Introduction to Dynamical Systems and Chaos
Homework for Unit Eight: Strange Attractors

Santa Fe Institute

http://www.complexityexplorer.org

Beginner

1. Consider the Hénon map:

$$x_{n+1} = y_n + 1 - ax^2 , \quad y_{n+1} = bx_n . \tag{1}$$

For each of the following pairs of parameter values, do you observe a strange attractor?

(a) $a = 1.1$, $b = 0.3$
(b) $a = 0.5$, $b = 0.4$
(c) $a = 1.0$, $b = 0.5$

2. For each of the following parameter values, describe the behavior of the Lorenz equations. Is there a strange attractor? An attracting cycle? Stable fixed point(s)?

(a) $\sigma = 20$, $\rho = 10$, $\beta = 2.66$
(b) $\sigma = 30$, $\rho = 40$, $\beta = 2.66$
(c) $\sigma = 30$, $\rho = 22$, $\beta = 2.66$

Intermediate

1. Consider the Lorenz equations with $\sigma = 10.0$, $\rho = 28.0$, and $\beta = 2.667$. These parameter values give the strange attractor that has two spirals. At the center of each spiral is an unstable fixed point. Verify that the coordinates of these fixed points are:

$$x = 6\sqrt{2} , \ \ y = 6\sqrt{2} , \ \ z = 27 , \tag{2}$$

and

$$x = -6\sqrt{2} , \ \ y = -6\sqrt{2} , \ \ z = 27 . \tag{3}$$

That is, plug these values into the Lorenz equations and show that all derivatives are zero.

Advanced

As in Unit 7, there are many programs one could write to explore the dynamical systems introduced in this unit. Some options are below.

1. Write a program that plots the Lorenz attractor in phase space.

2. Write a program that plots the Rössler attractor in phase space.