6.1 Introduction to Vectors

Not all physical qualities can be expressed by magnitude alone. Gravitational pull has magnitude, but it also has downward direction. A pilot needs to set both the speed and direction of flight. Police at an accident scene need to consider the momentum of cars of different masses travelling in different directions.

A **scalar** is a quantity that describes magnitude or size only (with or without units). It does not include direction.

A **vector** is a quantity that has both magnitude and direction.

Scalars	Examples	Vectors	Examples
numbers	1, 3.2, -5, $\sqrt{2}$		
temperature	-5°C, 72°F		
area	24 m^2, 15 cm^2		
distance	1 cm, 5.3 km	**displacement**	1 cm at an angle of 30°, 5.3 km north
speed	10 m/s, 80 km/h	**velocity**	10 m/s upward, 80 km/h west
mass	0.5 g, 23 kg	**force**	10 N downward, 35 N to the left

Example 1 Vector or Scalar?

State whether each of the following is an example of a vector or a scalar quantity.

a) a car travelling at 50 km/h to the east

b) a child pulling a wagon with a force of 100 N at 30° to the horizontal

c) a man's mass of 88 kg

d) a woman skiing at a speed of 25 km/h

e) a parachutist falling at 20 km/h downward

f) acceleration due to gravity on Earth of 9.8 m/s^2 downward

g) the number 5

h) your weight on a bathroom scale

Solution

a) The magnitude is 50 km/h, and the direction is east. This is a vector.

b) The magnitude is 100 N, and the direction is 30° to the horizontal. This is a vector.

c) The magnitude is 88 kg, but there is no direction. This is a scalar.

d) The magnitude is 25 km/h, but no direction is given. This is a scalar.

e) The magnitude is 20 km/h, and the direction is downward. This is a vector.

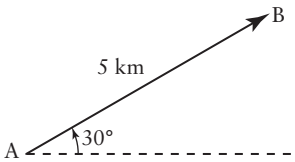
f) The magnitude is 9.8 m/s^2, and the direction is downward. This is a vector.

g) The number 5 has magnitude only, so it is a scalar. It does not matter that it has no units.

h) A scale uses the downward acceleration of gravity to calculate your weight. So, weight on a scale is a vector. Weight is sometimes used as a synonym for force. Your weight, in newtons, is your mass, in kilograms, multiplied by the acceleration due to gravity, which is 9.8 m/s^2 downward on Earth. Although your mass remains constant, your weight would be different on another planet because gravity is different on other planets.

A vector can be represented in several ways:

- In words, for example, as 5 km at an angle of 30° to the horizontal
- In a diagram, as a **geometric vector**, which is a representation of a vector using an arrow diagram, or directed line segment, that shows both magnitude (or size) and direction. The length of the arrow represents, and is proportional to, the vector's magnitude.



- In symbols, using the endpoints of the arrow: $\overrightarrow{AB}$

  Point A is the starting or initial point of the vector (also known as the "tail").

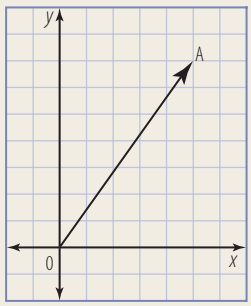
  Point B is the end or terminal point of the vector (also known as the "tip" or "head").

- In symbols, using a single letter: $\vec{v}$

The **magnitude**, or size, of a vector is designated using absolute value brackets. The magnitude of vector $\overrightarrow{AB}$ or $\vec{v}$ is written as $|\overrightarrow{AB}|$ or $|\vec{v}|$.

CONNECTIONS

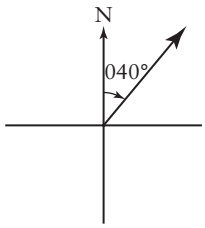
A position vector is a vector whose tail is at the origin, O, of a Cartesian coordinate system. For example, $\overrightarrow{OA}$ is a position vector. It describes the position of the point A relative to the origin. You will make extensive use of this concept in Chapters 7 and 8.



