

Breathing Classroom Life into the 8 Mathematics Teaching Practices

Steve Leinwand

American Institutes for Research

NCTM San Antonio April 6, 2017

sleinwand@air.org www.steveleinwand.com

Good Morning!

Let's define our terms:

Principles to Actions:

Ensuring Mathematical Success for All

Check out www.nctm.org/ptatoolkit



Principles to Actions:

Ensuring Mathematical Success for All

- **Teaching and Learning**
- **Access and Equity**
- **Curriculum**
- **Tools and Technology**
- **Assessment**
- **Professionalism**

Principles to Actions:

Ensuring Mathematical Success for All

Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Breathing life into all of these words....

Let us play!

TASKS

QUESTIONS

DISCOURSE

What do you notice?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3



© Can Stock Photo - csp6983401

What's the question? What's the answer?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3



© Can Stock Photo - csp6983401

Will you make it?

**Omaha
244 miles**

**Car Manual
Gas Tank Capacity:
21 gallons**

**Average MPG:
28.3**



© Can Stock Photo - csp6983401

About how far will you get?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3



© Can Stock Photo - csp6983401

How much gas needs to be in the tank to be sure you make it?

**Omaha
244 miles**

**Car Manual
Gas Tank Capacity:
21 gallons**

**Average MPG:
28.3**



© Can Stock Photo - csp6983401

Turn and Talk: What made that experience effective?

The Big Picture

Our collective LYNCHPIN:

**Daily classroom instruction
strengthened with collaborative
structures and coaching.**

Shapes

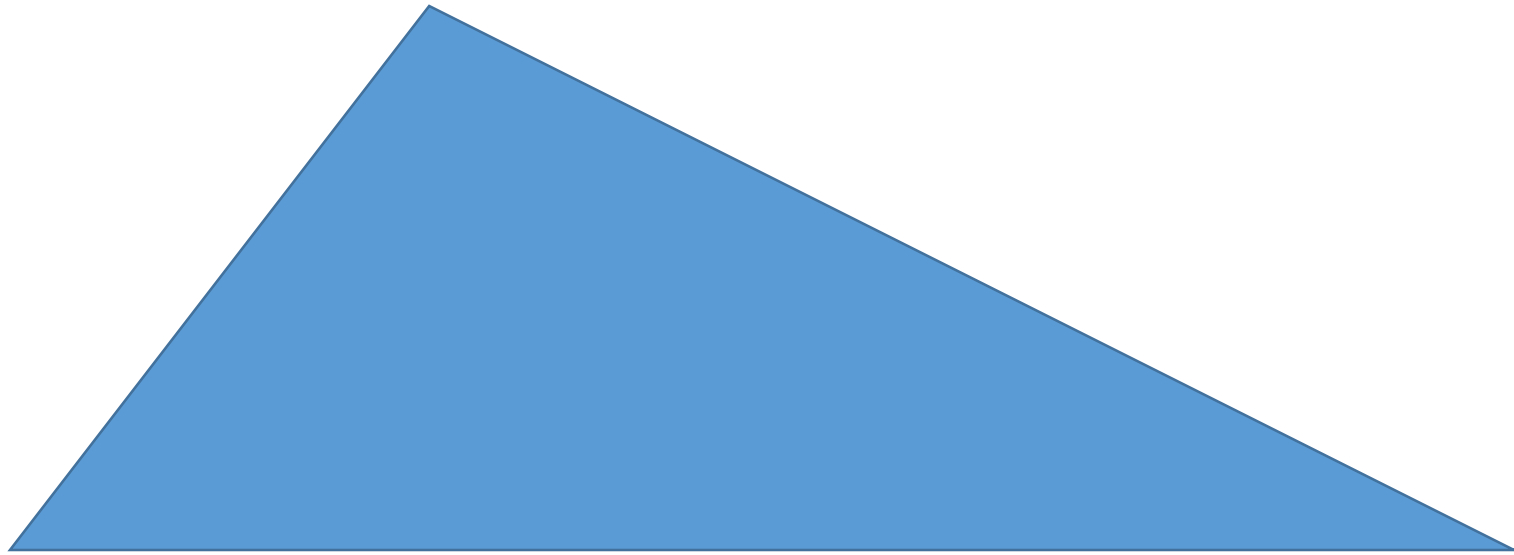
QUESTIONS DISCOURSE PRODUCTIVE STRUGGLE

LEARNING GOAL: Identify Shapes

VS.

