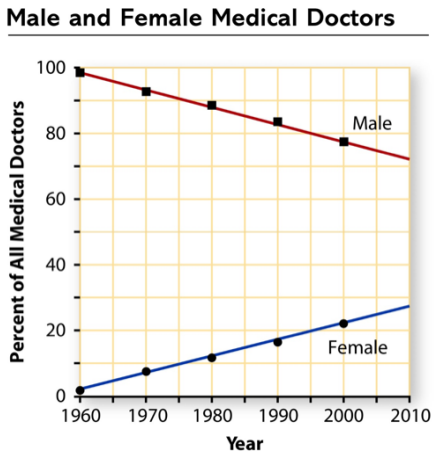
#NoticeWonder with Textbooks

Mike had 3 puzzles.
Now he has 5 puzzles.

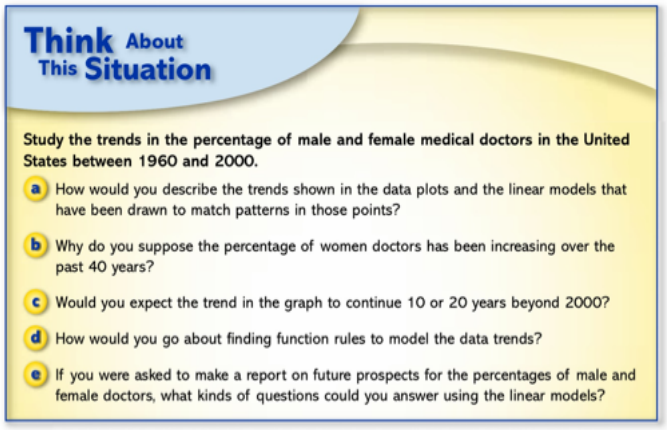
A store has the floor plan shown. The area of the women's department is [illegible]

The Math Forum
PEOPLE LEARNING MATH TOGETHER
Annie Fetter
@MFAnnie
#NoticeWonder

#NoticeWonder with Textbooks



#NoticeWonder with Textbooks



Think About This Situation

Study the trends in the percentage of male and female medical doctors in the United States between 1960 and 2000.

a. How would you describe the trends shown in the data plots and the linear models that have been drawn to match patterns in those points?

b. Why do you suppose the percentage of women doctors has been increasing over the past 40 years?

c. Would you expect the trend in the graph to continue 10 or 20 years beyond 2000?

d. How would you go about finding function rules to model the data trends?

e. If you were asked to make a report on future prospects for the percentages of male and female doctors, what kinds of questions could you answer using the linear models?

Using NW to Figure Out Rules

Math Message Follow-Up

WHOLE-CLASS ACTIVITY

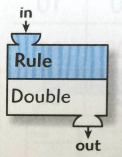
Draw or display a function machine and "What's My Rule?" table. (See Advance Preparation.)

Ask children to imagine that the **function machine** works like this:

- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

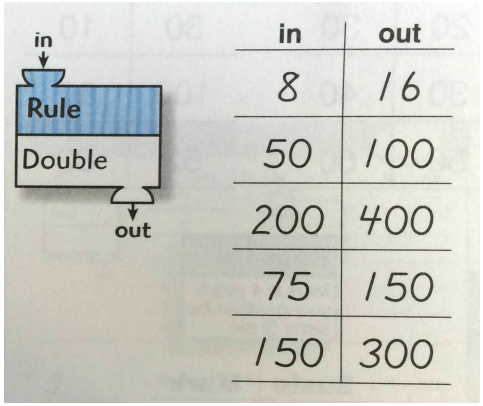
The **rule** for the Math Message problem is "Double the number." Write the word *Double* in the function machine.

Point out the **"What's My Rule?"** table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.



in	out
8	16
50	100
200	400
75	150
150	300

Using NW to Figure Out Rules

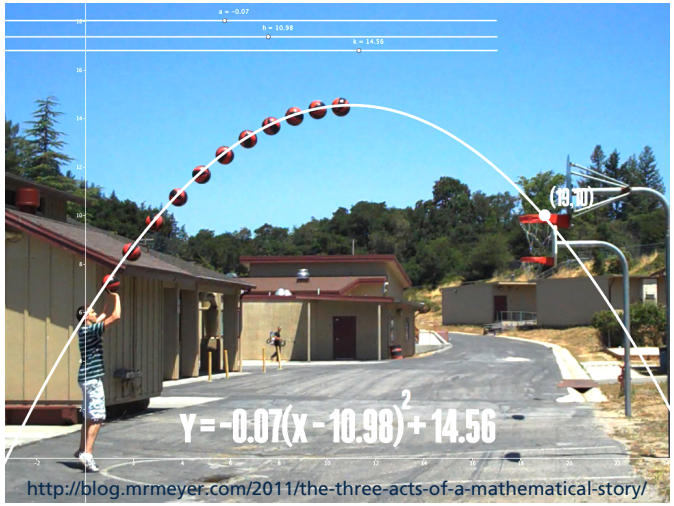


in	out
8	16
50	100
200	400
75	150
150	300

#NoticeWonder & 3-Act Tasks

http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

#NoticeWonder & 3-Act Tasks



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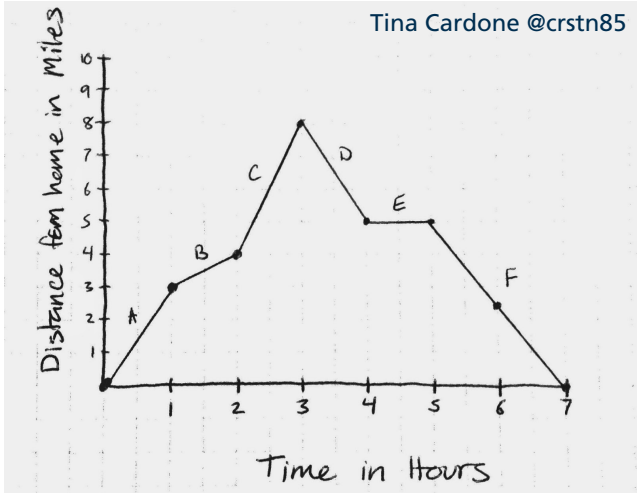
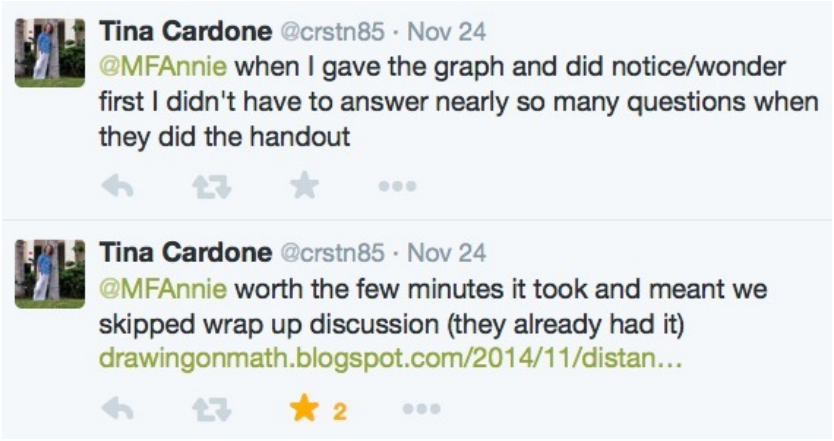
#NoticeWonder & 3-Act Tasks

http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

Doesn't It Take a Lot of Time? Time We Don't Have?

#NoticeWonder as a Launch



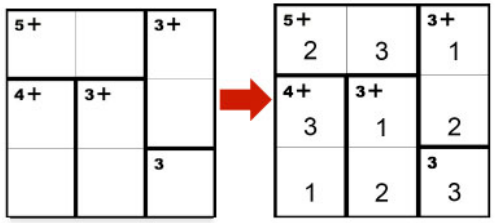


http://drawingonmath.blogspot.com/2014/11/distance-graph.html

Using NW to Figure Out Rules

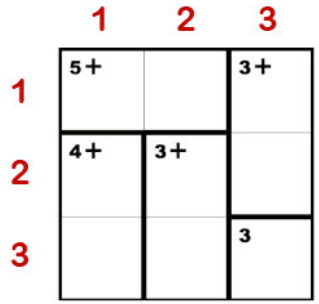
How to Play KenKen®

Your goal is to fill in the whole grid with numbers, making sure no number is repeated in any row or column.



