An International Perspective on Modeling: Implications for What We Teach

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Essential Questions

1. How is modeling defined?
2. How do teachers in other countries engage their students in modeling?
3. In what ways do pre-service teacher preparation programs and professional development prepare teachers to work with their students on modeling tasks?

TSG 21: Mathematical applications and modelling in the teaching and learning of mathematics

Modeling is…

Quick Chat

- How do you define Modeling?
- How do you use Modeling in your classroom?

Modelling Task

Pizza Task:
How much water is needed to make Pizza?

Group 1 : Camping

"How much water do we need to bring with us camping?"

Water

Dough
Sauce
Wash Hands
Wash Vegetables

Answer: Number of Gallons

Group 2: Home Cooks

"How much water is needed to make dough for any number of pizzas?"

Water

Dough

Answer: Proportion

Group 3: Water Conservation

"How much water is used to make/grow the ingredients for pizza?"

Water

Flour
Cheese
Vegetables

Answer: Range of Gallons

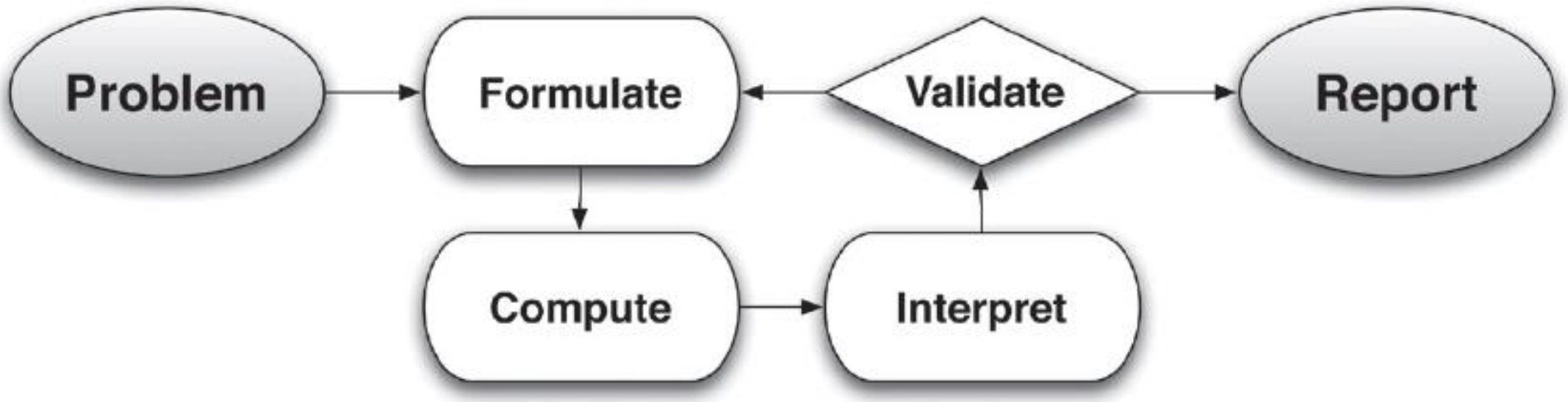
Modeling is…

USA - Common Core

- **SMP 4: Model with mathematics. -** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- **Domain**- Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions.