**Recognize and make conjectures about the
characteristics of shapes**

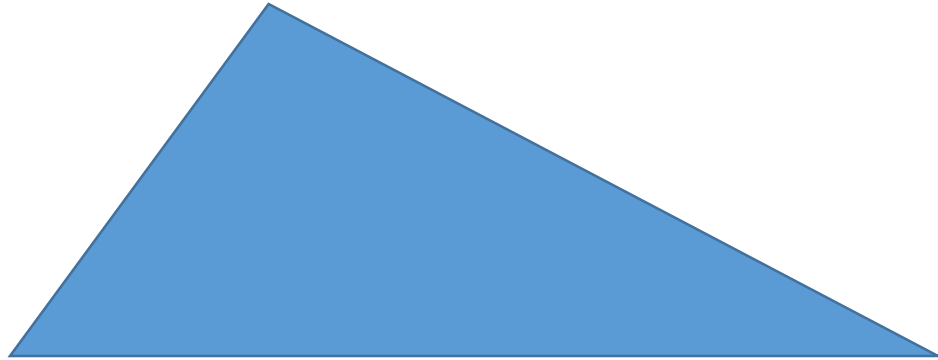
What do you see?



What do you see?



Versus



Identify three things you see.

Convince us.

On your white boards, A triangle is:

Compare to google/wikipedia

What is a triangle? Let's see how close we came.

- a plane figure with three straight sides and three angles.

"an equilateral triangle"

- a thing shaped like a triangle.

"a small triangle of grass"

- a situation involving three people or things, especially an emotional relationship involving a couple and a third person with whom one of them is involved.
- noun: **eternal triangle**; plural noun: **eternal triangles**

Sarah and her Apples

QUESTIONS

DISCOURSE

REPRESENTATIONS

OR

TURNING LEMONS INTO LEMONADE

Mindless and of Limited Value

Sarah has 91 empty boxes. She had 2605 apples to pack into the boxes. How many apples will Sarah put in each box?

Versus:

Adapting what the text bestows

Sarah has 91 empty boxes.

- What can you infer about Sarah?

She had 2605 apples to pack into the boxes.

- Now what can you say about Sarah?
- What do you think the question is?
- So what does the 91 tell us, what about the 2605?
- About how many apples do you think would be in each box? More than 100? Less than 100? Convince us
- Can you draw a picture?
- Can you create a number sentence?
- Why did you divide and not multiply?
- So about how many apples would be in each box if...

Care to join me in 6th grade?

Grade 6 SA Harlem Central Tues Dec 8, 2015

- Lesson 6 in the Expressions unit (6.EE standards)
- Ally and Mabubar co-teaching
- 19 Scholars
- Driven by a number strings mini-lesson, a Math Workshop task and an exit ticket
- “Our goal for today is to “identify, create and understand equivalent expressions.”
- “Zayasia, can you please repeat our learning goal?”
- “Let’s begin with our number strings.”

Number strings for today's Mini Lesson

Are they equivalent? How do you know?

1. $4(8) = 4(3 + 5)$

2. $4(8) = 4(a + 5)$

3. $4(8) = 4(3 + b)$

4. $3x + 3y = 3(x + y)$

Let's summarize: For each: Always, sometimes, never equivalent?

Math Workshop Task

Jan normally rides her bike to and from work.

Her normal route is 18 miles from home to work.

