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Session 574

Supporting Productive Struggle in Mathematics Classrooms

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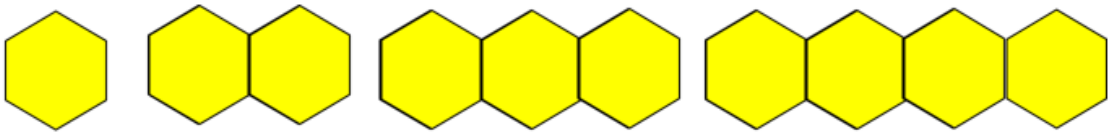
Handout Packet

Hexagon Pattern Trains

Trains 1, 2, 3 and 4 (shown below) are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



Train 1 Train 2 Train 3 Train 4

1. Compute the perimeter for each of the first four trains.

2. Draw the fifth train and compute the perimeter of the train.

3. Determine the perimeter of the 10th train without constructing it.

4. Write a description that could be used to compute the perimeter of any train in the pattern and explain why it works.

5. Determine which train has a perimeter of 110.

Ms. Rossman's Class

Student 1: Twenty-two plus 4 is 26; 26 plus 4 is 30; and 30 plus 4 is 34, 34 plus 4 is 38; and 38 plus 4 is 42.
Teacher: Okay. So you're telling me you saw a pattern here in the numbers?
Student 1: Yeah.
Teacher: Well, how could you find the perimeter of the tenth train if you didn't have this information? Would there be another way to find the perimeter of the train? Like you're telling me that this perimeter is four more (*points to the fourth train*) than this one (*points to the third train*). What's another way to find the perimeter if you don't know this?
Student 1: The-we start with one, and we know that's six, and then we put a two in (inaudible) and then we think that kinda we can get it. (*Student pointing to the hexagon*.)
Teacher: Why do you think it is that you add four from the picture?
Student 1: Because right here, we count six, and then we count like this, all the way, and then we – he said that count by four, and you get all the answers.

Teacher: I'm wondering where this thing that you're talking about, the four all the time, where is the four in the picture?
Student 1: Right here. One, two, three-
[crosstalk]
Teacher: Like this is – this is (*points to the third train*) four more than this one (*points to the second train*), right?
Student: Yes.
Teacher: But where in the picture is it four more than this one?
Student 2: In the middle?
Teacher: What do you mean in the middle? What do you see?
Student 1: Oh, yeah, because right here, when that is-
Student 2: Right here is five (*points to the hexagons at the beginning and end of a train*), and right here is four (*point to sides on the middle hexagons*)
Student 2: Five, and then five, then four, four, five…(points to the sides of a hexagon)
Student 2: Because we have to put in another one right here, and this one has got to be one, two, three, four.
Teacher: Ah. What do you think?
Student 1: Yeah. He is right.
Teacher: What does he mean, where' the four in the picture?
Student 1: That because if we-
Teacher: How much is on this one, on the end?
Students: Five.
Teacher: How much on this one?
Students: Five.
Teacher: How much here?
Students: Four.
Students: Four.
Teacher: So how could you think about that for the tenth one?
Student 1: The-
Teacher: Can you imagine in your mind what it looks like?
Students: Yes.
Student 2: Yeah. No, no, no, no.
Student 3: The first and the last one is going to be –
Student 3: Five
[crosstalk]
Student 3: And the other one is going to be four.
[crosstalk]
Student: In the middle.
Teacher: You should write about tenth train, because that's what it say to do here. Without building the tenth grain, write about how you find the perimeter. Can you write that?
Student: Yeah.
Student: Yes.
Teacher: Good. Go ahead.

These materials appear in: Smith, Margaret, Michael D. Steele, and Mary Lynn Raith. *Taking Action: Implementing the Effective Mathematics Teaching Practices Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics.

Deep Dark Secret

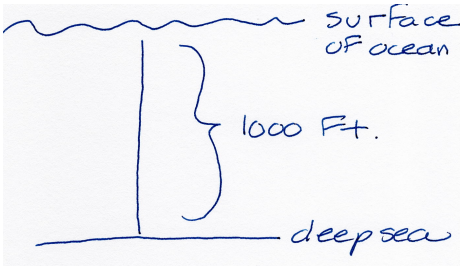
The deep sea is the lowest level of the ocean floor. Sunlight does not reach the deep sea, and scientists are discovering that it is home to amazing creatures that live in total darkness. Deep-sea creatures live more than 1000 feet below the water's surface. Marine biologists in an underwater vessel are descending to study a new species of fish that was discovered in the deep sea. Their vessel is currently located 100 feet below the surface (-100 feet). From this location, the biologists start their timer at t = 0 and begin their descent. They descend at a rate of 25 feet per minute. The vessel continues descending at this constant rate until it has reached a depth where they can study the deep sea creatures.