USA - Diagram of Basic Modeling Cycle



Modeling is…

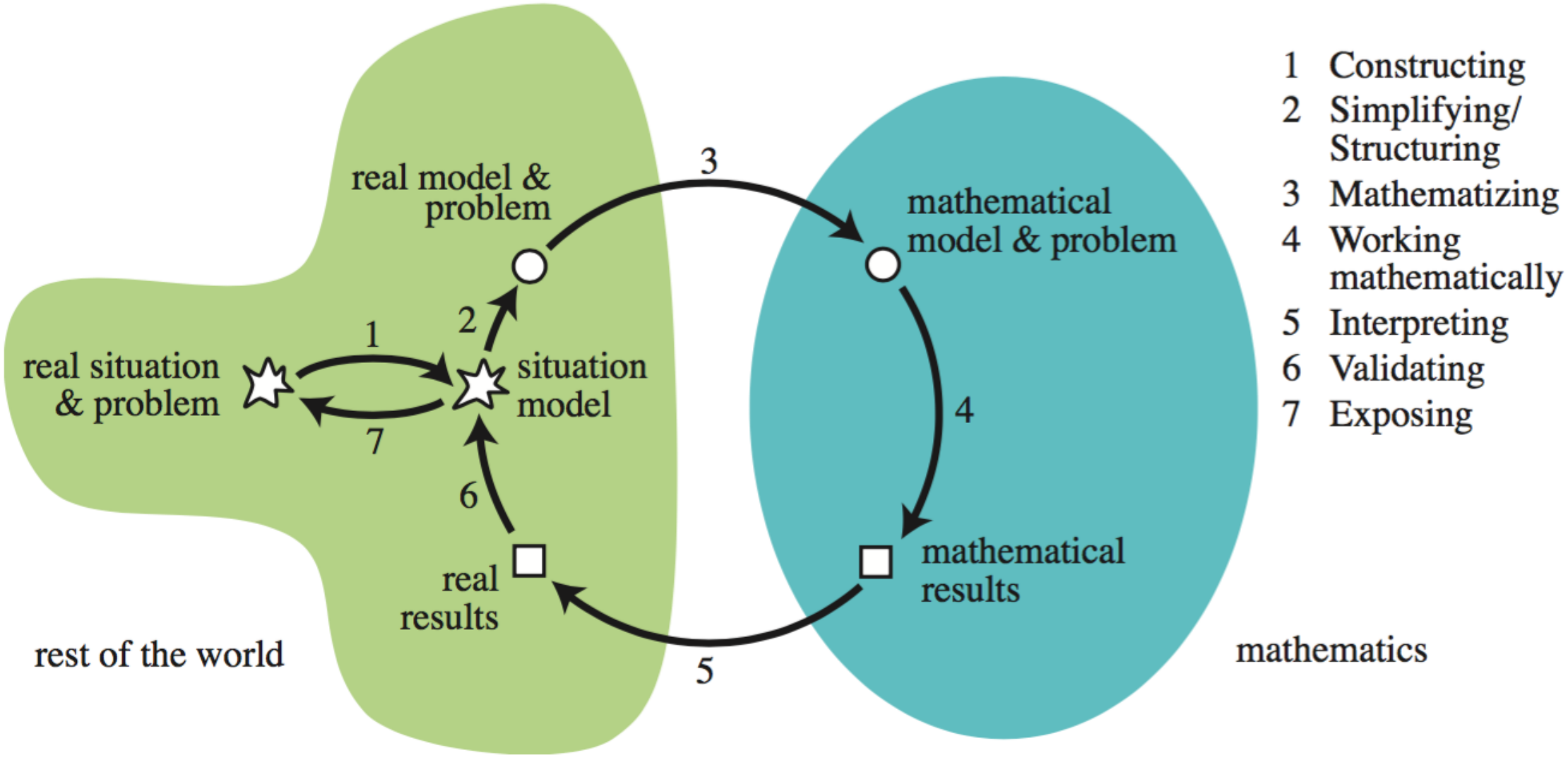
- The process of applying mathematics to a real world problem
- Problem Posing

Mathematical Modelling is…

- A problem from outside of mathematics occurs.
- Simplified and then translated into a mathematical problem that is worked on.
- The solution found is translated back into the real-world.
- The solution is validated where the result answers the primary problem adequately.
- Repeat modelling cycle until a satisfactory solution is produced.

Pollak, Rest of the World, 1979

Figure B-1. A Typical Modeling Process



Source: Blum and Ferri 2009, 46.

Mathematical Modelling Competency

1. Characteristic process of modelling
2. Modelling sub-competencies
3. Metacognitive ability

Mathematical Modelling Competency

1. Characteristic process of modelling - *students need to realize that modelling requires new mathematical habits.*
   - Analyzing a real-world situation strictly.
   - Simplifying a problem to make it mathematically solvable.

Mathematical Modelling Competency

2. Modelling sub-competencies:
   - Clarifying a real-world problem
   - Setting up assumptions
   - Generating and selecting variables
   - Generating and selecting relations
   - Interpreting the mathematical answer
   - Justifying the mathematical model
   - Eliminating errors.

Mathematical Modelling Competency

3. Metacognitive ability
   - Awareness and control of one's own thinking
   - Promotes how to monitor and control modelling sub-competency in an overall modelling process.

PreK-12…16

How do teachers in other countries engage their students in modeling?

How can mathematics teachers create meaningful modeling activities that are at the level of the student?