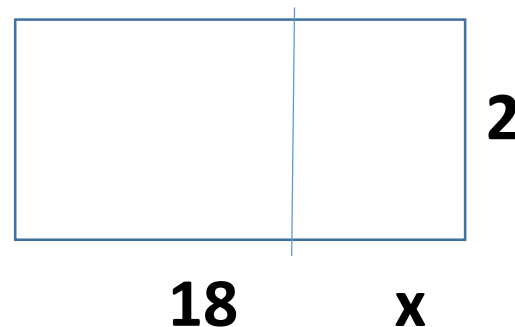
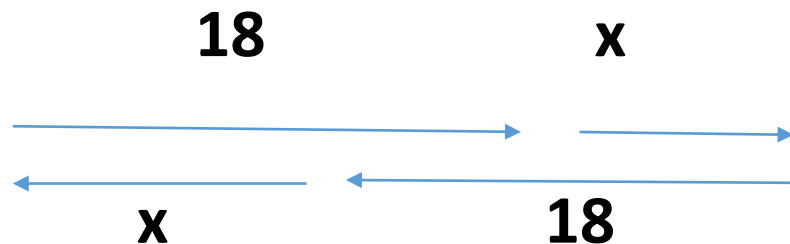
One day she goes to a coffee shop on her way to work and on her way home.

This adds x miles to her trip each way.

(“What do you notice?” “What’s the question?”)

Great: Write and show the distance Jan travels using a diagram or picture and two different, but equivalent, expressions.

Resulting in:



- $18 + x + 18 + x$
- $2(18 + x)$
- $2(x + 18)$
- $x + x + 18 + 18$
- $36 + 2x$

“Is everyone correct? Turn and tell your partner why?”

“What do the numbers and variables represent?”

“Which expression is simplest or easiest to use? Why?”

Lesson 6 Exit Ticket

Which of the following represent equivalent expressions?

Explain or show your process of determining which expressions ARE equivalent.

Select all that apply:

a. $x + x + x + x = 4x$

b. $15y + 5x = 3(5y + x)$

c. $6(2 + x) = 12 + 6$

d. $3(x + y) = 3x + y$

Your turn: One by one, where did each of these live in this lesson?

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

Establish mathematics goals to focus learning.

A verbatim common core standard vs.:

- A specific learning goal;
- An I-can statement;
- I understand that....statements
- NOT SWBAT;
- What is posted may not be what we put in our planning documents.

How do you determine lesson learning goals?

How do you balance skills, concepts and applications?

**Implement tasks that promote reasoning
and problem solving.**

**Find them, steal them, adapt them, revise them,
reflect on them**

Why are “tasks” so important?

- Tasks represent the meat of instruction.
- They are how we engage students and support the development of mathematical understanding.
- They connect one’s learning goals to the actual classroom opportunity to learn.
- Exercises are opportunities to use procedures to get answers to simple problems (they have value) BUT tasks are opportunities to develop deeper and broader understanding and application of mathematics.

How do we find good tasks?

Great On-line Math Resources

Learn Zillion: www.learnzillion.com

Inside Mathematics: www.insidemathematics.org

Illustrative Mathematics: www.illustrativemathematics.org

NCTM Illuminations: <http://illuminations.nctm.org>

Balanced Assessment: <http://balancedassessment.concord.org>

Mathalicious: <http://www.mathalicious.com>

Dan Meyer's three act lessons:

<https://docs.google.com/spreadsheet/cc?key=0AjlqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE>

Thinking blocks: <http://www.thinkingblocks.com>

Math Assessment Project <http://map.mathshell.org/materials/index.php>

Yummy Math: www.yummymath.com

See www.steveleinwand.com/publications

(publications; great online resources)

And best of all:

Emergent Math: <http://emergentmath.com/my-problem-based-curriculum-maps>

How do we decide if a task is good?

- Accessible** – clear and has multiple entry points
- Fair** – all students are able to complete it
- Reasonable** – not too complex, familiar context, not too arcane
- Aligned** – matches standards and current learning goals
- Comprehensive** – integrates key understandings
– big enough bang for the time
- Engaging** – uses graphics
– has an intriguing or familiar context
- Divergent** – provides multiple pathways to solve

**Use and connect mathematical
representations.**

The power of models and representations

Siti packs her clothes into a suitcase and it weighs 29 kg.

Rahim packs his clothes into an identical suitcase and it weighs 11 kg.

Siti's clothes are three times as heavy as Rahims.

What is the mass of Rahim's clothes?

What is the mass of the suitcase?

The old (only) way:

Let S = the weight of Siti's clothes

Let R = the weight of Rahim's clothes

Let X = the weight of the suitcase

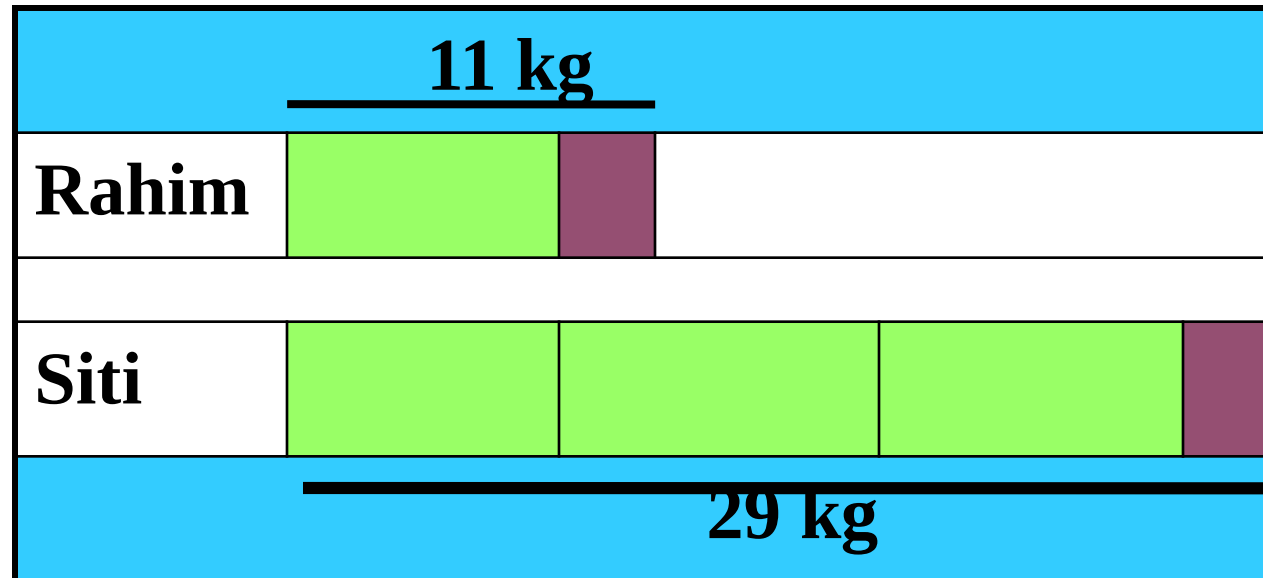
$$S = 3R \quad S + X = 29 \quad R + X = 11$$

so by substitution: $3R + X = 29$

and by subtraction: $2R = 18$

so $R = 9$ and $X = 2$

Or using a model:



Remember How

$$5 + (-9)$$

“To find the difference of two integers, subtract the absolute value of the two integers and then assign the sign of the integer with the greatest absolute value”

Understand Why

$$5 + (-9)$$

- Have \$5, lost \$9
- Gained 5 yards, lost 9
- 5 degrees above zero, gets 9 degrees colder
- Decompose $5 + (-5 + -4)$
- Zero pairs: $x\ x\ x\ x\ x$ $o\ o\ o\ o\ o\ o\ o\ o\ o$
- On number line, start at 5 and move 9 to the left

**Facilitate meaningful mathematical
discourse.**



Classroom Posters that Drive Discourse

- **Why?**
- **How do you know?**
- **Can you explain that? (and who did it differently?)**
- **Convince us.**
- **How did you picture that? (and who did it differently?)**
- **What do you notice?**
- **What do you wonder?**
- **How are they the same?**
- **How are they different?**

Pose purposeful questions.

So:

Answer getting

Vs.

**Explanations, alternatives,
connections**

**And look at the power of our
questions!**

**Build procedural fluency from
conceptual understanding.**

Ready??

What is $8 + 9$?

17 Bing Bang Done!

Vs.

Convince me that $9 + 8 = 17$.

Hmmmm....

$$8 + 9 =$$

17 – know it cold

10 + 7 – add 1 to 9, subtract 1 from 8

7 + 1 + 9 – decompose the 8 into 7 and 1

18 – 1 – add 10 and adjust

16 + 1 – double plus 1

20 – 3 – round up and adjust

Who's right? Does it matter?

Convince us that 7×6 is 42.

What is $1/10$ of 450?

Convince us that $1/10$ of 450 is 45.

Support productive struggle in learning mathematics.

Ready!

At Central Middle School, there are 3 scholars in the Science Club for every 8 scholars in the Math Club.

If there are a total of 45 scholars in one or both of these clubs, how many scholars are in both clubs?

- Math $8x$ Science $3x$ Total: $11x$ $11 \times 5 = 55$, so 10 in both
 Ergo:

<u>Math</u>	<u>Science</u>	<u>(Both)</u>	<u>Total (45)</u>
- 8	3		11?
- 16	6		22?
24	9		33?
32	12		44?
40	15	10	55

Elicit and use evidence of student thinking.

How difficult is it to appoint two “5-minutes-left time monitors”?

With 5 minutes to go:

- turn and tell your partner what you learned today**
- quick “tweets” to share**
- Ok, let’s see. Turn your exit slip over and complete the task**

**Four examples of putting it all together
and giving us fodder for further
discussion**

Design Questions

- 1. What are you trying to accomplish in this lesson? OK frame your opening lesson goal slide.**
- 2. What would students be able to do to convince us the lesson worked. OK build your exit ticket slide.**
- 3. What tasks, examples, problems, activities will students be engaged in to maximize the chance they will successfully meet the lesson goals and complete the exit ticket? OK craft your core lesson slides.**
- 4. What pre-requisite understandings are likely to be essential for the tasks, examples, etc. to work? OK craft your warm-up slides.**

Collaboratively developing 6- to 8-slide lesson guides

- **Objective or Lesson Goal slide student friendly with underlining to focus on understanding of vocabulary and goal**
- **Warm-up slide(s) – to get the blood flowing and focus on key prerequisite understandings**
- **Tasks with Questions and all necessary graphics – so others can use without additional support**
- **WDYLT**
- **Exit Slip**

Nido Grade 1B

March 3, 2017

**Today we are going to solve
problems using addition and
subtraction.**

Let's see what we already know.

On your white boards, number for 1 to 3.

1. How many tens and how many ones are in 17?

2. $3 + 5 =$ _____

3. About how many pens tall am I? _____

Task #1

Fruit Sale

Apples 13 pesos

Pears 7 pesos

- **What do you notice about the Fruit Sale?**
- **Sammy wants one apple and one pear.**
- **How much more does an apple cost than a pear?**
- **Show how you know?**

Task 2 – return to your desks and get a blank piece of paper.

Salad for the Festival

Carrots	5
Tomatoes	17
Cucumbers	9

Can you tell us three different things about our Festival Salad.

How many carrots and cucumbers do we need? (do we add or subtract and why)

How many more tomatoes than cucumbers do we need? (do we add or subtract and why)

If this is pretty easy: how many vegetables do you buy in all. Convince me.

Great work today Grade 1B

Turn and tell your partner what you learned today.

Let's see if we really learned it

Fruit Sale

Apples 11 pesos

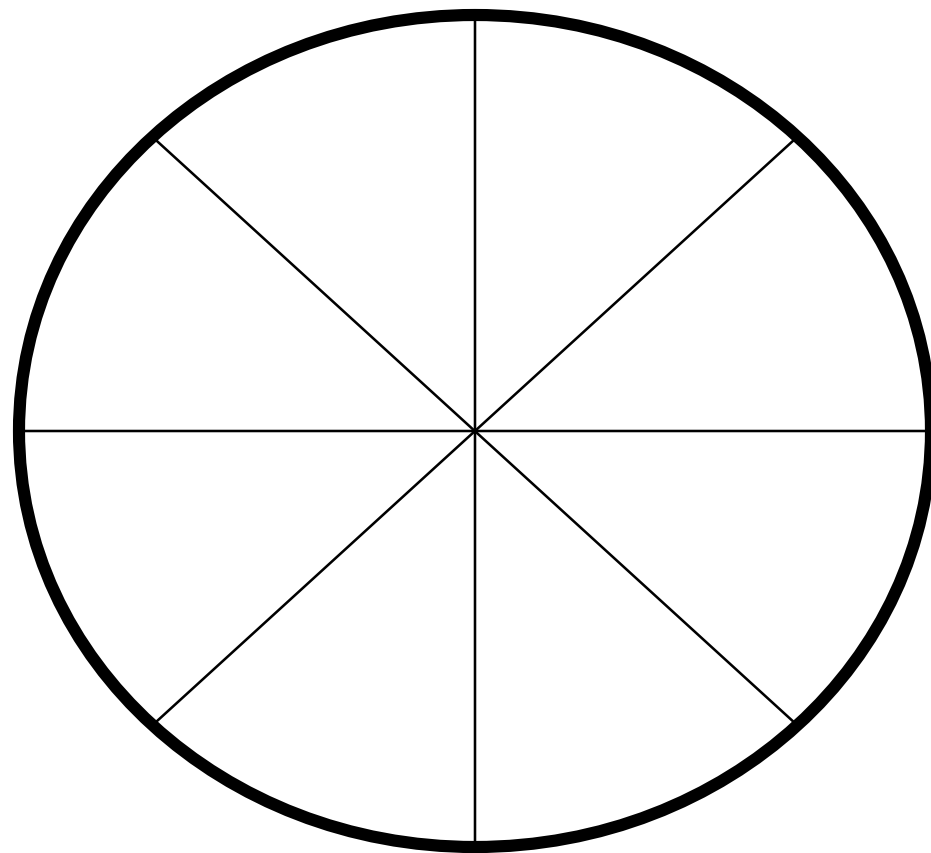
Pears 6 pesos

1. How much will it cost for one apple and one pear?
2. How much more does an apple cost than a pear?

Grade 3

Learning Goal: Solidify our understanding of fractions, including equivalent fractions, ordering fractions, and being sure we don't fall victim to common misconceptions.

In your groups, write down 5 things you see



Tim and Sari

Tim eats $\frac{1}{2}$ of a pizza.

Sari eats $\frac{2}{4}$ of a pizza that is the same size.

Who eats more pizza? How do you know? Can you use a drawing to convince us?

Tim and Sari 2

Tim eats $\frac{1}{4}$ of a pizza.

Sari eats $\frac{1}{5}$ of a pizza that is the same size.

Who eats more pizza? How do you know? Can you use a drawing to convince us?

Tim and Sari 3

Tim and Sari each eat $\frac{1}{2}$ of a pizza.

Tim says they both ate the same amount.

Sari says she ate more than Tim.

Show with pizza drawings for Tim and Sari that both can be right!!!

Tim and Sari 4

Tim eats $\frac{3}{4}$ of a pizza. Show this.

Sari's pizza has 8 slices and is the same size of Tim's. Show this.

How many slices does Sari need to eat so that she and Tim eat the same amount?

Show with pizza drawings how you know.

Math 7

ZIS – March 13, 2017

Today we have a guest teacher: Mr. Steve

**Our goal today is to extend our
understanding of numerical expressions to
algebraic expressions**

Getting the blood back into our brains

- 1. What is 8×500 ?**
- 2. There are 4 boys for every 3 girls in the 6th grade.
There are more than 50 6th graders. How many boys and how many girls could there be?**
- 3. What is the length of the segment that connects $(-4, 9)$ and $(7, 9)$?**

Warm-up

1. Take 6 away from 5 times 3

- Answers?**
- How did you get that?**
- What's the equivalent numerical expression?**

Warm-up

2. Add two times one-third and one-third

- Answer?**
- Did anyone get another answer?**
- How did you get that?**
- What's the equivalent numerical expression?**

What's the difference?

a) The difference of 2 times 10 and 8

b) The difference of 2 times a number and 8

What's the difference?

a) $3(2 + 5)$

b) $3(x + 5)$

Numerical and Algebraic Expressions

Turn and tell your partner what is the difference between a numeric expression and an algebraic expression.

Time for a Card Match

Find a partner.

Cut out the rectangles

Match.

Which two expressions have no match?

Reverse aka Inverse

17 fewer than 3 times a number is the same as _____

$3(x + 5)$ is the same as _____

Your turn: Convert to words:

$$x + 2$$

$$3x + 2$$

$$3(x + 2)$$

How many equivalent sentences can we create?

$$4x - 12$$

Why do we need to know this?

Amazing!!!

My son is eight years fewer than $\frac{1}{2}$ my age.

What would you like to know?

OK: Answer the question.

Nice work!

**Turn and tell your partner what you
learned today.**

Exit Ticket

- 1. Convert to an algebraic expression:**
Two times the difference of 7 and a number
- 2. Convert to an English sentence:**
 $5x - 19$
- 3. Convert to an English sentence: $25 - 6M$**



Tell your partner three things you see here.



What is your guess? Share your guess with your neighbor and justify your guess.



What information is important here?
How would you get it?

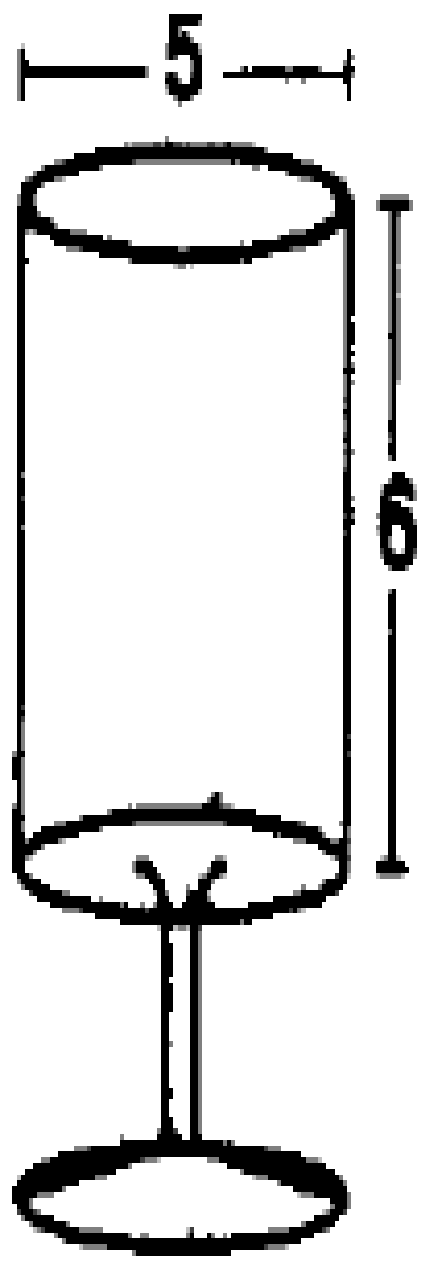




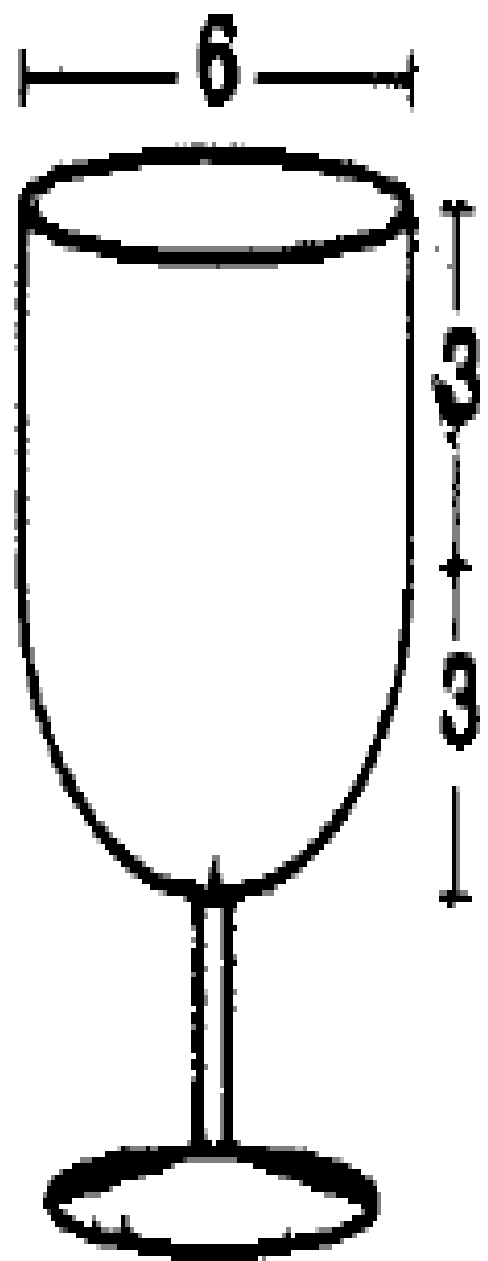
Was anything learned?