Using NW to Figure Out Rules

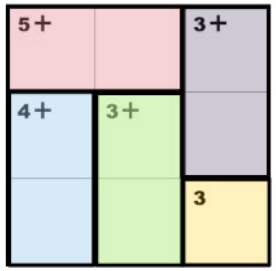
In a 3x3 puzzle, use the numbers 1 – 3.



In a 4x4 puzzle, use the numbers 1 – 4.
In a 5x5, use the numbers 1 – 5, and so on.

Using NW to Figure Out Rules

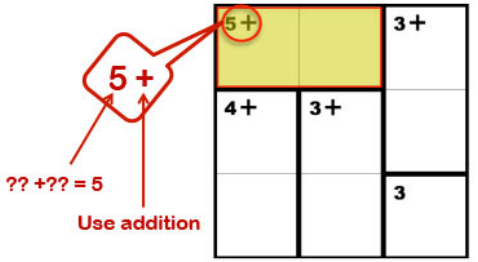
The heavily-outlined areas are called "cages."



This puzzle has 5 cages.

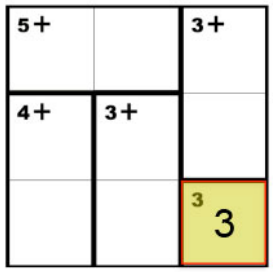
Using NW to Figure Out Rules

In this cage, the math operation to use is addition, and the numbers must add up to 5. Since this cage has 2 squares, the only possibilities are 2 and 3, in either order (2+3 or 3+2 = 5).



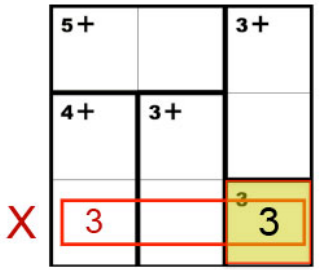
Using NW to Figure Out Rules

A cage with one square is a "Freebie"... just fill in the number you're given.



Using NW to Figure Out Rules

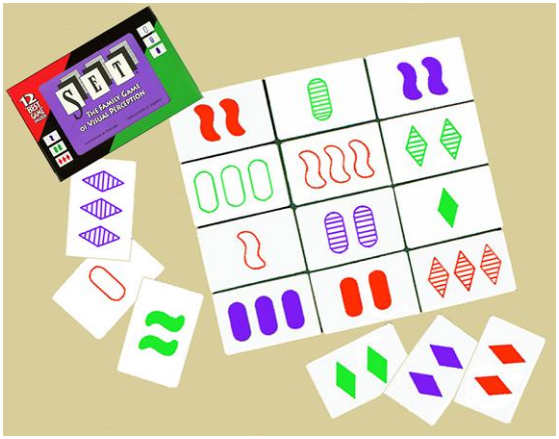
A number cannot be repeated within the same row or column.



Using NW to Figure Out Rules

4+ 1	3	3+ 2	6+ 4
7+ 3	4	1	2
3+ 2	1	7+ 4	3
4 4	6+ 2	3	1

Using NW to Figure Out Rules



Using NW with Naked Problems

20. $3\frac{5}{6} - \left(-\frac{2}{3}\right)$ 21. $\frac{3}{4} - \left(-2\frac{5}{12}\right)$ 22. $1\frac{1}{15} - \left(-\frac{5}{60}\right)$ 23. $-2\frac{1}{8} - 4\frac{1}{4}$

24. $-4\frac{2}{3} - 6\frac{1}{4}$ 25. $-5\frac{1}{2} + 8\frac{2}{3}$ 26. $-7\frac{2}{5} + \left(-\frac{3}{4}\right)$ 27. $7\frac{4}{5} + 11\frac{1}{3}$

How Long Does it Take?

What If It Doesn't Work?

(meaning they don't wonder the thing you want them to)

What If It Doesn't Work?

(meaning they don't wonder the thing you want them to)

Fawn Nguyen
@fawnpnguyen

Replying to @MrsGoytia @MFAnnie

I'd want to honor that they don't. But in a class of 35 Ss, I can't remember a time when they collectively did not.

