

# **Summary of Unit 5**

## **Bifurcations: Part II**

### **The Logistic Map**

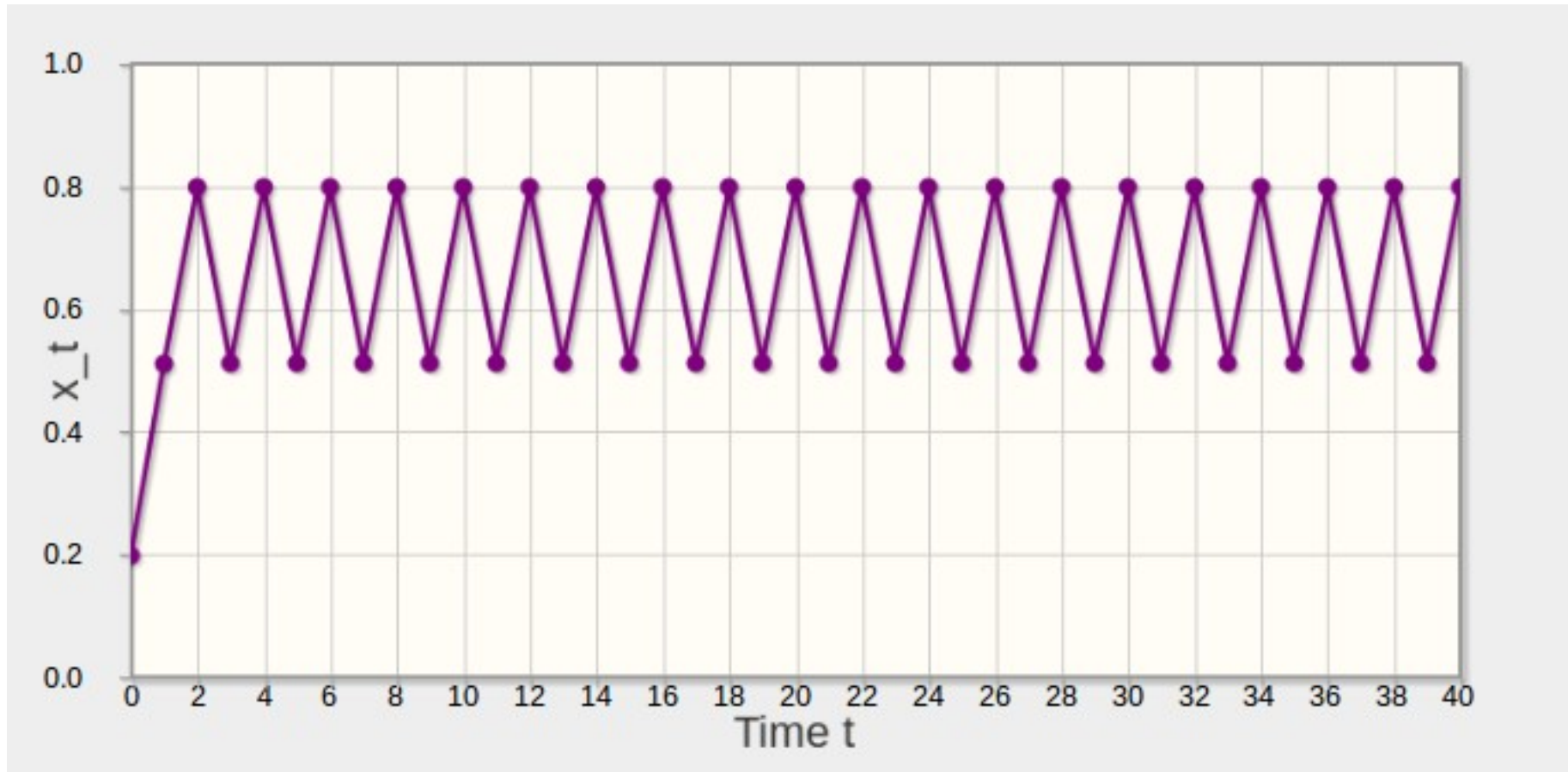
**Introduction to  
Dynamical Systems and Chaos**