1. After how many minutes will the vessel be positioned to observe deep-sea creatures? Explain how you know.
2. Write an inequality to represent the depth of the vessel at any point in time, t, after the vessel has reached a depth at which the marine biologists can observe deep sea creatures, where t represents the number of minutes since the vessel has left its original position of -100 feet.

Adapted from Seventh Grade Set of Related Lessons: Investigating Inequalities, Institute for Learning, University of Pittsburgh.

Ms. Kaufmann's Class

Ms. K: So how long do you think it will take before they will see the creatures? Kaila?
Kaila: We think it would take 40 minutes.
Ms. K: It will take 40 minutes to do what? Jake?
Jake: To get to the deep sea.
Ms. K: How did you get 40? Chris?
Chris: We divided 1000 by 25.
Ms. K: Why did you do that? Jerome?
Jerome: It says that deep-sea creatures live more than 1000 feet below the surface. So if the vessel goes down 25 feet per minute, you just divide 1000 by 25.
Ms. K: Can you draw a picture of the situation to help explain what you did? I will be back in a few minutes to see what you came up with.
[Teacher leaves to respond to other students and returns five minutes later. Members of the group work discuss what the picture should look like. Kaila offers to draw.]
Ms. K: So, tell me about your picture.
Kaila: Okay, see *[referring to the drawing below]*, we started with the surface of the ocean and then showed where the deep sea started, which is 1000 feet below the surface.



Ms. K: So, how does this picture explain what you did? Jerome?

Jerome: The vessel kept going down 25 feet every minute. So, you need to find out how many 25's it takes to get to 1000. That will tell you how many minutes it would take to get to 1000. So, it takes 40 minutes, just like Kaila said.

Ms. K: Okay. So, what else do we know about the situation? Jake, can you reread the problem?

[Jake reads the problem aloud and stops when he reaches the part that says "The vessels starts 100 feet below the surface."]

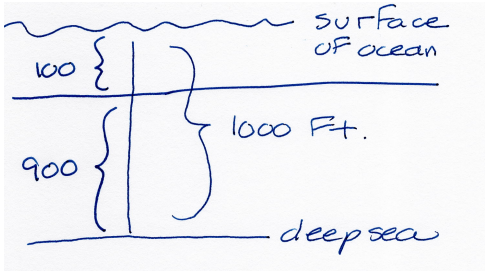
Chris: Oh wait…our drawing isn't right…the vessel starts 100 feet below the surface. We need to fix our picture.

Ms. K: Does everyone see what Chris is saying?

Jake: Yeah. The vessel was already 100 feet below the surface when they decided to go see the sea creatures.

Jerome: Okay, so that means that they only went down 900 feet, not 1000.

Kaila: *[Quickly making a change, as in the sketch shown below.]* Now this shows what Jerome just said.



Ms. K: So does the starting point make a difference? Chris?

Chris: Yeah, because like Jerome said, they only went down 900 feet from where they started, not 1000 feet.

Ms. K: So, what does the drawing suggest to you about your initial approach to the problem?

Jake: So, it should be 900 divided by 25. *[Jerome quickly divides 900 by 25].*

Jerome: Which is 36.

Ms. K: Okay, so what does 36 mean in this problem? Kaila?

Kaila: It is the number of minutes it took to get to deep sea.

Ms. K: Does everyone agree?

Chis: I think it takes 36 to get to 1000 feet but you need to get deeper than that to actually see sea creatures because the creatures are more than 1000 feet below the surface.

Ms. K: So how long will that take?

Kaila: More than 36 minutes.

Ms. K: Does everyone agree?

Students: Yeah!

Ms. K: I liked the way you all worked together on this. I knew you could figure it out! Now you are ready to take on the second part of the task. I will check in with you a little later.

These materials appear in: Smith, Margaret, Michael D. Steele, and Mary Lynn Raith. *Taking Action: Implementing the Effective Mathematics Teaching Practices Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics.

Ducklings

The local nature club is carrying out a survey of the number of ducklings in each family of ducks in the lake. Here are the results of the survey:

4, 7, 6, 5, 8, 7, 5, 4, 10, 4, 9, 6, 5, 4, 4, 5, 9, 8, 4

How many ducks are in a typical family? Using tables, graphs, and/or arithmetic to justify your answer.

Adapted from http://www.insidemathematics.org/assets/common-core-math-tasks/ducklings.pdf (Noyce Foundation, 2012)

This task appears in: Smith, Margaret, Michael D. Steele, and Mary Lynn Raith. *Taking Action: Implementing the Effective Mathematics Teaching Practices Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics.