

map, reduce, and Type Inference

Lecture 13 – COMP110 – Spring 2018

Announcements

- Review Session: Tomorrow at 5pm in SN14
 - If you were dissatisfied with your midterm grade, you should come!
- Office hours close Friday at 12pm for Spring Break
 - If you anticipate needing to ask questions about PS03 you should do so before Friday afternoon
- No tutoring this week because of the early Friday close
 - Come to office hours with conceptual help questions

1. Which of these functions is an implementation of the Transform<T, U> functional interface?

```
interface Transform<T, U> {  
    (item: T): U;  
}
```

- A: (m: number, n: number): number => { /* ... */ };
- B: (m: number): number => { /* ... */ };
- C: (m: string): number => { /* ... */ };
- D: (m: string): string => { /* ... */ };
- E: None of the above
- F: All except A
- G: All of the above

2. Given the definitions on the left, after the code below completes, what is the result variable's type and what is its value?

```
interface Transform<T, U> {
  (item: T): U;
}

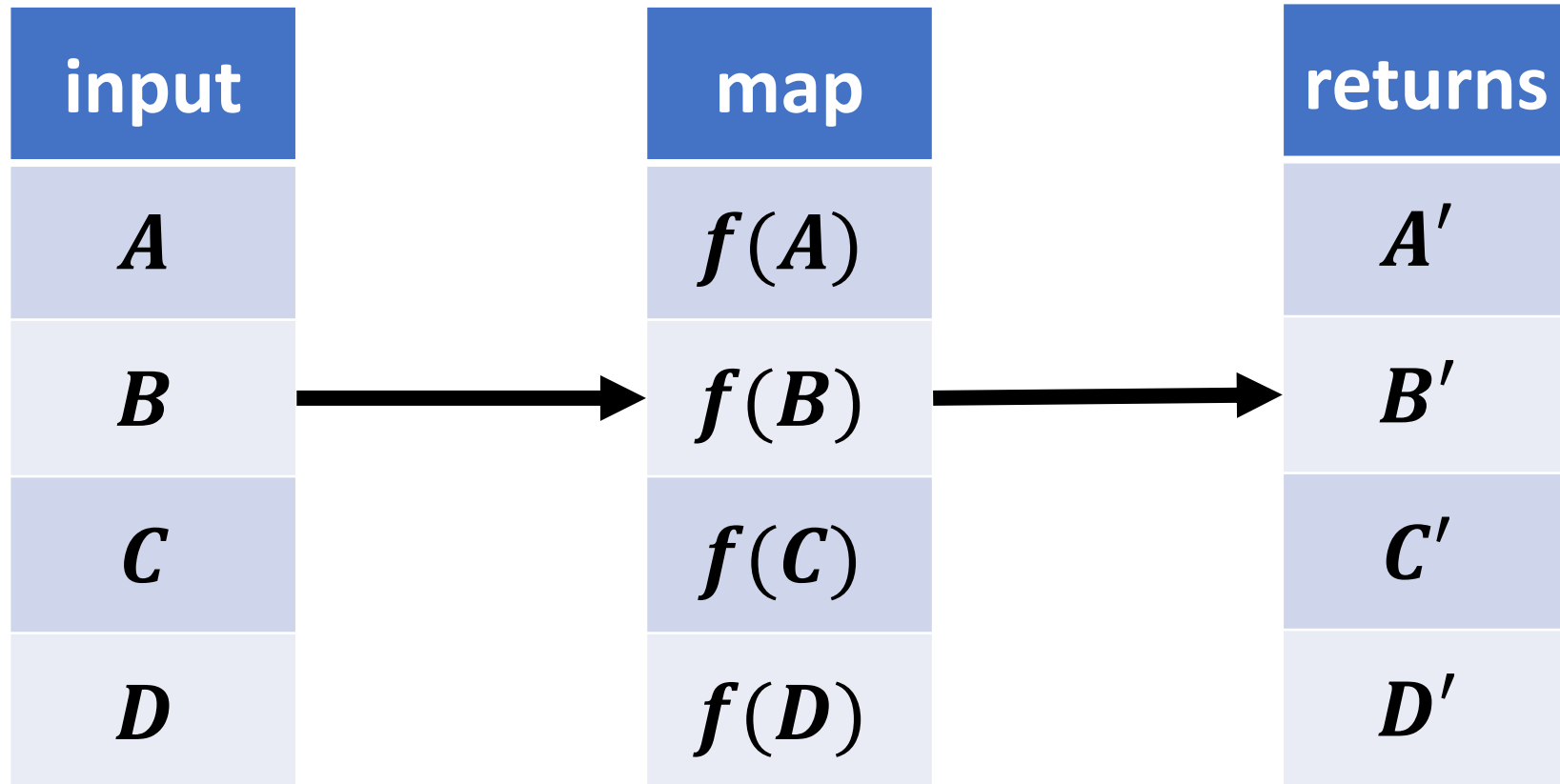
let map = <T, U> (xs: List<T>, f: Transform<T, U>): List<U> => {
  if (xs === null) {
    return null;
  } else {
    return cons(f(first(xs)), map(rest(xs), f));
  }
};

let strToLen: Transform<string, number> = (s: string): number => {
  return s.length;
};
```

```
let words: List<string> = listify("lets", "go", "unc");
let result: _____ = map(words, strToLen);
```

The `map` Function

Given an *input* list and a *transform function* f , returns a new list with f applied to every element in the input list.



The `Transform<T, U>` Functional Interface

- What if we wanted to generically describe a function that given an argument of any type *T* returned a value of any type *U*?

```
interface Transform<T, U> {  
    (element: T): U;  
}
```

- Examples:

1. a function that takes an argument of type string and returns a number
2. takes string and returns a string... *it's ok for T and U to be the same type!*

1


```
(s: string): number => {  
    return s.length;  
}
```

2

```
(s: string): string => {  
    return s.toUpperCase();  
}
```

An implementation of **map**

```
let map = <T, U> (xs: List<T>, f: Transform<T, U>): List<U> => {  
  if (xs === null) {  
    return null;  
  } else {  
    return cons(f(first(xs)), map(rest(xs), f));  
  }  
};
```



- Notice this is the recursive List building pattern we've used all along, just with the function f being passed in and called on each element as we *cons* it onto the resulting List.

Hands-on: Writing a Transform function and using map

- Open lec13 / 00-map-app.ts
- Goal: After loading Berry's game data from data/berry-stats-2018.csv, produce a list of *only* Berry's points per game values
- 1. **TODO #1)** Declare a (Transform) function named ***gameToPoints*** that is given a ***Game*** object named ***g*** as a parameter and returns a ***number***. It should simply return the ***points*** property of the Game parameter: ***g.points***
- 2. **TODO #2)** In the main function, **assign** to the variable ***points*** the result of mapping the ***gameToPoints*** function over the ***games*** list.
 1. `map(games, gameToPoints)`
- You should see a list of points values printed after loading your data. Can you trace through how the map function is calling gameToPoints?
- Check-in on PollEv.com/compunc when this is working


```
// TODO #1: Define a function named gameToPoints
// It should take in a Game object as a parameter and return a number
// The number it returns should be the game's points property
let gameToPoints = (g: Game): number => {
  return g.points;
};
```

```
// TODO #2 - Assign to points the result of calling map with
// the games List and the gameToPoints function you wrote below.
points = map(games, gameToPoints);
```

3. Which of these functions is an implementation of the `Reducer<T, U>` functional interface?

```
interface Reducer<T, U> {  
    (memo: U, item: T): U;  
}
```

- A: `(m: number, n: number): number => { /* ... */ }`;
- B: `(m: number, n: string): number => { /* ... */ }`;
- C: `(m: string, n: number): number => { /* ... */ }`;
- D: `(m: string, n: string): string => { /* ... */ }`;
- E: All except B
- F: All except C

4. What is the output?

```
let add = (m: number, n: number): number => {  
    return m + n;  
};
```

```
let xs: List<number> = listify(3, 4, 5);  
let s: number = 0;  
s = add(s, first(xs));  
s = add(s, first(rest(xs)));  
s = add(s, first(rest(rest(xs))));  
print(s);
```

The reduce Function

Given an *input* list, a *reducer function* f , and an initial *memo* value, **reduce** gives f the memo and the next value. Whatever f returns is used as the next memo for the next element until the final value returned is the solution.

```
let reduce = <T, U> (xs: List<T>, f: Reducer<T, U>, memo: U): U => {
  if (xs === null) {
    return memo;
  } else {
    return reduce(rest(xs), f, f(memo, first(xs)));
  }
};
```

The `reduce` Function's Intuition

List: `1 → 2 → 3 → null`

How can we reduce a list using the following *add* reducer?

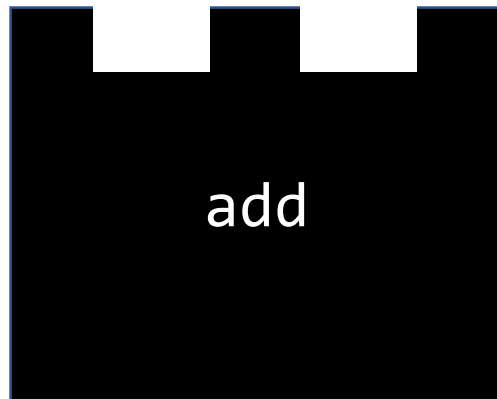
```
let add = (memo: number, item: number): number => {  
  return memo + item;  
};
```

It's like scanning down a list and keeping track of some "*reduced*" value (like a total) as you continue each step of the way...

The `reduce` Function's Intuition

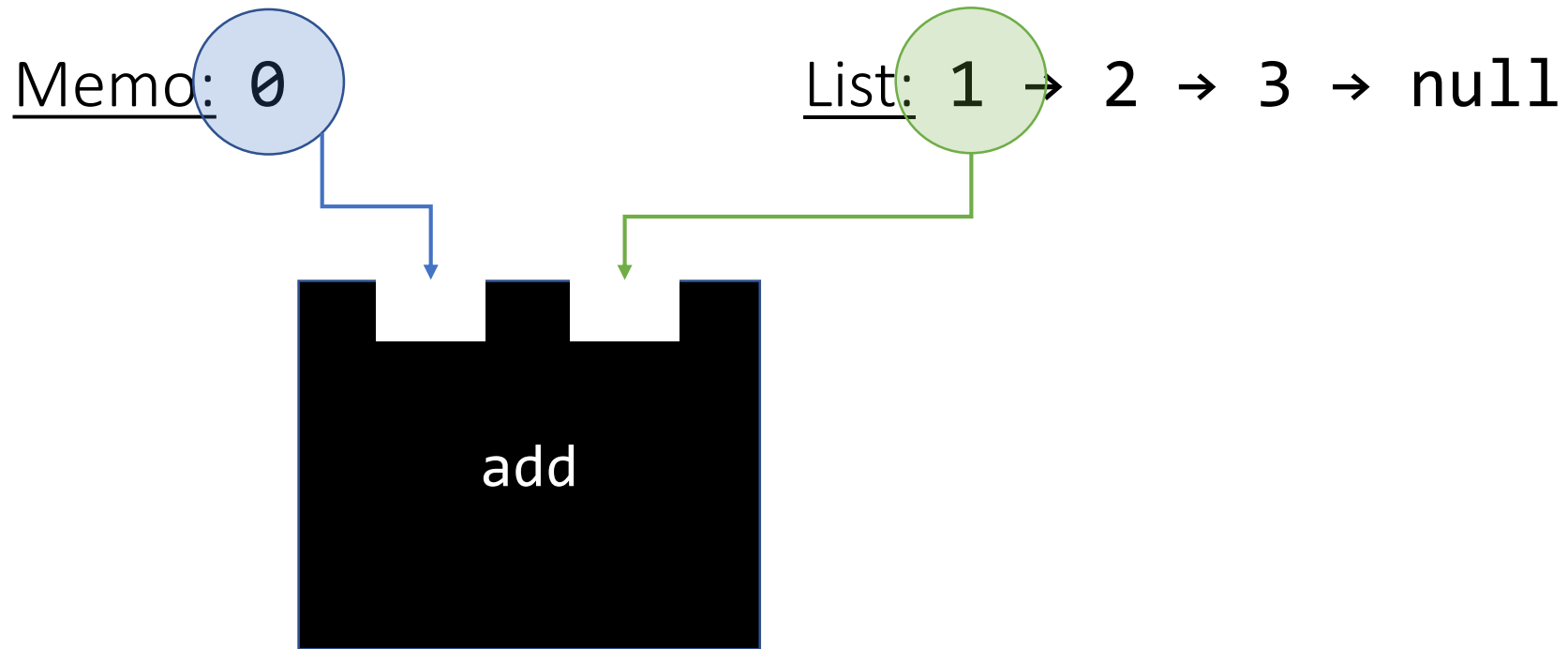
Memo: 0

List: 1 → 2 → 3 → null



Reduce's initial "memo" is the starting value. Here, since we're trying to add up all the numbers, we'll start with a memo of 0.

The reduce Function's Intuition

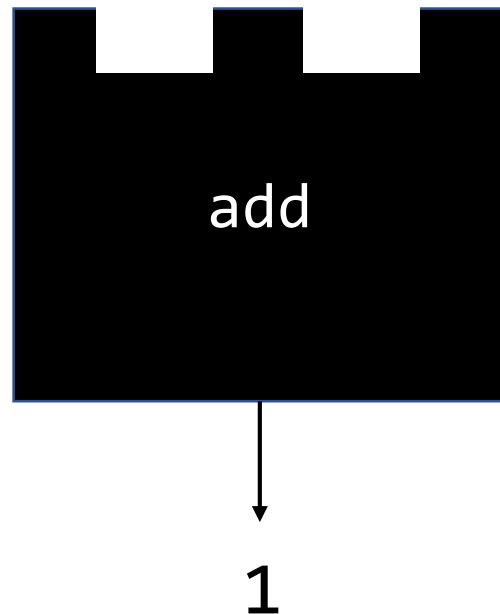


The reduce function calls add (the reducer) with memo and the next value from the list.

The `reduce` Function's Intuition

Memo: \emptyset

List: 1 → 2 → 3 → null

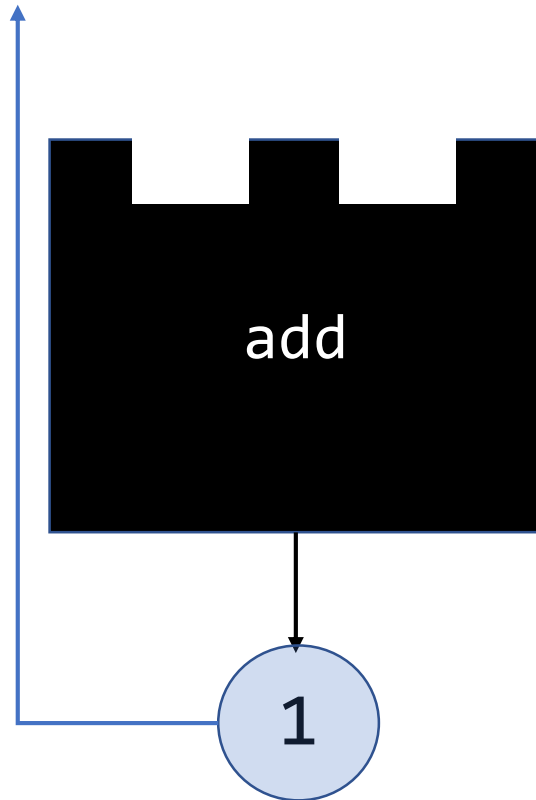


The reducer produces a return value.