Three orientations to use real world contexts in mathematics

- Mathematics as a tool for everyday life
- The real world as a vehicle for learning mathematics
- Engagement with Real World as a motivation to learn mathematics

Why should we do modeling?

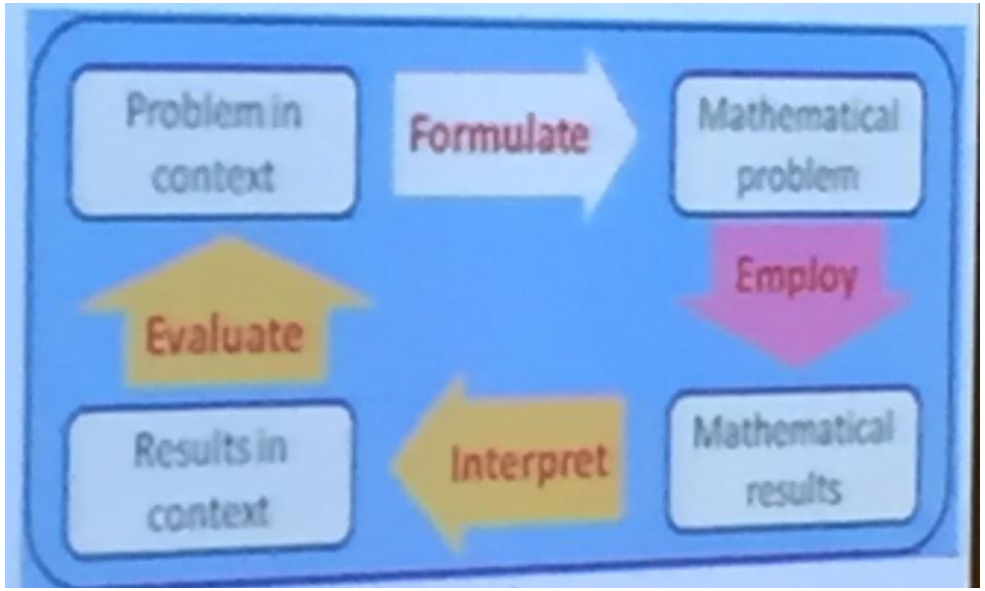
- Prescriptive modeling
  - Taking action in order to change the world
- Teaching math for social justice
  - Math can be used to read *and write* the world
  - Move toward solutions to social and political problems (poverty, equity)
- Build on students' funds of knowledge
- Place-based learning
  - Lens of storytelling

How do teachers in other countries engage their students in modeling?

- Schools in Hamburg had Modelling Days, 3 days out of the school year devoted to engaging student in modeling tasks
- Scientific process & how to apply mathematics to science (e.g., Denmark)
  - Students can collect data based on experiments in the classroom
  - Students involved in mathematical model & realize limitations of the model