**<http://www.complexityexplorer.org>**

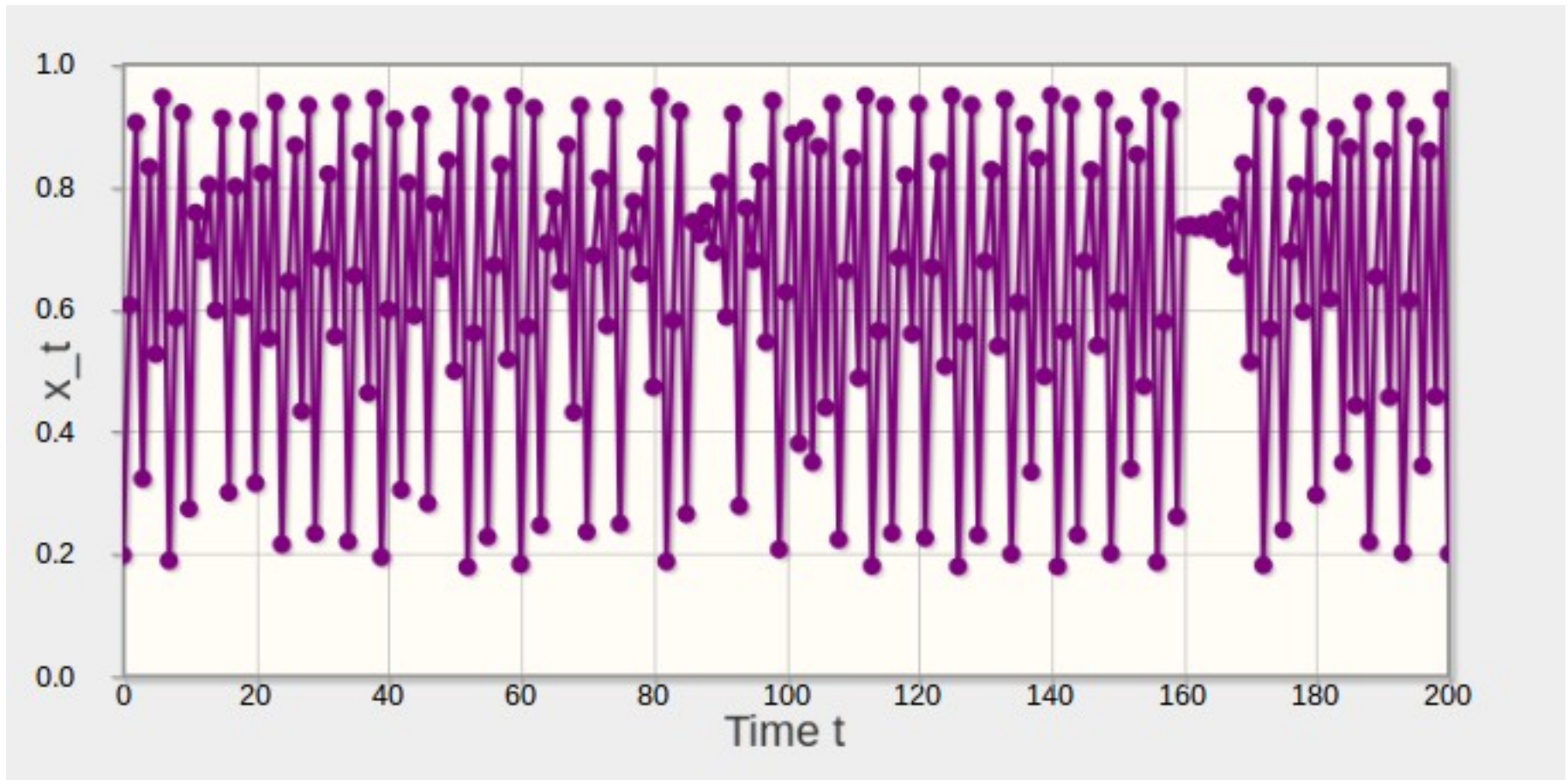
# Final State Diagrams

- For a given  $r$  value:
- Iterate 100 times
- Then plot the next 200 iterates on the unit interval
- The result is a final-state diagram, similar to the phase line for a differential equation