10:50pm · 4 Apr 2017 · TweetDeck

Student-Led #NoticeWonder

Bradley Smith
@gauchobrad

Replying to @MNmMath @MFAnnie and 3 others

Having Ss bring in their own visuals & lead a #noticewonder empowers Ss & takes it 2 a new level. Our Ts love your work

10:32am · 1 Apr 2017 · Twitter for iPhone

Annie Fetter
@MFAnnie

Replying to @gauchobrad @MNmMath and 3 others

Have seen Ss lead #NoticeWonder - it is amazing. Haven't seen or done it with S-provided visual. Neat!

11:07am · 1 Apr 2017 · TweetDeck

Bradley Smith
@gauchobrad

Replying to @MFAnnie @MNmMath and 3 others

I did it w a MS summer class. Treated it like preschool show and tell. Ss loved it & they got more creative as time went on

11:12am · 1 Apr 2017 · Twitter for iPhone

#NoticeWonder School Culture

Annie Fetter @MFAnnie · 2h
I know I'll hear from @MNmMath @bkdidact @JSchwartz10a @Brandeli1974 as vocal devotees of #NoticeWonder

> **Annie Fetter** @MFAnnie
> Talking #NoticeWonder at #NCSM17 + #NCTMannual. What's something thing you'd want me to be sure to tell/show/share with folks? #mtbos

4 1

Beth Brandenburg
@Brandeli1974

Following

Replying to @MFAnnie @MNmMath and 2 others

there is no better strategy than #NoticeWonder to tackle MP1! We use #noticewonder from PK-5!

10:46 AM - 1 Apr 2017

#NoticeWonder School Culture

#NoticeWonder School Culture

Other Tips from Twitter

Melynee Naegele
@MNmMath

Replying to @MFAnnie @bkdidact and 2 others
#NoticeWonder is for everyone! Given real think time ALL can & do think critically It is life changing for everyone involved. POWERFULSTUFF!

10:22am · 1 Apr 2017 · Twitter for Android
Verdigris, OK, United States

Andrew Gael
@bkdidact

Replying to @MFAnnie @MNmMath and 2 others
#noticewonder creates access for all Ss by focusing on sense-making and not answer-getting. Levels the playing field. Creates ownership!

11:05am · 1 Apr 2017 · Twitter for iPhone

Joe Schwartz
@JSchwartz10a

Replying to @MFAnnie @MNmMath and 2 others
I'd say: Be sure to read Max's book. N/W isn't just an end in itself, it's a means to an end: problem solving/mathematizing @maxmathforum

5:56pm · 1 Apr 2017 · Twitter Web Client

Beth Brandenburg
@Brandeli1974

Replying to @MFAnnie @MNmMath and 2 others
#noticewonder also levels the playing field so that ALL students have an entry point into problems.

Other Tips from Twitter

Amie Albrecht
@nomad_penguin

Replying to @MFAnnie
Non-mathematical #NoticeWonder are part of the process and shouldn't be dismissed.

10:10am · 1 Apr 2017 · Tweetbot for iOS

Christine Newell
@MrsNewell22

Replying to @MFAnnie @MNmMath and 3 others
Honor all noticings/wonderings but discuss mathematical vs. "Other"

2:26am · 2 Apr 2017 · Twitter for Android

Debster
@hartmannd12

Replying to @MFAnnie @HCDSB
Ts are loving #noticewonder and we present again on April 5 to another group. @HCDSB loves #noticewonder for math as well as other subjects!

2:44pm · 1 Apr 2017 · Twitter Web Client

Debster
@hartmannd12

Replying to @MFAnnie
How to use #noticewonder for assessment for, of, as learning? @HCDSB #hcdsbmath

8:43am · 1 Apr 2017 · Twitter for iPad

Other Tips from Twitter

Kristin Gray
@MathMinds

Replying to @bkdidact @MFAnnie and 3 others

Just as powerful in teaching students as it is when working w/teachers.