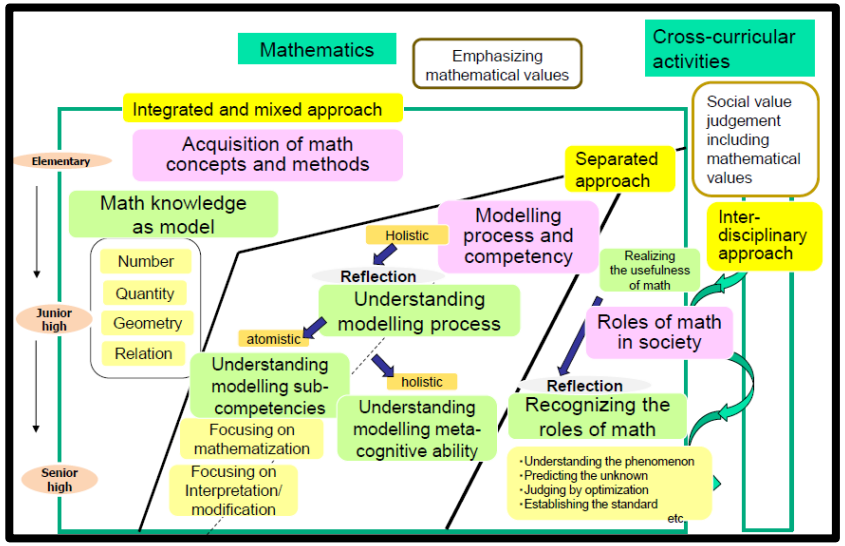
TIMSS Video Study

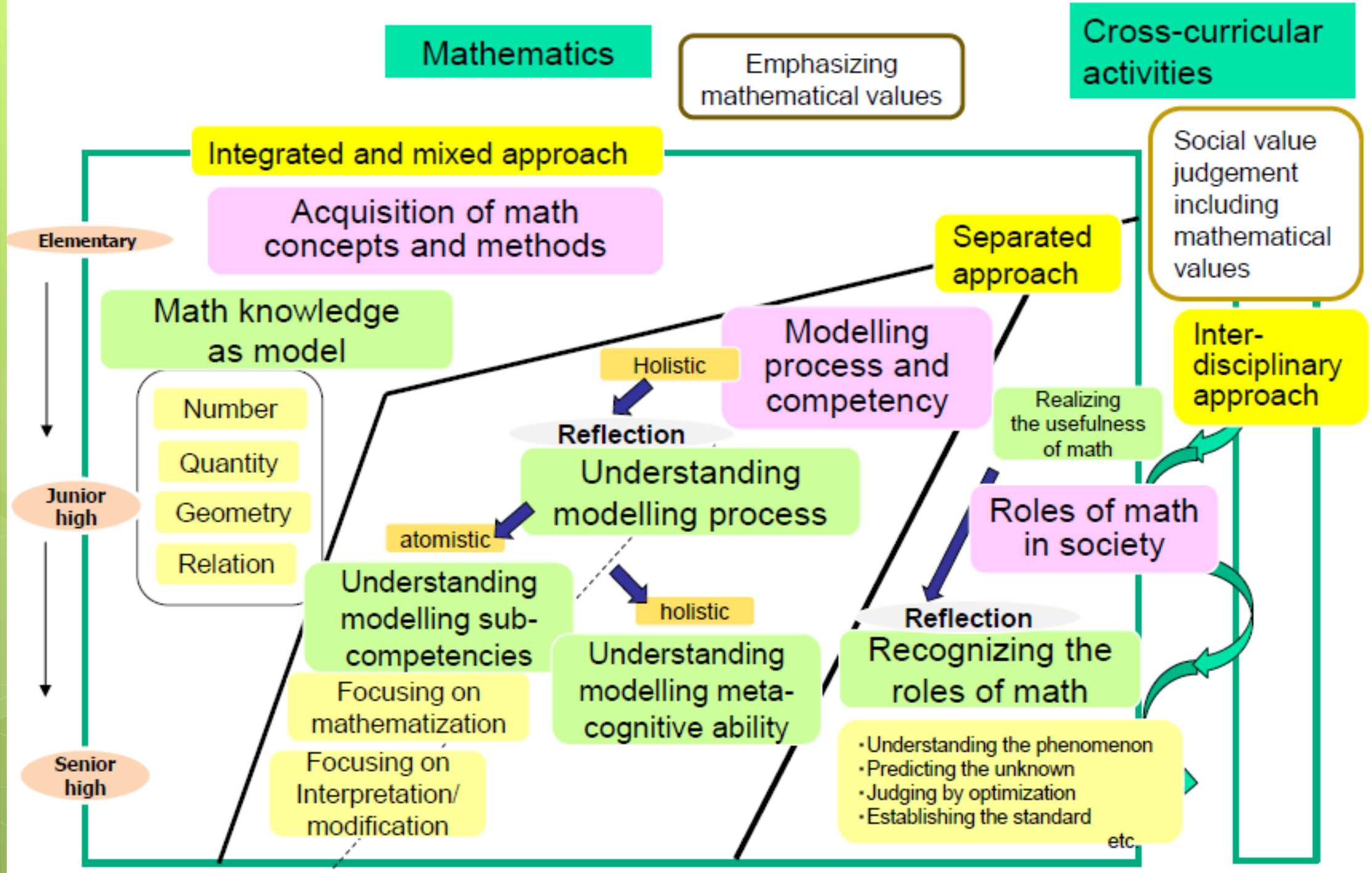
- Different countries focus on different parts of the modeling cycle
- Students in Asian countries have strength in the "formulate" part of the modeling cycle