# Final State Diagrams

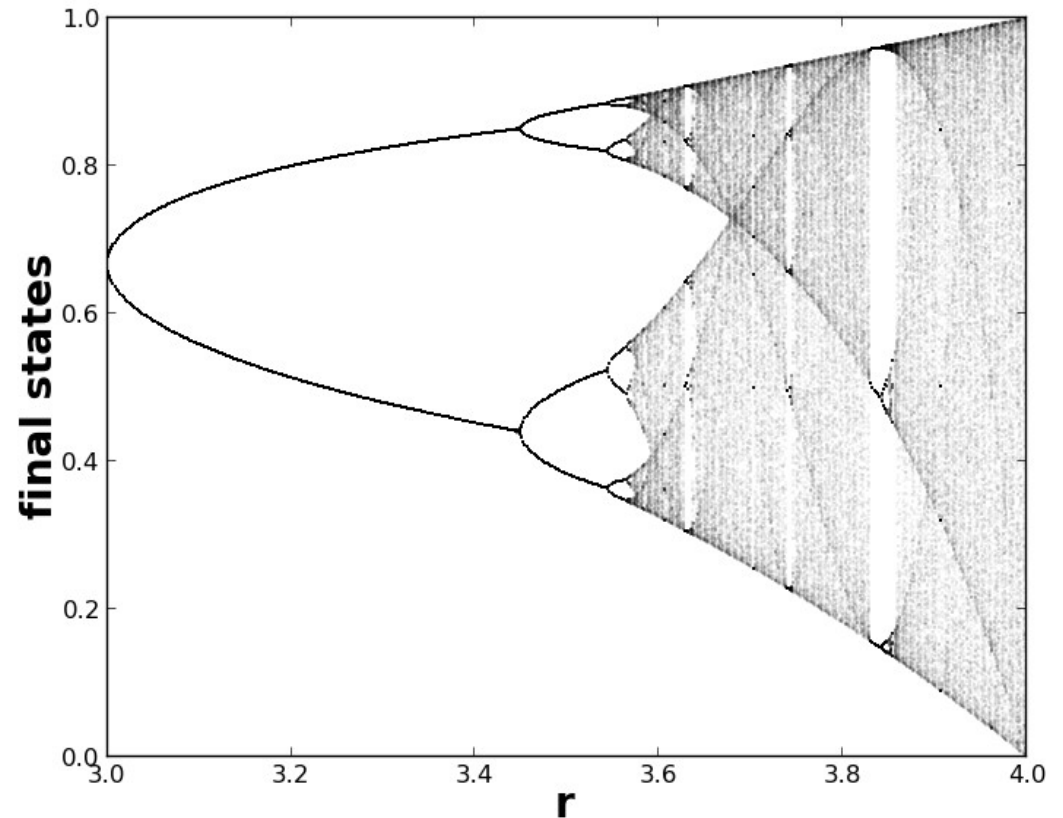


# Final State Diagrams



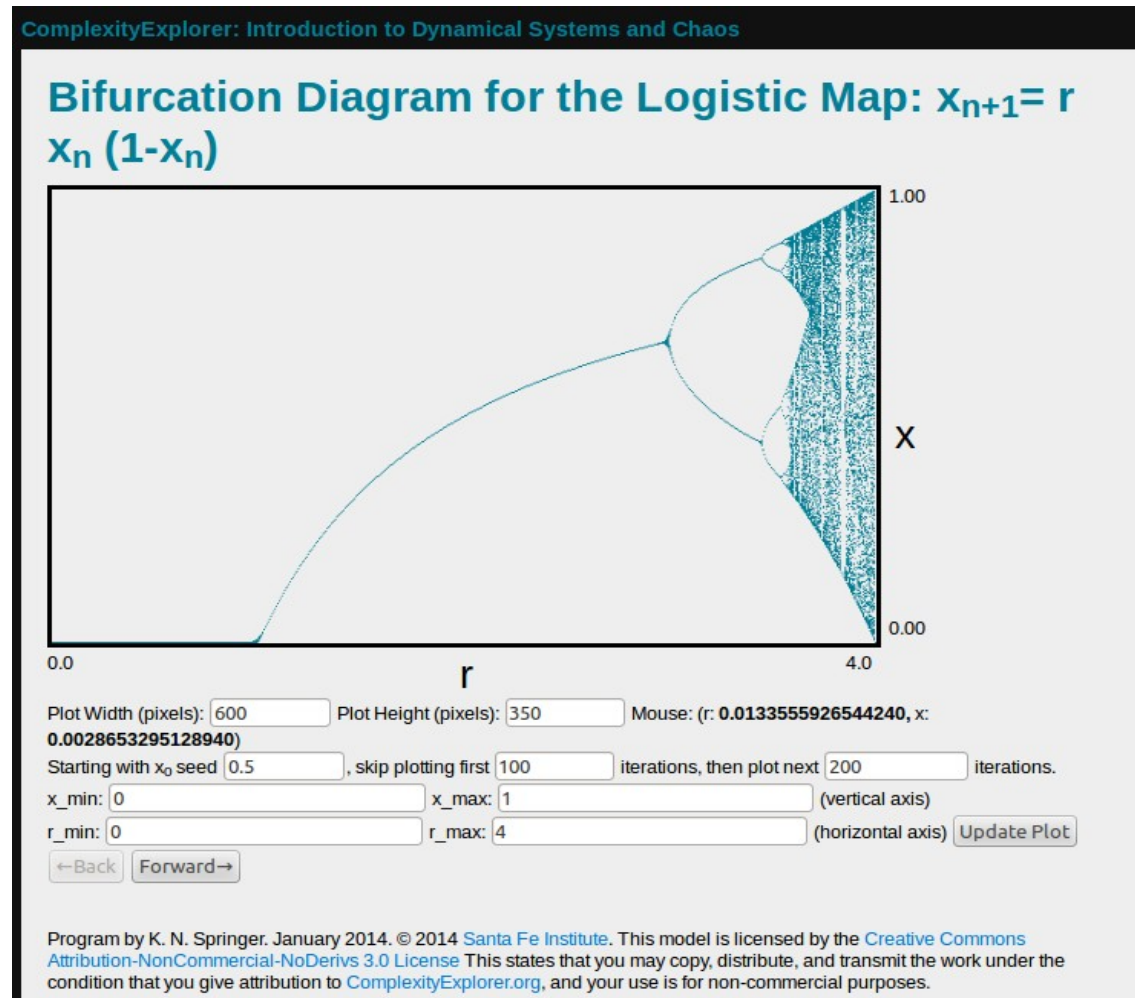
# Bifurcation Diagram

- “Glue” lots of final-state diagrams together to make a bifurcation diagram.
- The bifurcation diagram is like a dictionary that lets you look up behaviors for different  $r$  values.



# Bifurcation Diagram Program

- Can zoom in and explore
- Need to adjust number of orbits skipped and number of orbits plotted



# Bifurcation Diagram

- The bifurcation diagram comes from the simple equation:  $f(x) = rx(1-x)$
- The code to make a bifurcation diagram is very simple.
- The python program I used is just 47 lines.