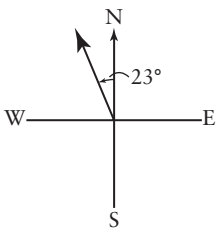
A vector's direction can be expressed using several different methods. In the diagram of $\overrightarrow{AB}$, it is expressed as an angle, moving counterclockwise with respect to a horizontal line. In navigation, vector directions are expressed as bearings.

A **true bearing** (or **azimuth bearing**) is a compass measurement where the angle is measured from north in a clockwise direction.

True bearings are expressed as three-digit numbers, including leading zeros. Thus, north is a bearing of 000°, east is 090°, south is 180°, and west is 270°. For example, a bearing of 040° is an angle of 40° in a clockwise direction from due north. For simplicity, we will use the word *bearing* to refer to a true bearing.



Directions can also be expressed using a **quadrant bearing**, which is a measurement between 0° and 90° east or west of the north-south line. The quadrant bearing N23°W is shown in the diagram.

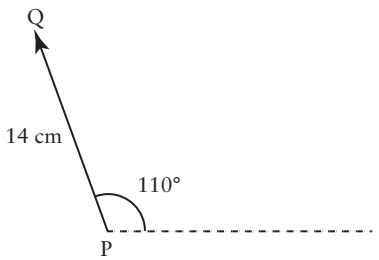


A quadrant bearing always has three components: the direction it is measured from (north in this case), the angle (23°), and the direction toward which it is measured (west). The quadrant bearing N23°W is read as 23° west of north, whereas S20°E is read as 20° east of south. All quadrant bearings are referenced from north or south, not from west or east.

Example 2 Describe Vectors

Describe each vector in words.

a)



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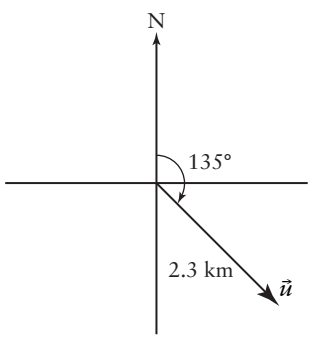

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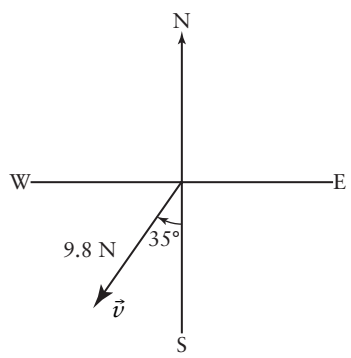


Solution

a) 14 cm at 110° to the horizontal

b) 2.3 km at a true bearing of 135°

c) 9.8 N at a quadrant bearing of S35°W

Example 3 Draw Bearings

Draw a geometric vector with each bearing. Show the scale that you used on each diagram.

a) $\vec{v} = 2$ km at a bearing of 020°

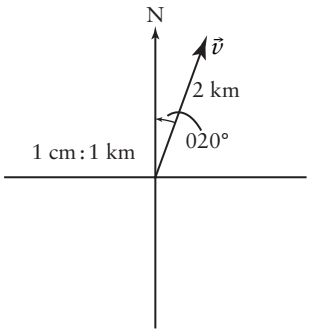
b) $\vec{v} = 4$ km at a bearing of 295°

c) $\vec{u} = 30$ km/h at a quadrant bearing of N40°E

d) $\vec{u} = 40$ km/h at a quadrant bearing of S70°W

Solution

a)



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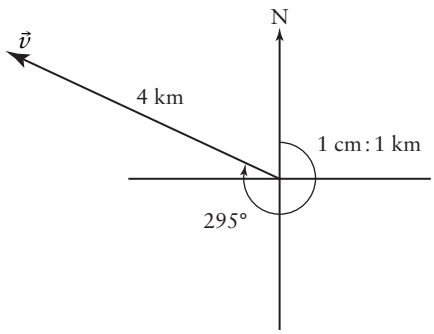

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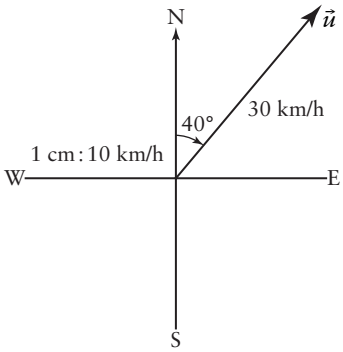
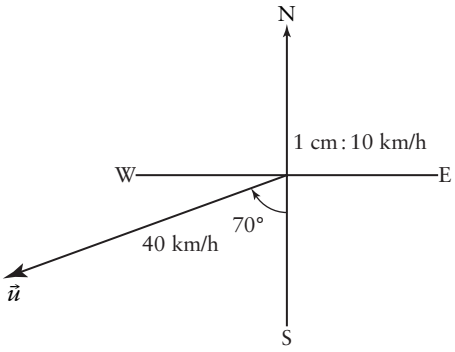


d)



CONNECTIONS

When a compass is installed on an aircraft, it will often give false readings due to magnetic fields generated by radios and other aircraft components. The compass must be tested, and a "compass correction card" is attached to the instrument panel. This process is known as "swinging the compass." True bearings are used universally in aviation, rather than quadrant bearings. When planning a flight, the pilot draws the route and then determines the true track. The local magnetic variation is applied to the true track to obtain the magnetic track. Since the pilot steers the airplane using a magnetic compass, the pilot needs the magnetic track.

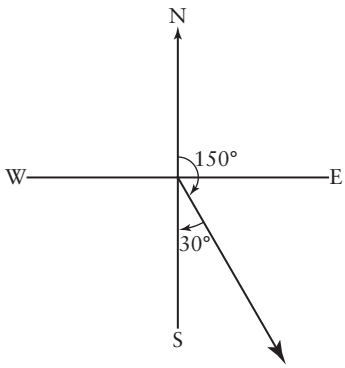
Example 4 Convert Between True Bearings and Quadrant Bearings

a) Write the true bearing 150° as a quadrant bearing.

b) Write the quadrant bearing N50°W as a true bearing.

Solution

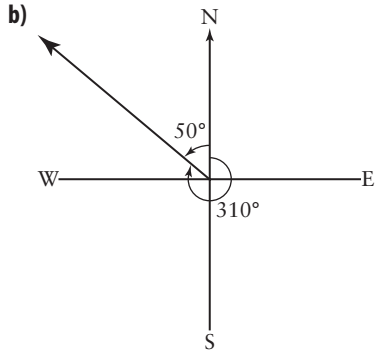
a)



$180° - 150° = 30°$

A bearing of 150° is equivalent to a quadrant bearing of S30°E.

b)

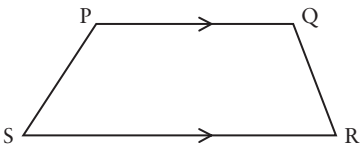


$360° - 50° = 310°$

A quadrant bearing of N50°W is equivalent to a true bearing of 310°.

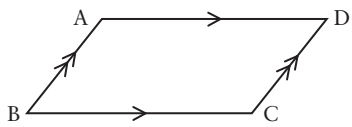
Parallel vectors have the same or opposite direction, but not necessarily the same magnitude.

In trapezoid PQRS, $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ and $\overrightarrow{PQ} \parallel \overrightarrow{SR}$.



Equivalent vectors have the same magnitude and the same direction. The location of the vectors does not matter.

Opposite vectors have the same magnitude but opposite direction. Again, the location of the vectors does not matter. The opposite of a vector $\overrightarrow{AB}$ is written as $-\overrightarrow{AB}$.

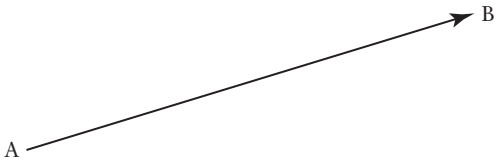


Consider parallelogram ABCD. The table on the next page shows the pairs of equivalent and opposite vectors.

	Vectors	Vector Equation
Equivalent	$\overrightarrow{AB}$ and $\overrightarrow{DC}$	$\overrightarrow{AB} = \overrightarrow{DC}$
	$\overrightarrow{BA}$ and $\overrightarrow{CD}$	$\overrightarrow{BA} = \overrightarrow{CD}$
	$\overrightarrow{AD}$ and $\overrightarrow{BC}$	$\overrightarrow{AD} = \overrightarrow{BC}$
	$\overrightarrow{DA}$ and $\overrightarrow{CB}$	$\overrightarrow{DA} = \overrightarrow{CB}$
Opposite	$\overrightarrow{AB}$ and $\overrightarrow{CD}$	$\overrightarrow{AB} = -\overrightarrow{CD}$
	$\overrightarrow{BA}$ and $\overrightarrow{DC}$	$\overrightarrow{BA} = -\overrightarrow{DC}$
	$\overrightarrow{AD}$ and $\overrightarrow{CB}$	$\overrightarrow{AD} = -\overrightarrow{CB}$
	$\overrightarrow{DA}$ and $\overrightarrow{BC}$	$\overrightarrow{DA} = -\overrightarrow{BC}$

Example 5 Equivalent and Opposite Vectors

a) Draw a vector equivalent to $\overrightarrow{AB}$, labelled $\overrightarrow{EF}$.



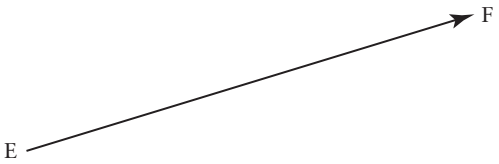
b) Write an expression to show how $\overrightarrow{AB}$ and $\overrightarrow{EF}$ are related.

c) Draw a vector opposite to $\overrightarrow{AB}$, labelled $\overrightarrow{GH}$.

d) Write an expression to show how $\overrightarrow{AB}$ and $\overrightarrow{GH}$ are related.

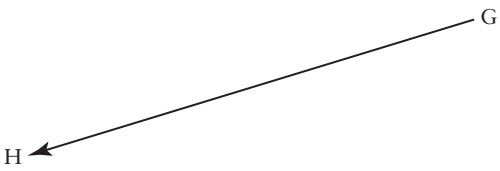
Solution

a)



b) $\overrightarrow{AB} = \overrightarrow{EF}$, because they have the same direction and magnitude.

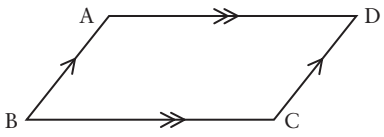
c)



d) $\overrightarrow{AB} = -\overrightarrow{GH}$, because they have the same magnitude but opposite directions.

KEY CONCEPTS

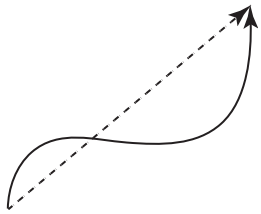
- A vector is a quantity that has both magnitude and direction.
- A scalar is a quantity that describes only magnitude.
- A vector can be represented in words, in a diagram, or in symbols.
- A true bearing (or bearing) is a directed compass measurement, beginning at north and rotating clockwise.
- A quadrant bearing is a compass measurement east or west of the north-south line.
- Equivalent vectors are equal in magnitude and direction. In parallelogram ABCD, $\overrightarrow{AB} = \overrightarrow{DC}$.
- Opposite vectors are equal in magnitude but opposite in direction. In parallelogram ABCD, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are opposite vectors, and $\overrightarrow{AB} = -\overrightarrow{CD}$.



Communicate Your Understanding

C1 Friction causes an ice skater to slow down. Explain why friction is considered a vector.

C2 The curved arrow shows the path of a cyclist. Which represents the displacement, the curved arrow or the dotted arrow? Explain.



A Practise

1. For which of the following situations would a vector be a suitable mathematical model? Provide a reason for your decision.

a) A boat is travelling at 35 km/h east.

b) A boat is travelling at 10 knots.

c) A line segment of length 6 cm is drawn at 30° to the horizontal.

d) A racecar goes around an oval track at 220 km/h.

e) A baby's mass is 2.9 kg.

f) A box is pushed 10 m across the floor.

g) A chair has a weight of 50 N.

h) A cup of coffee has a temperature of 90°C.

i) A pulley system uses a force of 1000 N to lift a container.