The reduce Function's Intuition

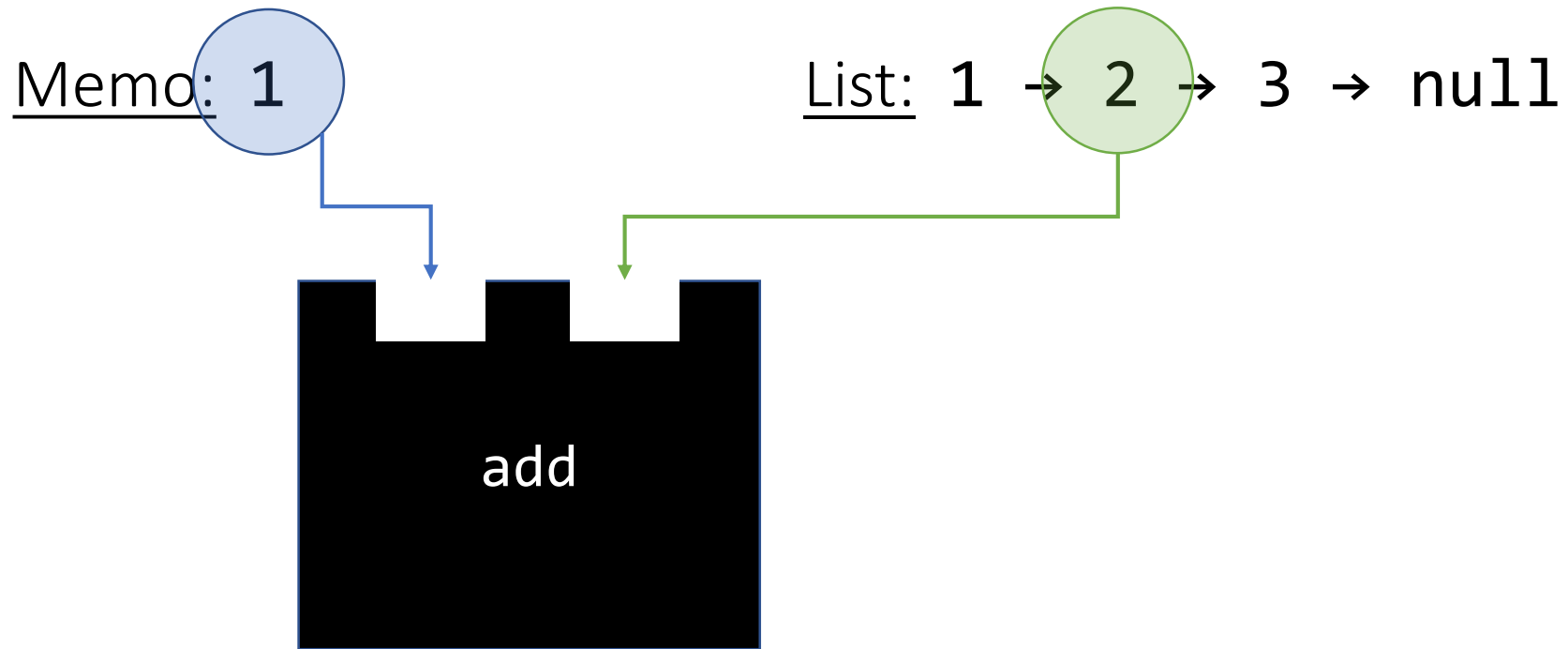
Memo: 1

List: 1 → 2 → 3 → null



This return value then becomes the next memo!

The reduce Function's Intuition

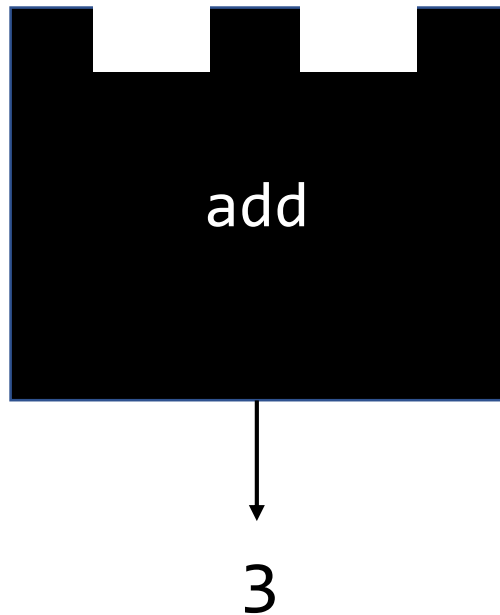


The reduce function calls add (the reducer) with memo and the next value from the list.

The `reduce` Function's Intuition

Memo: 1

List: 1 → 2 → 3 → null

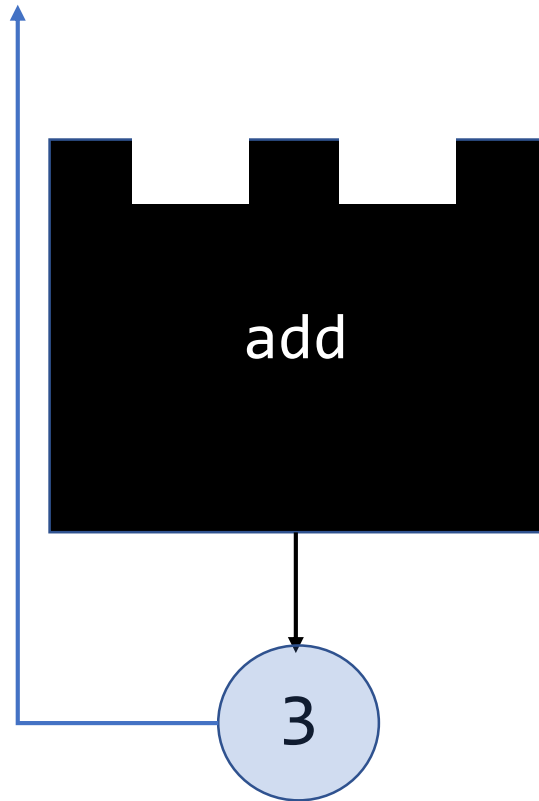


The reducer produces a return value.

The reduce Function's Intuition

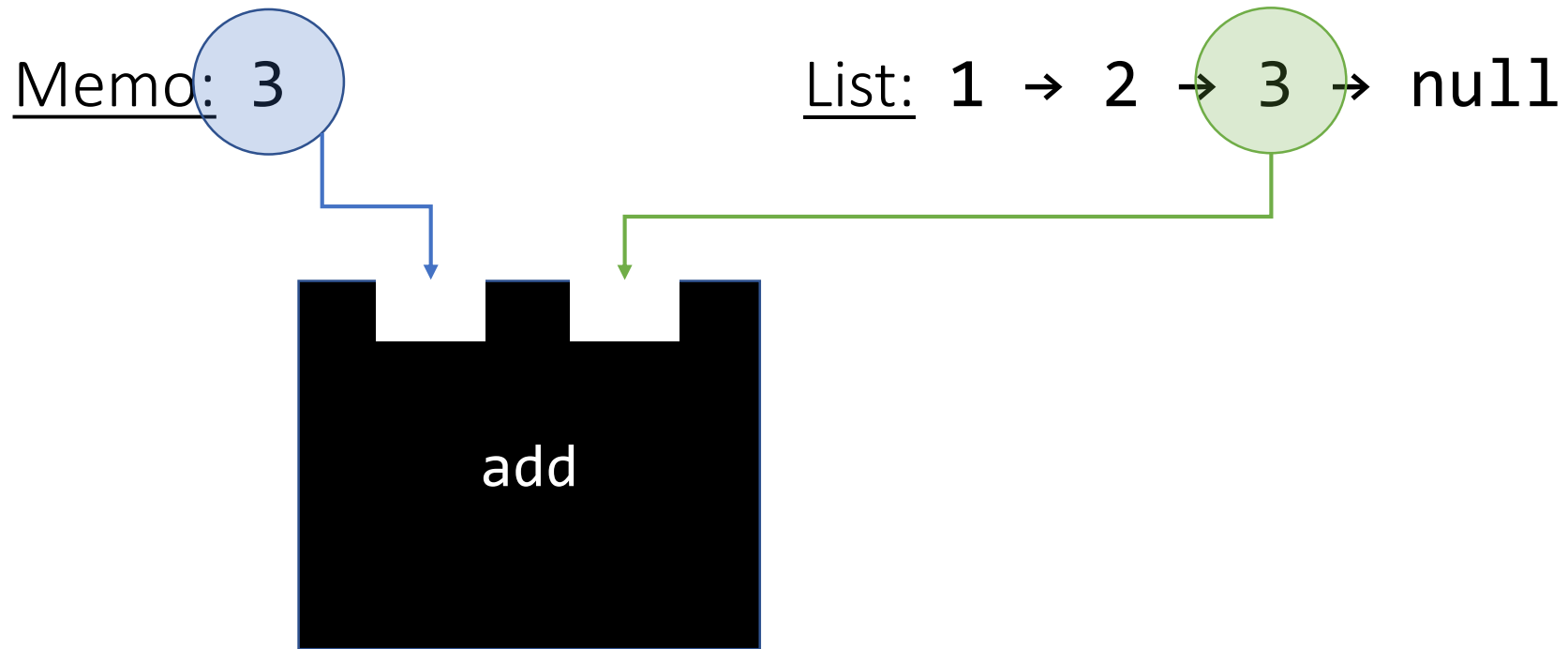
Memo: 3

List: 1 → 2 → 3 → null



This return value then becomes the next memo!

The `reduce` Function's Intuition

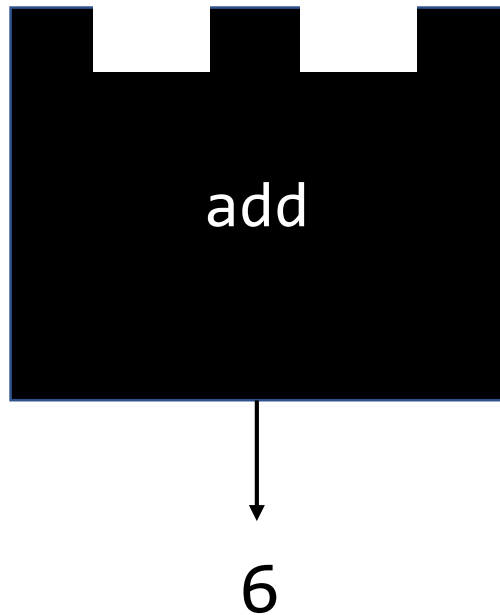


The reduce function calls `add` (the reducer) with memo and the next value from the list.

The `reduce` Function's Intuition

Memo: 3

List: 1 → 2 → 3 → null

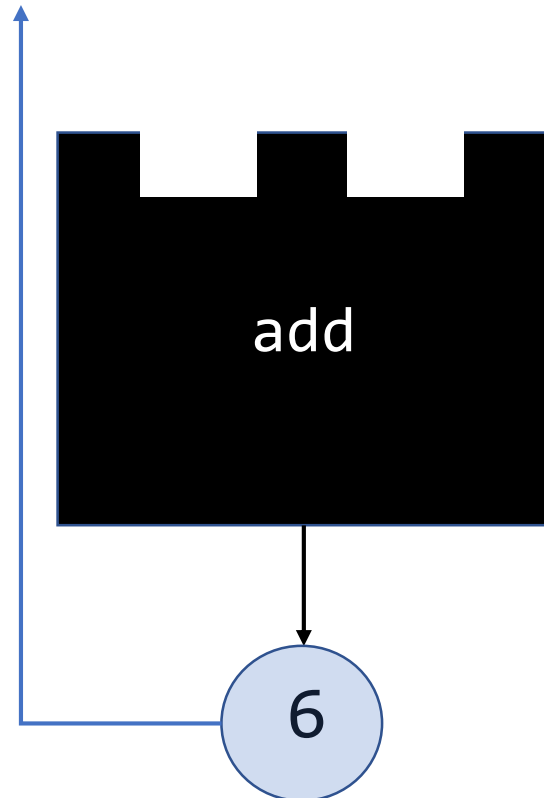


The reducer produces a return value.

The reduce Function's Intuition

Memo: 6

List: 1 → 2 → 3 → null

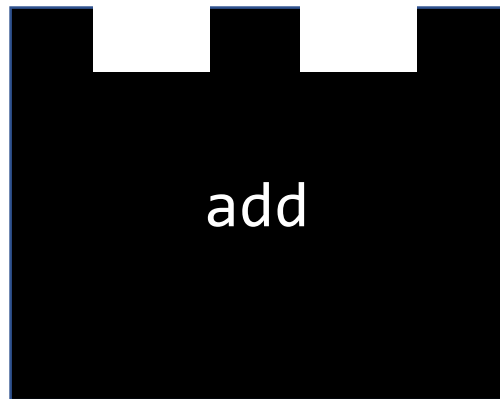


This return value then becomes the next memo!

The `reduce` Function's Intuition

Memo: 6

List: 1 → 2 → 3 → null



When the end of the list is reached, `reduce`'s returned value is memo.

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

In the following slides we will trace through this program line-by-line.


*Note that to fit the **reduce** function on this slide it is not the fully generic form of reduce we'll otherwise use.*

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

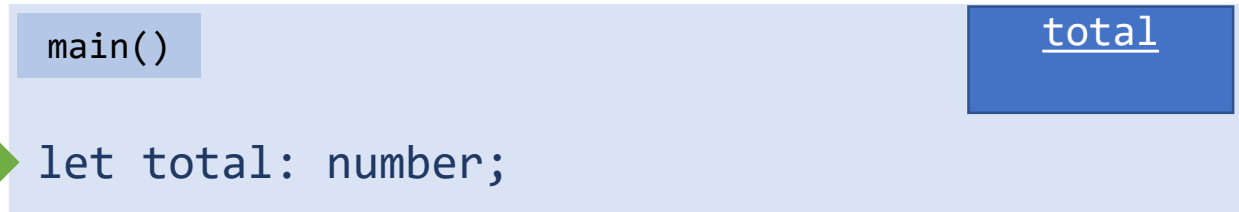


Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

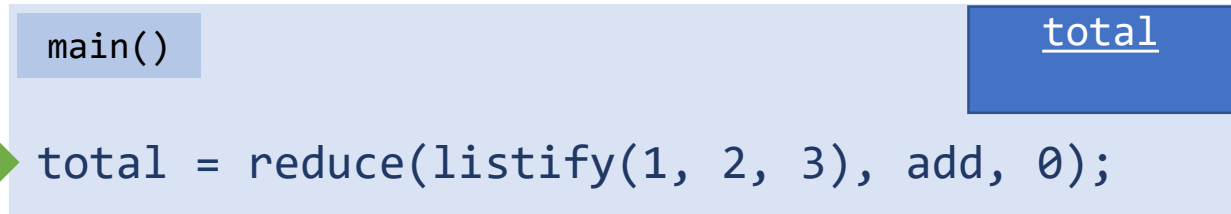


Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

if (xs === null)

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

if (false)


main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(rest(xs), f, f(memo, first(xs)));



main()

total

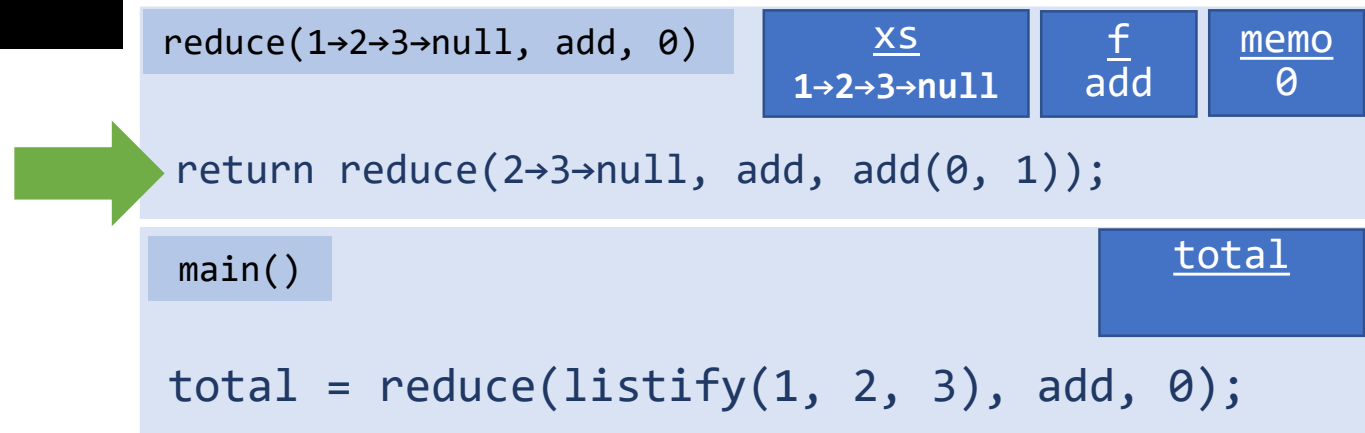
total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```


```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```




Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



add(0, 1)

$\frac{m}{0}$

$\frac{n}{1}$

return m + n;

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, add(0, 1));


main()

total

total = reduce(listify(1, 2, 3), add, 0);


Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



add(0, 1)

$\frac{m}{0}$

$\frac{n}{1}$

return 0 + 1;

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, add(0, 1));


main()

total

total = reduce(listify(1, 2, 3), add, 0);


Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



add(0, 1)

$\frac{m}{0}$

$\frac{n}{1}$

return 1;

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, add(0, 1));

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

if (xs === null)

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

if (false)

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);


main()

total

total = reduce(listify(1, 2, 3), add, 0);


Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(rest(xs), f, f(memo, first(xs)));

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);


main()

total

total = reduce(listify(1, 2, 3), add, 0);


Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(3→null, add, add(1, 2));

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

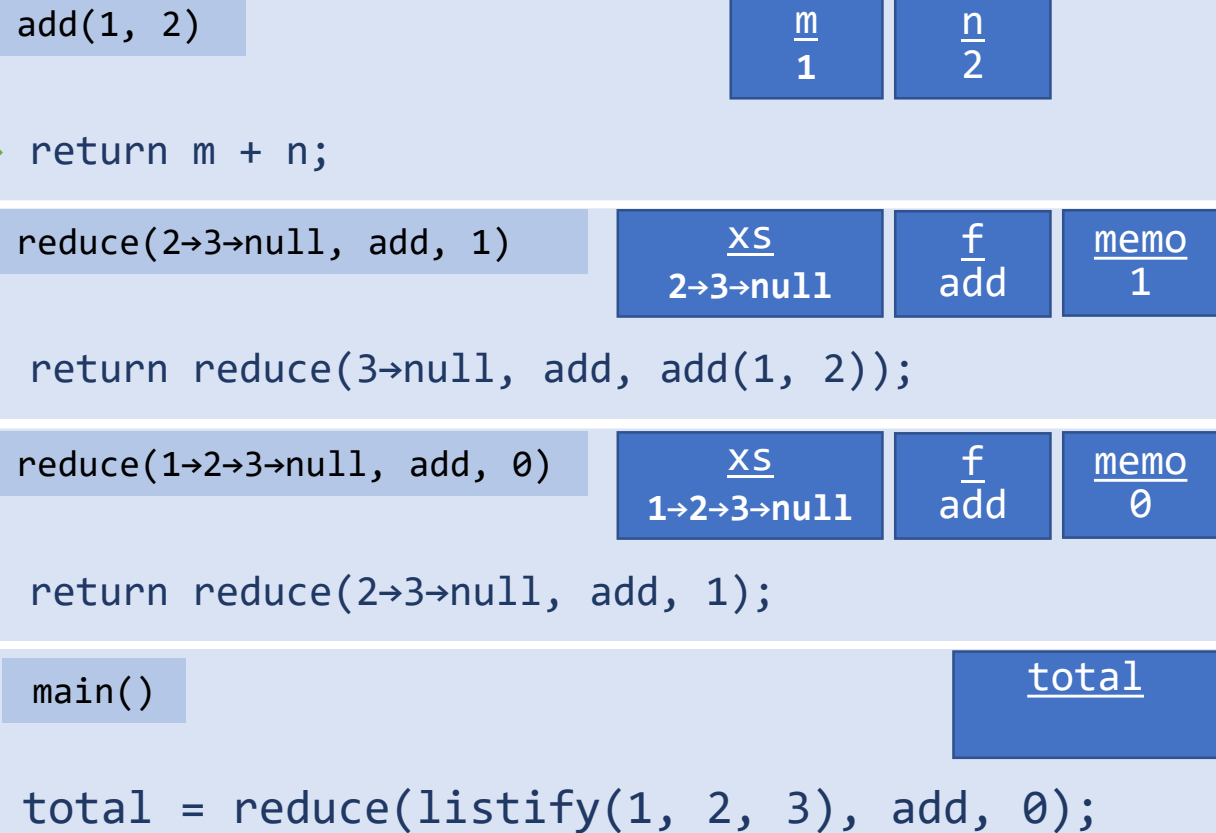
total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
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```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
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main();
```

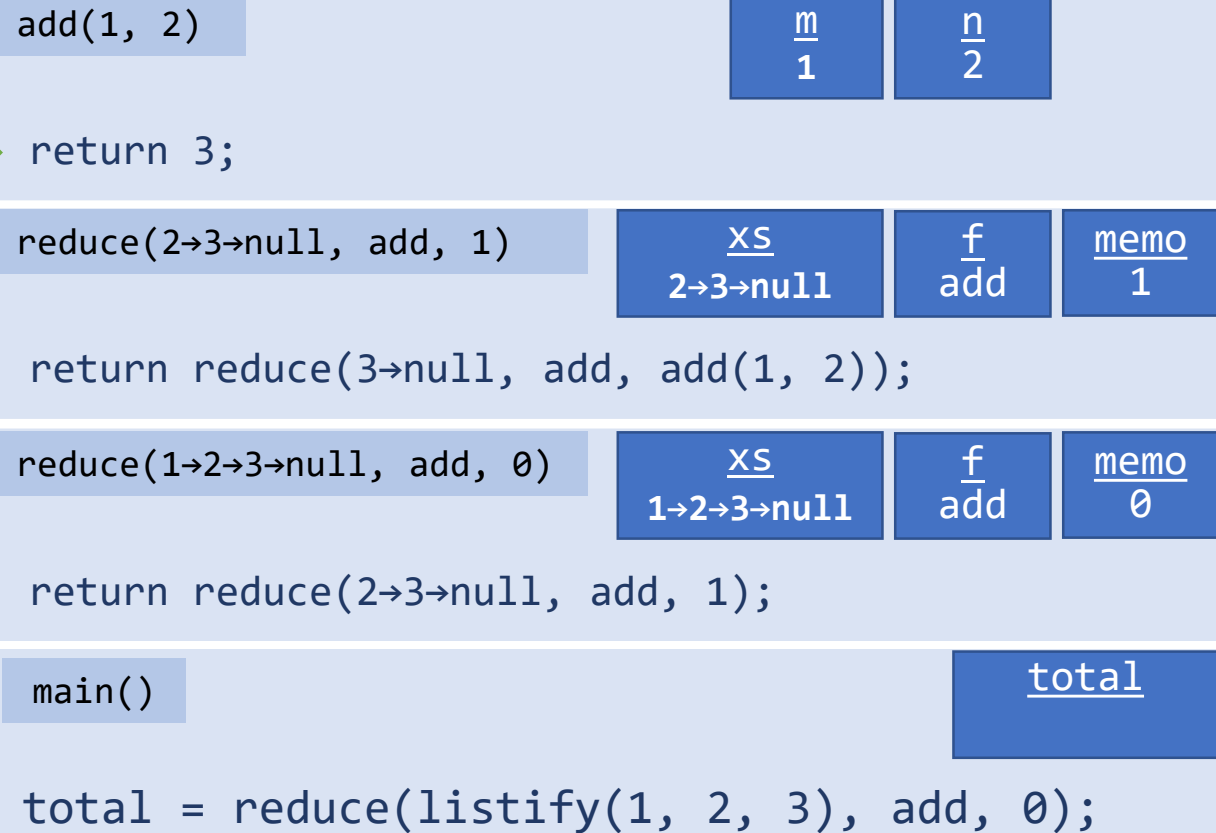
add(1, 2)	$\frac{m}{1}$	$\frac{n}{2}$	
return 1 + 2;			
reduce(2→3→null, add, 1)	$\frac{xs}{2 \rightarrow 3 \rightarrow null}$	$\frac{f}{add}$	$\frac{memo}{1}$
return reduce(3→null, add, add(1, 2));			
reduce(1→2→3→null, add, 0)	$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$	$\frac{f}{add}$	$\frac{memo}{0}$
return reduce(2→3→null, add, 1);			
main()			<u>total</u>
total = reduce(listify(1, 2, 3), add, 0);			

