Mathematics for Complex Systems

Unit 2: Introduction to Differential Equations (David Feldman)

Homework

Note: Homework for this course is not turned in, but we highly recommend that you do it! Homework solutions will be posted to the Lectures page. Feel free to use any course materials or other resources to help you do the problems, and you are encouraged to discuss the problems with other students on the course forum.

1. Consider the differential equation that describes the temperature T of an object in a 20-degree room:

$$\frac{dT}{dt} = 0.2\,(20 - T).$$

Suppose that the initial temperature of the object is $T = 10$ degrees. Use Euler's method with $\Delta t = 2$ to come up with estimates for the object's temperature at $t = 2$ and $t = 4$.

2. Consider the differential equation

$$\frac{dY}{dt} = -\frac{1}{2}Y.$$

Let $Y(0) = 100$.

Use Euler's method with $\Delta t = 2$ to determine estimates for $Y(2)$ and $Y(4)$.

3. (Advanced) Write a program that implements Euler's method for the main example of this unit,

$$\frac{dT}{dt} = 0.2\,(20 - T).$$

Some things to try out or experiment with:

- Have your program produce a plot of your Euler solution.
- Make plots of the Euler solutions for several different values of Δt.

• Compare the Euler solution with the exact solution

$$T(t) = 20 - 5e^{-0.2t}.$$

(This analytic result is obtained via calculus, using T(0) = 15.)

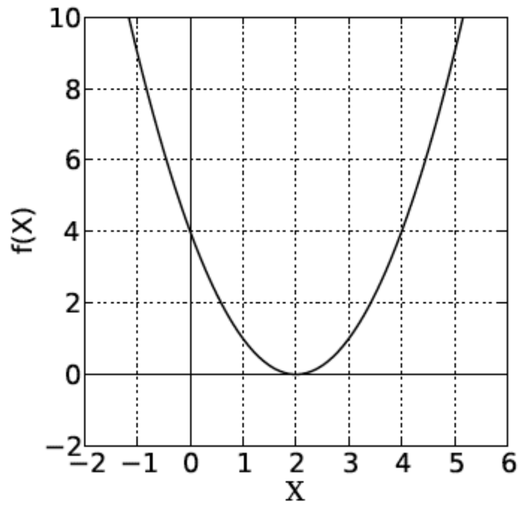
• Generalize your program so that it can solve any differential equation of the form

$$\frac{dX}{dt} = f(X).$$

4. (Advanced): Consider again the differential equation

$$\frac{dX}{dt} = f(X)$$

where $f(X)$ is plotted below:



Suppose the initial X value is 1. If you used Euler's method with $\Delta t = 1$ to figure out the value of X at $t = 1$, would your result be above or below the exact value for $X(1)$? Why?

5. (Advanced): Euler's method is an approximation that becomes better and better as Δt approaches zero. Under what circumstances would Euler's method yield an exact solution without letting Δt approach zero?