- **Now draw two glasses with different diameters and show the heights of equal amounts of liquid. Explain your reasoning.**
- **And on the unit test.....**

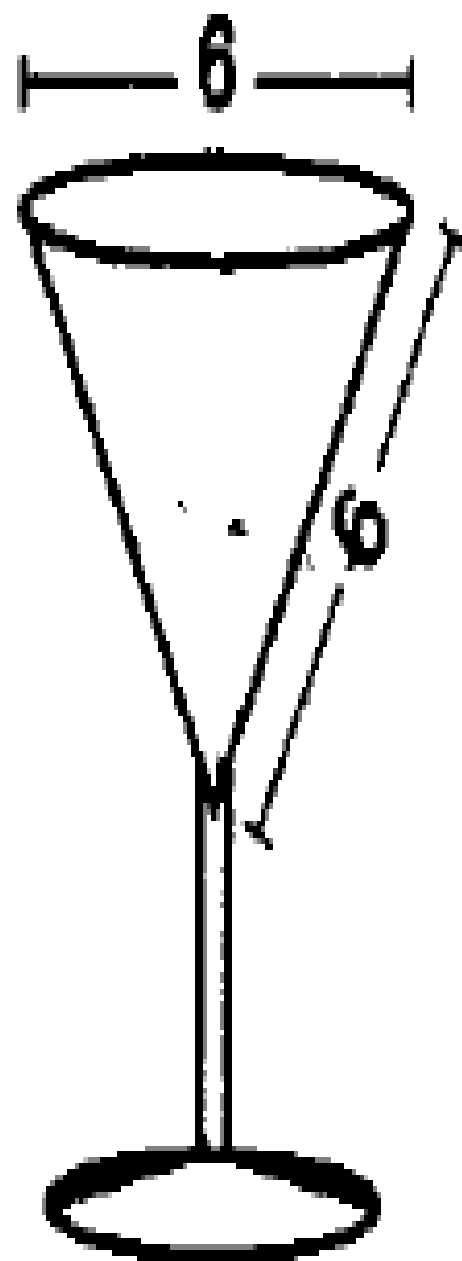
Your turn



1



2



3

So?

- **Order from smallest to largest and justify**
- **What is the height of Glass 3?**
- **What is the volume of each?**
- **If Glass 1 has volume V , express volume of Glasses 2 and 3 in terms of V**
- **When Glass 1 is $\frac{1}{2}$ full, the height of the liquid is 3 cm. What are the heights of the liquid in Glasses 2 and 3 when they are $\frac{1}{2}$ full?**

A Template for all Lesson Planning and Implementation

Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

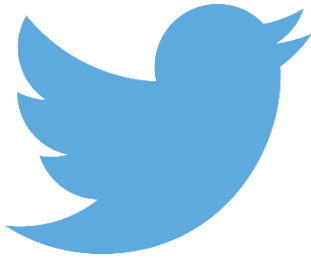
Thank you!

**Go forth and make effective use of
the 8 MTPs.**

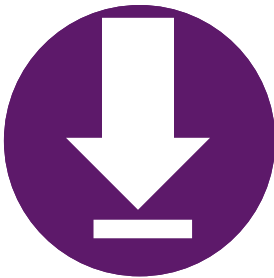


Rate this presentation on the conference app!

Search “NCTM” in your app store or follow the link at nctm.org/confapp to download



Join in the conversation! [#NCTMannual](https://twitter.com/NCTMannual)



Download available presentation handouts from the online planner at nctm.org/planner