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```


```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



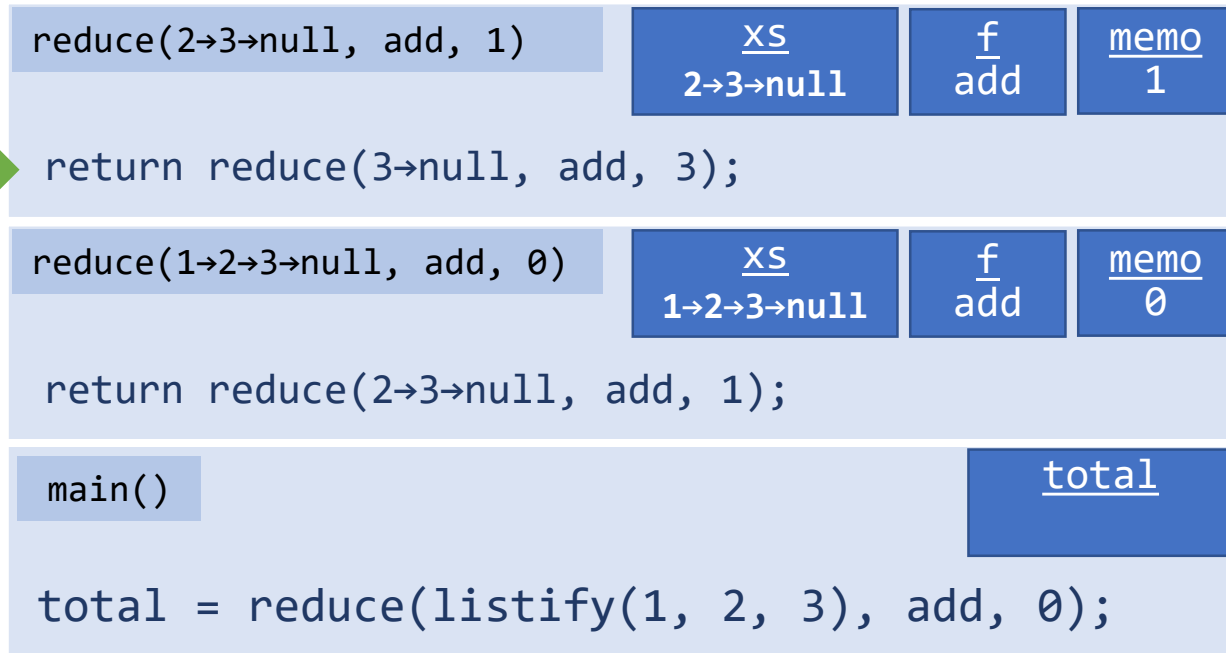

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

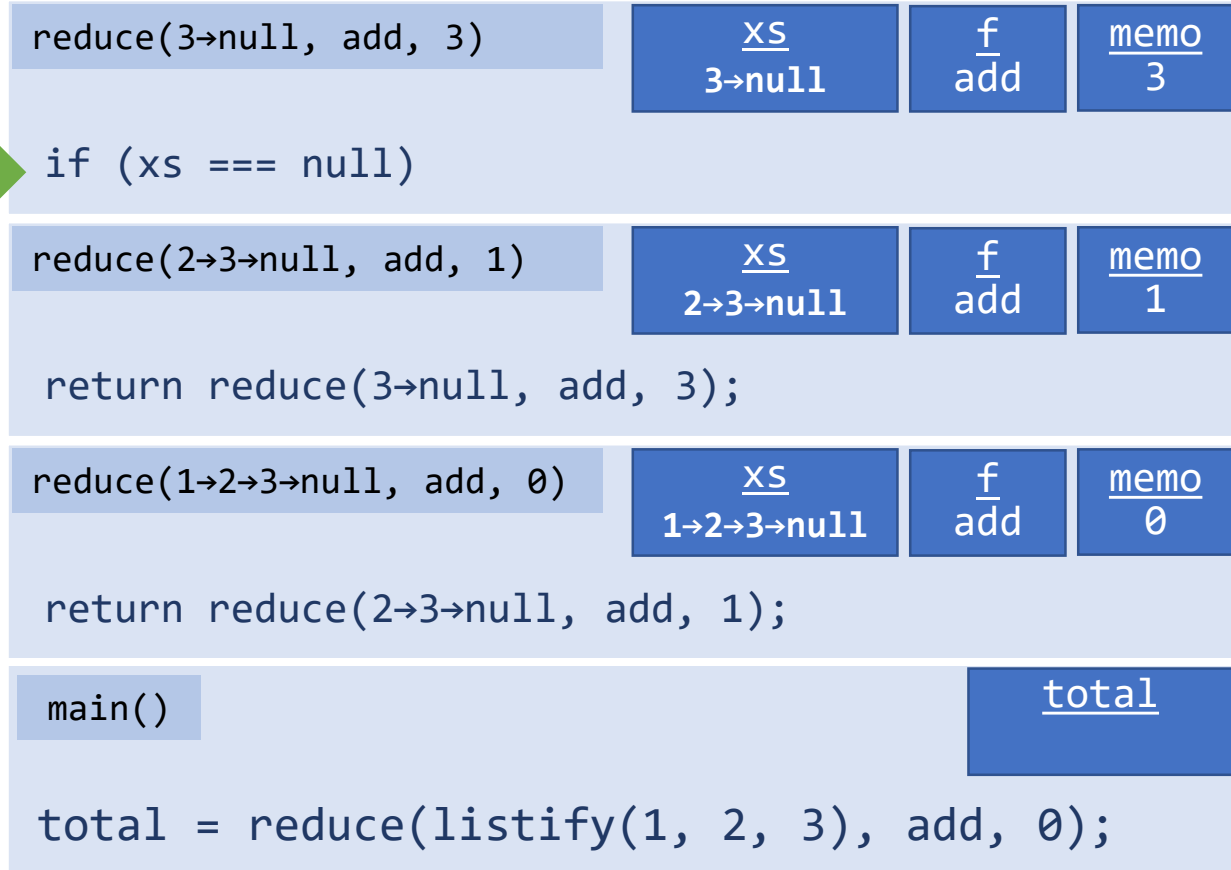


Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

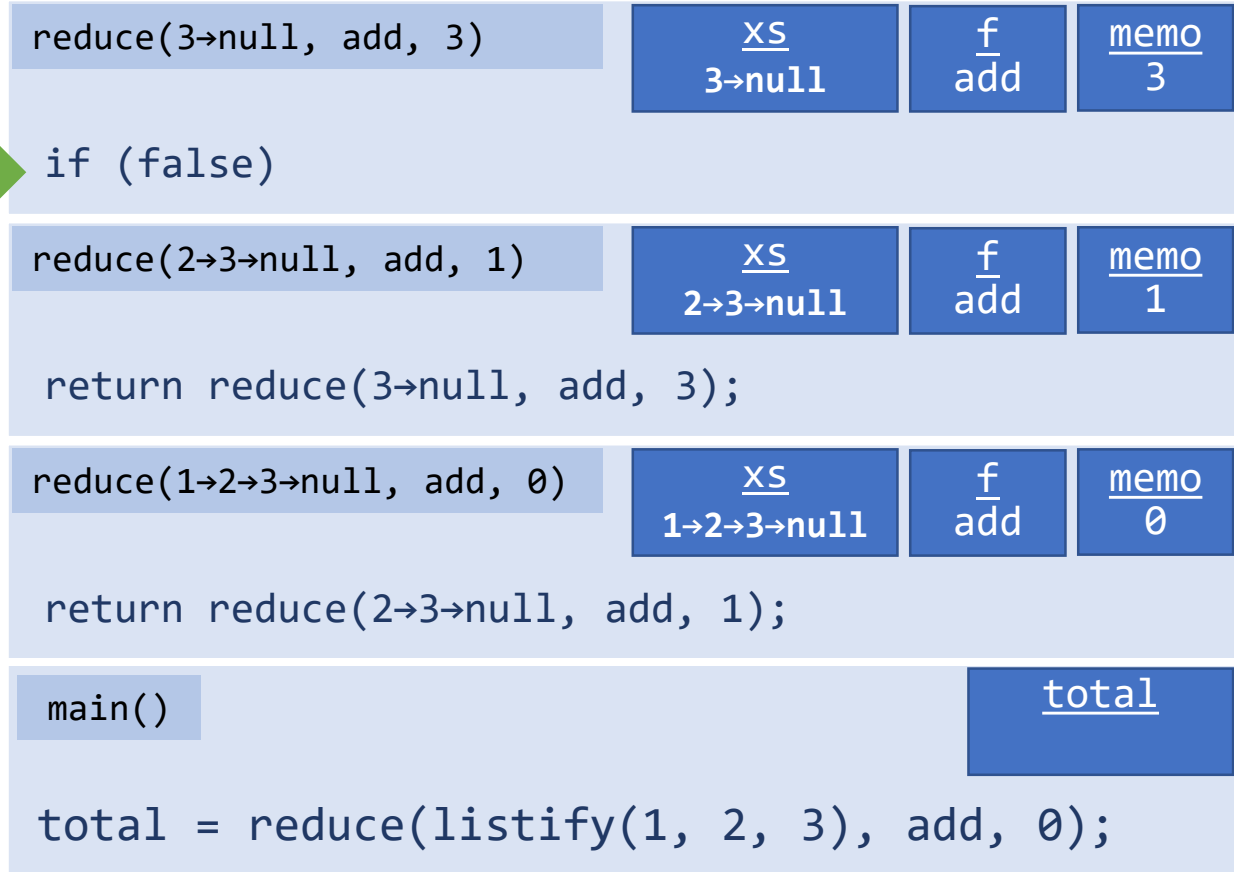


Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```


```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```




Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



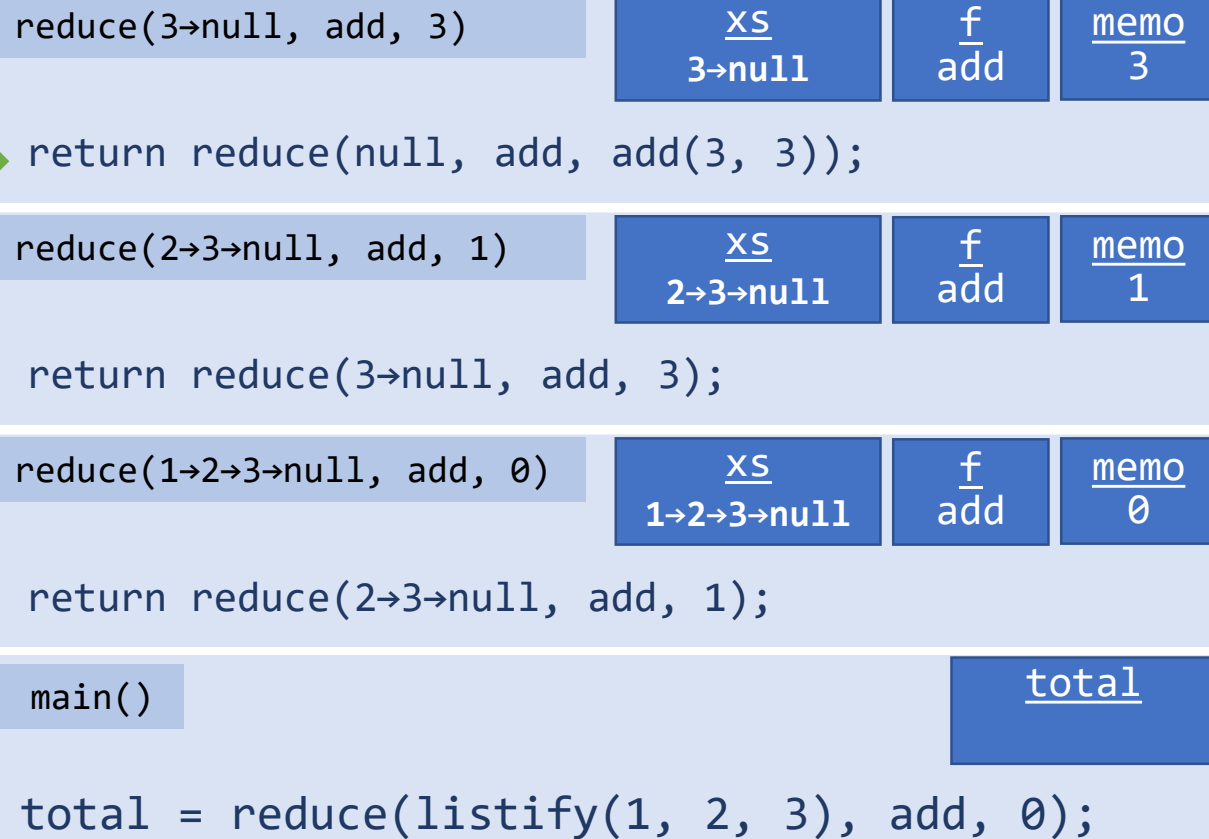
reduce(3→null, add, 3)	<u>xs</u> 3→null	<u>f</u> add	<u>memo</u> 3
return reduce(rest(xs), f, f(memo, first(xs)));			
reduce(2→3→null, add, 1)	<u>xs</u> 2→3→null	<u>f</u> add	<u>memo</u> 1
return reduce(3→null, add, 3);			
reduce(1→2→3→null, add, 0)	<u>xs</u> 1→2→3→null	<u>f</u> add	<u>memo</u> 0
return reduce(2→3→null, add, 1);			
main()			<u>total</u>
total = reduce(listify(1, 2, 3), add, 0);			

