

# Solutions

## Introduction to Dynamical Systems and Chaos Homework for Unit 5: Bifurcations Part II

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<http://www.complexityexplorer.org>

### Beginner

- There is a bifurcation from period-three to period-six behavior at  $r \approx 3.8415$ .
  - Near  $r = 3.9605$  there is a periodic window of period four.
  - Near  $r = 3.628$  there is a periodic window of period six.
- At  $r = 5.0$  orbits are pulled toward a fixed point at  $x \approx 0.75$ .
  - At  $r = 5.5$  orbits are periodic with period two.
  - At  $r = 6.0$  orbits are aperiodic. The orbit ranges from roughly 0.52 to 0.9.
- Use the bifurcation diagram web program (for the logistic map) to determine:
  - There is a transition from period 1 to period 2 at  $r = 3.00$ .
  - There is a transition from period 2 to period 4 at  $r \approx 3.44948$ .
  - There is a transition from period 4 to period 8 at  $r = 3.544089$ .
  - There is a transition from period 8 to period 16 at  $r = 3.564407$ .

### Intermediate

1.

$$\delta_1 = \frac{3.44948 - 3.00}{3.544089 - 3.44948} = \frac{0.44948}{0.94609} \approx 4.7509 . \quad (1)$$

2.

$$\delta_2 = \frac{3.544089 - 3.44948}{3.564407 - 3.544089} = \frac{0.094609}{0.020318} \approx 4.6564 . \quad (2)$$

### Advanced

Answers to these programming questions will vary. As usual, if you have a working program, I encourage you to share it with others on the course forum.