Modeling Approaches in Japan

- Integrated/mixed (Elementary)
- Separate (Secondary)
- Interdisciplinary




Mathematics
Emphasizing mathematical values
Cross-curricular activities
Integrated and mixed approach
Social value judgement including mathematical values
Acquisition of math concepts and methods
Elementary
Separated approach
Math knowledge as model
Modelling process and competency
Inter-disciplinary approach
Holistic
Number
Quantity
Geometry
Relation
Reflection
Realizing the usefulness of math
Understanding modelling process
Junior high
atomistic
Roles of math in society
Understanding modelling sub-competencies
holistic
Reflection
Understanding modelling meta-cognitive ability
Recognizing the roles of math
Focusing on mathematization
Understanding the phenomenon
Predicting the unknown
Judging by optimization
Establishing the standard
etc.
Focusing on Interpretation/ modification
Senior high

Contexts teachers use to engage students in modeling

- Deforestation
- Building a game to simulate interaction between wolves, elk, and bears in Yellowstone National Park.
- Planning a classroom party
- Building a goat cage
- The relationship between the height of a ball when dropped and the bounce height
- Roundabout vs Traffic Light

- Amount of water used in morning shower
- 100-meter sprint speed
- Decay of alcohol in body over time
- Predict date of cherry blossom in Japan

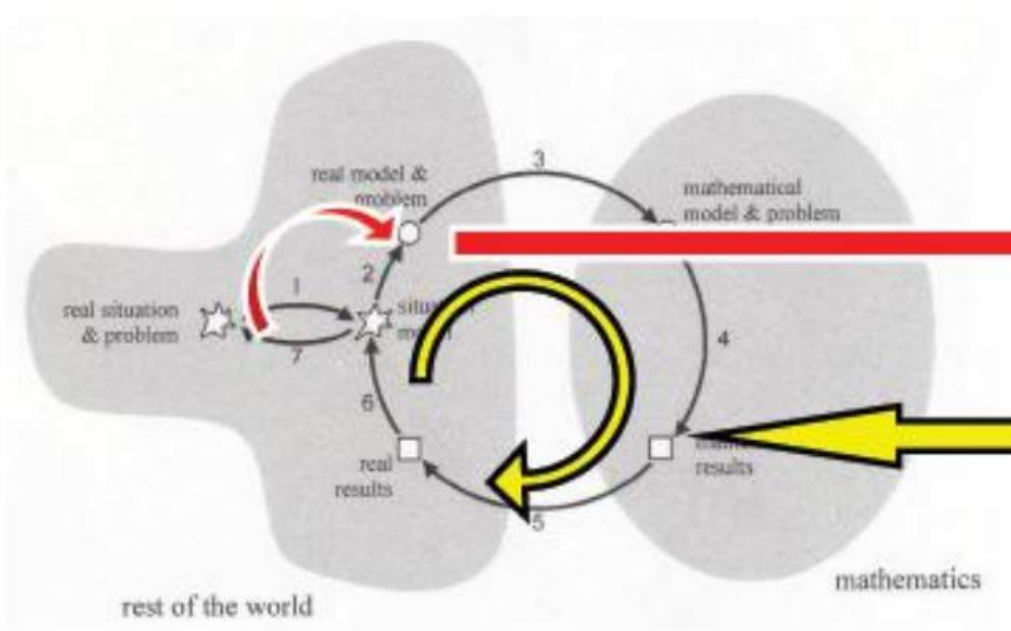
Calculus 1 and Physics 1 integration

Website with applications: www.scientix.eu

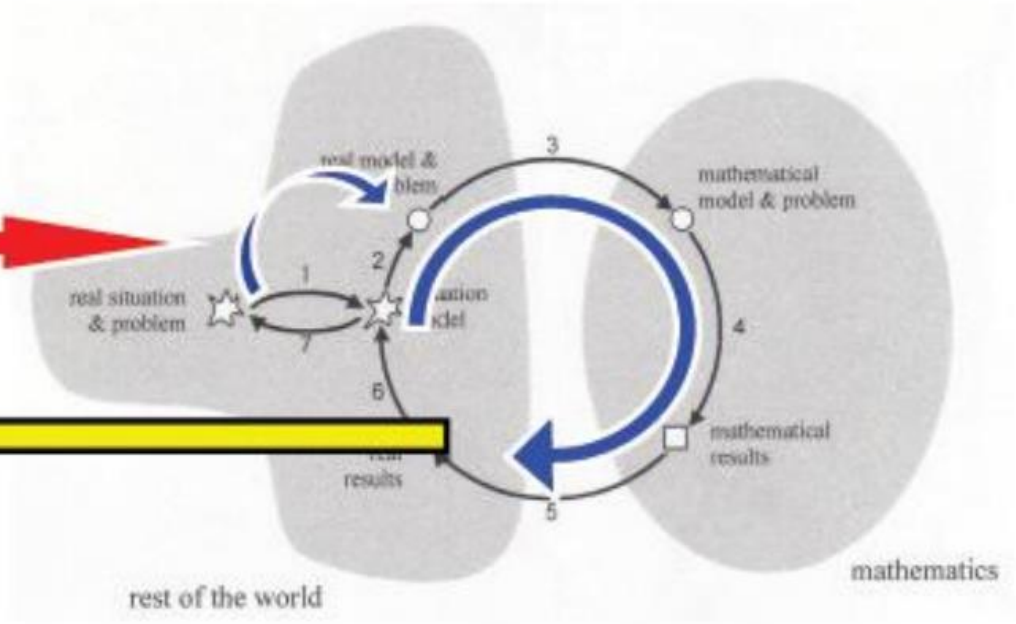
Promising teaching strategies for engaging students in modeling