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number =  
  return m + n;
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

add(3, 3)

$\frac{m}{3}$

$\frac{n}{3}$

return m + n;

reduce(3→null, add, 3)

$\frac{xs}{3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{3}$

return reduce(null, add, add(3, 3));

reduce(2→3→null, add, 1)

$\frac{xs}{2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{1}$

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number =  
  return m + n;
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

add(3, 3)

$\frac{m}{3}$

$\frac{n}{3}$

return 3 + 3;

reduce(3→null, add, 3)

$\frac{xs}{3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{3}$

return reduce(null, add, add(3, 3));

reduce(2→3→null, add, 1)

$\frac{xs}{2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{1}$

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number =  
  return m + n;
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

add(3, 3)

$\frac{m}{3}$

$\frac{n}{3}$

return 6;

reduce(3→null, add, 3)

$\frac{xs}{3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{3}$

return reduce(null, add, add(3, 3));

reduce(2→3→null, add, 1)

$\frac{xs}{2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{1}$

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

$\frac{xs}{1 \rightarrow 2 \rightarrow 3 \rightarrow null}$

$\frac{f}{add}$

$\frac{memo}{0}$

return reduce(2→3→null, add, 1);


main()

total

total = reduce(listify(1, 2, 3), add, 0);

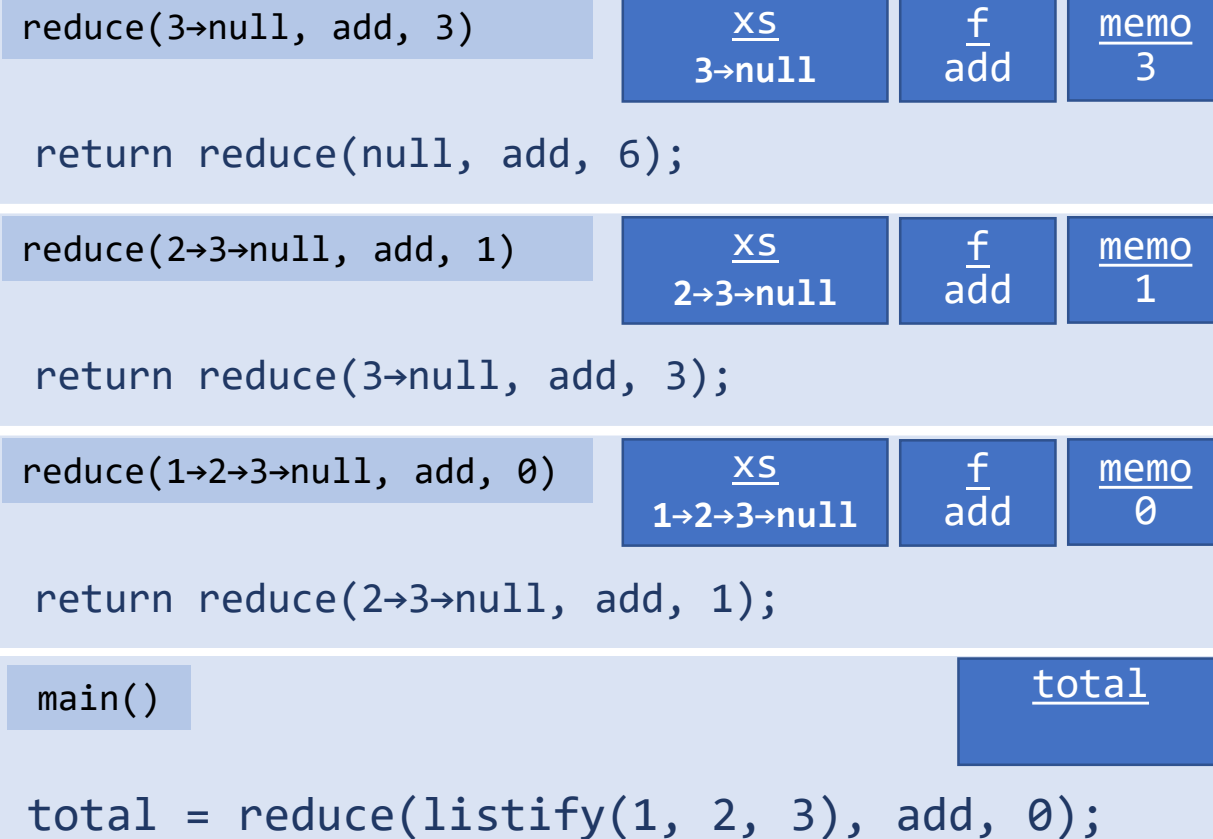
Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(null, add, 6)

xs
null

f
add

memo
6

if (xs === null)

reduce(3→null, add, 3)

xs
3→null

f
add

memo
3

return reduce(null, add, 6);

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(null, add, 6)

xs
null

f
add

memo
6

if (true)

reduce(3→null, add, 3)

xs
3→null

f
add

memo
3

return reduce(null, add, 6);

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(null, add, 6)

xs
null

f
add

memo
6

return memo;

reduce(3→null, add, 3)

xs
3→null

f
add

memo
3

return reduce(null, add, 6);

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(null, add, 6)

xs
null

f
add

memo
6

return 6;

reduce(3→null, add, 3)

xs
3→null

f
add

memo
3

return reduce(null, add, 6);

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return reduce(3→null, add, 3);

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);

main()

total

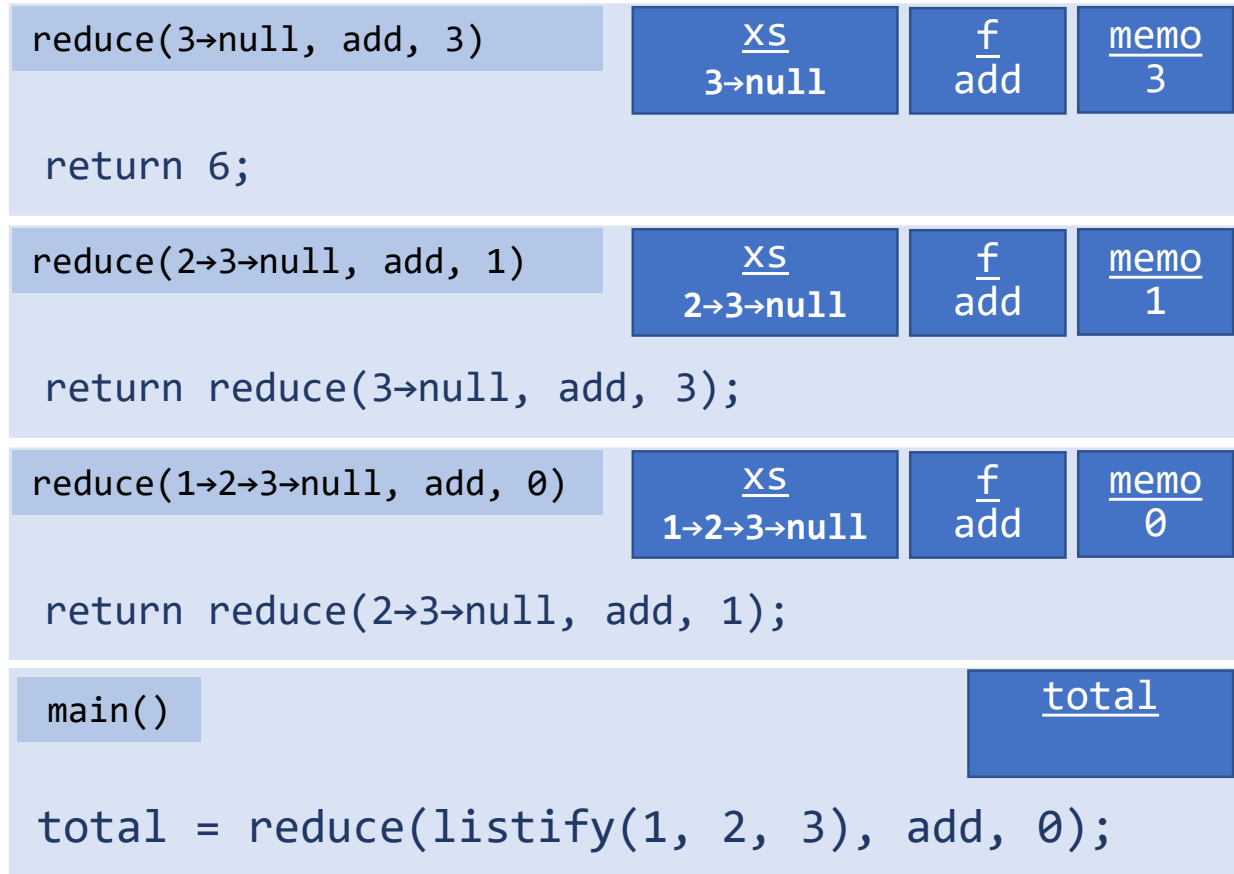
total = reduce(listify(1, 2, 3), add, 0);