5:00pm · 1 Apr 2017 · Twitter for iPhone

Trish Kepler
@KeplerTrish

Replying to @MathMinds @MFAnnie and 4 others

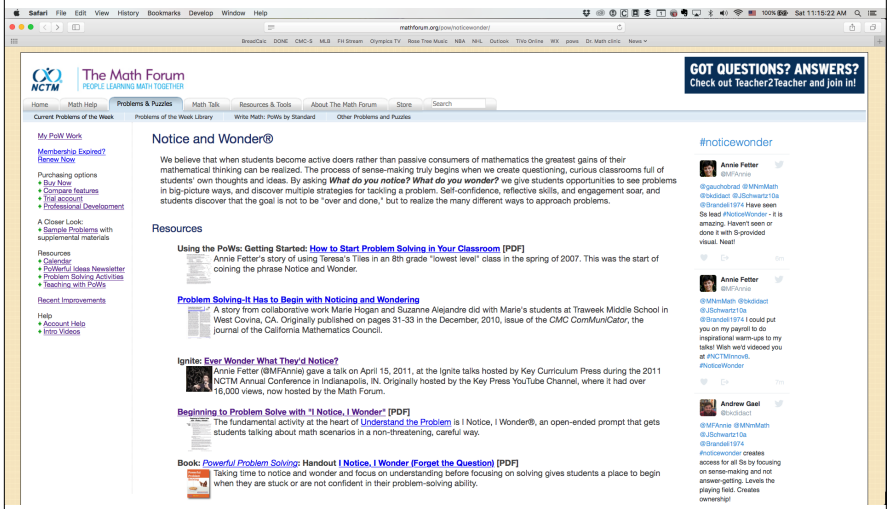
Excellent point - love opening with #noticewonder with Ts!

7:03am · 2 Apr 2017 · Twitter for iPhone

How & Where to Keep Learning

Twitter (duh)

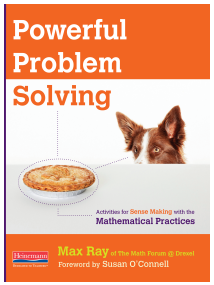
More Resources - mathforum.org



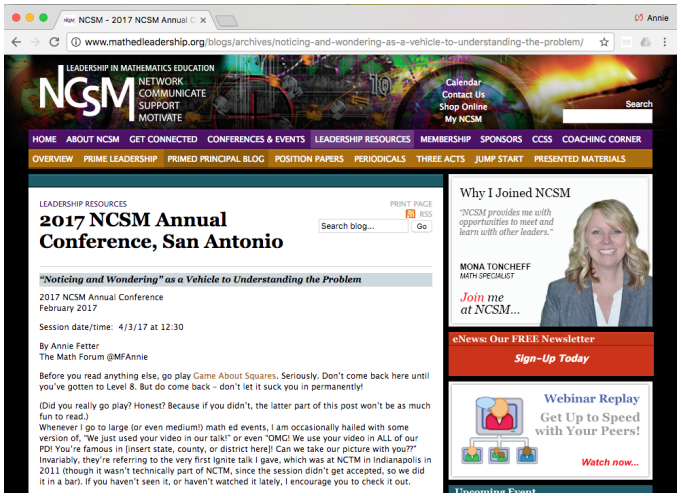
More Resources - Ignite Video

More Resources – Book + Videos

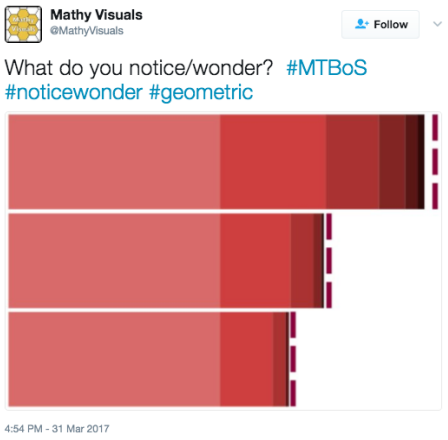
- *Powerful Problem Solving*, by Max Ray-Riek
- Videos of grades 3-8 doing Notice and Wonder (including Teresa's Tiles and Growing Worms) from http://mathforum.org/pps/



More Resources – Conf Web Site



More Resources - #NoticeWonder



More Resources - #NoticeWonder

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Next Steps

When could you use a "scenario" next week?

Where can you see this fitting in right away?

Next Steps

When could you use a "scenario" next week?

Where can you see this fitting in right away?

How could you launch this in your classroom? in your school?

Thanks!

Annie Fetter, The Math Forum at NCTM

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