- Students work as independently as possible while also achieving acceptable outcomes
  - Focus teacher intervention on heuristic strategies: solving a simpler problem; work backwards; use symmetry; multiple representation;, etc
  - Ask questions like, "What are you doing right now?"
- Students worked in groups to solve modeling tasks
- Technology was used in upper grades, e.g. one presentation featured the use of tablets to study function families.

Dual Modelling Cycle Framework



[The first modelling cycle]

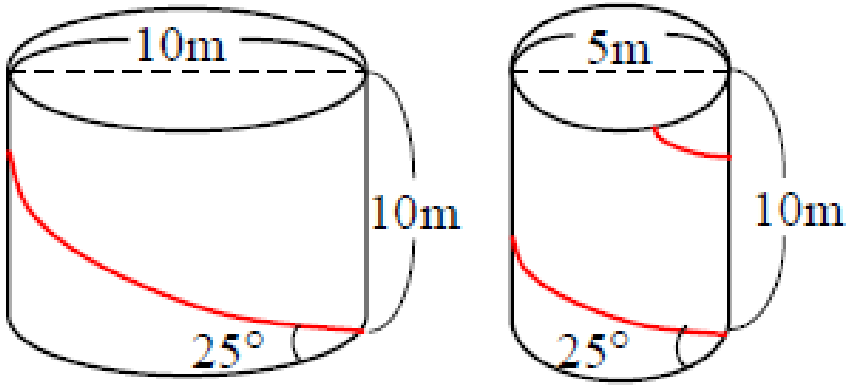


[The second modelling cycle]

Dual Modelling Cycle Framework (DMCF)

Oil Tank Task [Task 1] (Initial Task)

There are several types of oil tanks. Their heights are equal but their lengths of diameters are different. Are the lengths of the spiral banisters equal or not? As conditions, angles to go up spiral banisters are all the same.



Toilet Paper Tube Task [Task 2] (Similar Task)

It is impossible to open an tank along the spiral banister. We can instead take a toilet paper tube with spiral slit as a similar shape to an tank. Consider what the shape of an opened tube would be.

Pre-service and In-service

In what ways do pre-service teacher preparation programs and professional development prepare teachers to work with their students on modeling tasks?

Mathematical modeling with pre-service teachers

- In the U.S. there is ambiguity surrounding the term "Modeling" in elementary school mathematics classrooms
  - Modeling as a cyclic process of mathematizing authentic scenarios
  - Modeling as representation of math concepts (physical models, manipulatives, diagrams)
  - Modeling as a scaffolding technique - "I do, we do, you do"
- Examining pre- and in-service teachers' perceptions of modeling can help focus on a vision of modeling in a math classroom as an open-ended, student-centered process (Carlson 2016)

In what ways can WE prepare teachers to work with their students on modeling tasks?

- Helping teachers develop mathematical habits of mind such as:
  - generic pattern sniffing
  - visualizing
  - formulating
  - conjecturing
- Integrating courses about modeling into preservice programs
- Collectively analyzing videos of students engaged in the modeling process

How can PD help teachers evolve their teaching of modeling?

- Professional development opportunities where teachers engage in modeling tasks
- Focus on ambitious teaching practices that support modeling (e.g., student empowerment)
- A cohort group for teachers to share similar struggles

Questions?