Tracing reduce

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```


```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```



Tracing `reduce`

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(2→3→null, add, 1)

xs
2→3→null

f
add

memo
1

return 6;

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return reduce(2→3→null, add, 1);


main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```



```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

reduce(1→2→3→null, add, 0)

xs
1→2→3→null

f
add

memo
0

return 6;

main()

total

total = reduce(listify(1, 2, 3), add, 0);

Tracing **reduce**

```
let reduce = (xs: List<number>, f: Reducer, memo: number): number => {  
  if (xs === null) {  
    return memo;  
  } else {  
    return reduce(rest(xs), f, f(memo, first(xs)));  
  }  
};
```

```
let add = (m: number, n: number): number=> {  
  return m + n;  
};
```

```
export let main = async () => {  
  let total: number;  
  total = reduce(listify(1, 2, 3), add, 0);  
};  
  
main();
```

main()

total
6

total = 6;

Hands-on: Writing a Reducer function and using **reduce**

- Open lec13 / 01-reduce-app.ts
 - Goal: After loading Berry's game data from data/berry-stats-2018.csv, find the most points Joel scored in a game
1. **TODO #1)** Declare a (Reducer) function named **max** that is given two pare a **Game** object named **g** as a parameter and returns a **number**. It should simply return the **points** property of the Game parameter: **g.points**
 2. **TODO #2)** In the main function, assign to the variable **points** the result of mapping the **gamesToPoints** function over the **games** list.
 1. `map(games, gamesToPoints)`
- You should see a list of points values printed after loading your data. Can you trace through how the map function is calling gameToPoints?
 - Check-in on PollEv.com/compunc when this is working

```
// TODO #1 - Write a reducer named max that is given two numbers
// and will return the larger of the two numbers.
let max = (m: number, n: number): number => {
  if (m > n) {
    return m;
  } else {
    return n;
  }
};
```

```
// TODO #2 - Assign to high the result of calling reduce with arguments
// 1. the points array
// 2. your max reducer function
// 3. an initial memo value of 0
let high: number = reduce(points, max, 0);
```

Breaking a Project into Multiple Files

- You can **export** *functions* and *classes* from one TypeScript file

```
export let f = (...
```

```
export class {...
```

- And **import** them into *another* TypeScript file

```
import { <names>, <of>, <functions/classes> } from "./<file>";
```

- Example:

```
import { filter, map, reduce } from "./list-utils";
```

Referencing Imported Files

- Where you see the dot-slash in **"./library"**, this means **"from within the same folder I am in, import the library.ts file"**
 - Note: you do not specify the .ts file extension
- To move up a folder, the dot-dot-slash in **"../super"**, means, **"move up to my parent folder, and import *super.ts* from there"**
- To move up to a parent folder, and back down to a sibling folder: **"../ps01-adventure/index-app"** to reference the file `index-app.ts` in Problem Set 01.
- No stress: when you need to do imports/exports in problem sets, we'll guide you through it.

Library Files

- As we move further into the semester we will break our projects into library files whose classes and functions can be reused by many different apps.
- Let's take a look at an example library file in `lec13 / list-utils.ts`

Hands-on: filter/map/reduce Pipeline

- Open `02-stats-app.ts`

1. First, import the functions `filter`, `map`, and `reduce` from `./list-utils.ts`:

```
import { filter, map, reduce } from "./list-utils";
```

2. Assign to the **filtered** variable the result of calling the **filter** with the **games** List and one of **Predicate** functions below:

```
let filtered: List<Game> = filter(games, PREDICATE);
```

3. Assign to the **values** variable, the result of calling **map** with the **filtered** List and one of the **Transform** functions below:

```
let values: List<number> = map(filtered, TRANSFORM);
```

4. Assign to the result variable, the result of calling **reduce** with the **values** List and one of the **Reducer** functions below (what should the memo be?):

```
let result: number = reduce(values, REDUCER, MEMO);
```

5. Now change your code to find the max # of assists Joel Berry had in a game where he scored less than 15 points. Check-in on [PollEv.com/compunc](https://pollev.com/compunc) when you've got it.

```
import { filter, map, reduce } from "./list-utils";
```

```
// TODO #1  
let filtered: List<Game> = filter(games, fewPoints);  
// TODO #2  
let values: List<number> = map(filtered, toAssists);  
// TODO #3  
let result: number = reduce(values, max, 0);
```

filter-map-reduce Pipeline

Of games that UNC won, how many points did the player score in total?

Outcome	Points
L 76-67	4
W 95-75	20
W 97-57	13
L 103-100	9
L 77-62	22

List<Game>

Filter
→

Outcome	Points
W 95-75	20
W 97-57	13

List<Game>

Map
→

20
13

List<number>

Reduce
→

33

number

filter-map-reduce Data Processing Pipeline

Of games	<u>that UNC won</u>	, what was the	<u>points</u>	<u>total</u>
	<u>that UNC lost</u>		<u>assists</u>	<u>average</u>
	<u>with 3+ assists</u>		<u>fouls</u>	<u>min</u>
	<u>with a block</u>		<u>blocks</u>	<u>max</u>
	<u>etc</u>		<u>etc</u>	<u>etc</u>

Filter: $List<Game> \rightarrow List<Game>$

Map: $List<Game> \rightarrow List<number>$

Reduce: $List<number> \rightarrow number$

Big idea: We can **select any combo** of a filter, map, and reduce sequence.

Result: **(# Filters)** x **(# Maps)** x **(# Reduces)** different analyses.

Type Inference: Variables

- Can you fill in the blank correctly so that there is not an error?

```
let x: _____ = "hello, world";
```

- It's a *string!* As a human, we can *infer* this based on its context in a variable initialization statement.
- Why can't the programming language infer this?
- It turns out... *it can!*

Type Inference: Variables

- TypeScript, and most modern programming languages, can infer the type of a variable based on how it is initialized.
- When you write... **let x = 0;**
... TypeScript infers that you intended to write **let x: number = 0;**
- If you later tried to assign **x** a value that is not a number, you will get a type error.
- To use type inference with variables in COMP110:
You must initialize your variable in the same statement you declare it.
- If you do not declare and initialize at the same time, you should specify the type like you've been doing up until now.

Follow-along

- Open 03-type-inference-app.ts
- Under the **TODO**, remove the explicit type declarations from the variable declarations

```
let games = await csvToList("Game Data", Game);  
let filtered = filter(games, isWin);  
let values = map(filtered, toPoints);  
let result = reduce(values, sum, 0);
```

- Notice there are no errors!
- If you hover your cursor over any variable you will be told exactly what type TypeScript has inferred your variable to be.

Plot Twist: We've relied on variable Type Inference all semester long...

- Whenever we've declared a function, that function is assigned to a variable whose type TypeScript has inferred for us.
- Just like we previously explicitly typed data variables, we can explicitly type function variables, as well:

```
let isWin: Predicate<Game> = (g: Game): boolean => { ...
```

```
let toPoints: Transform<Game, number> = (g: Game): number => { ...
```

```
let sum: Reducer<number, number> = (m: number, n: number): number => { ...
```

- This even more *redundant and verbose* than variable declarations. Isn't type inference awesome?