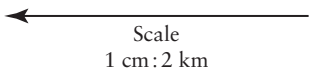
2. State three examples of vectors and three examples of scalars that are different from those in question 1.

3. Copy and complete the table. Explain your answers.

Quantity	Vector or Scalar?
$\vec{v}$	
$\lvert\vec{v}\rvert$	
6	
$-\overrightarrow{CD}$	
$-\lvert\overrightarrow{AB}\rvert$	
π	
$-\sqrt{7}$	

4. Describe the magnitude and direction of each vector. Describe each vector in words and in symbols.

a)



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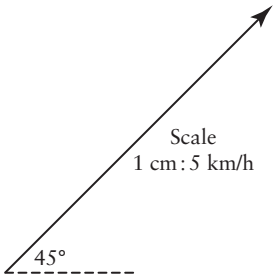

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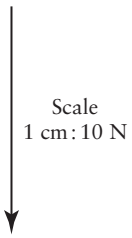


5. Convert each true bearing to its equivalent quadrant bearing.

a) 070° **b)** 180° **c)** 300°

d) 140° **e)** 210° **f)** 024°

6. Convert each quadrant bearing to its equivalent true bearing.

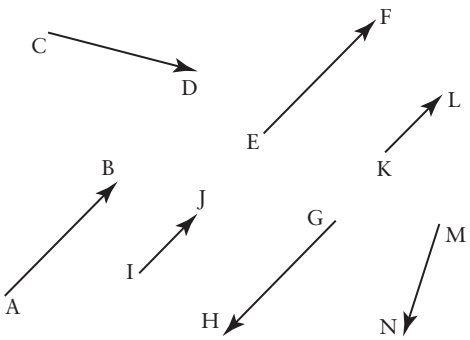
a) N35°E **b)** N70°W **c)** S10°W

d) S52°E **e)** S18°E **f)** N87°W

7. a) Which vectors are parallel to vector $\overrightarrow{AB}$?

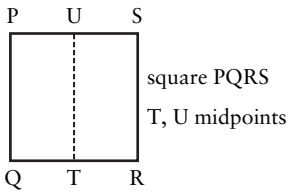
b) Which vectors are equivalent to vector $\overrightarrow{AB}$?

c) Which vectors are opposite to vector $\overrightarrow{AB}$?



8. Name all the equivalent vectors in each diagram.

a)



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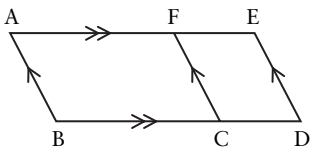
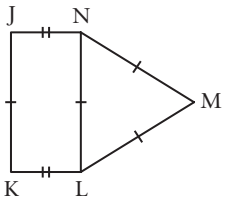


c)



9. State the opposite of each vector.

a) 200 km east

b) 500 N upward

c) 25 km/h on a bearing of 060°

d) 150 km/h on a quadrant bearing of S50°W

e) $\overrightarrow{AB}$

f) $\vec{v}$

10. Describe a vector that is parallel to each vector in parts a) to d) of question 9.

11. Use an appropriate scale to draw each vector. Label the magnitude, direction, and scale.

a) displacement of 40 m east

b) velocity of 100 km/h at a bearing of 035°

c) force of 5000 N upward

d) acceleration of 10 m/s^2 downward

e) velocity of 50 km/h at a quadrant bearing of S20°E

f) displacement of 2000 miles on a bearing of 250°

g) force of 600 N at 15° to the horizontal

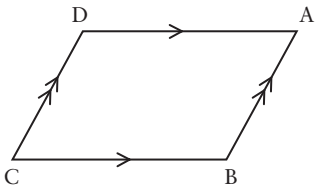
h) two forces of 500 N at 30° to each other

B Connect and Apply

12. The tread on a car's tires is worn down. Which is the most likely cause: distance, speed, displacement, or velocity? Explain.

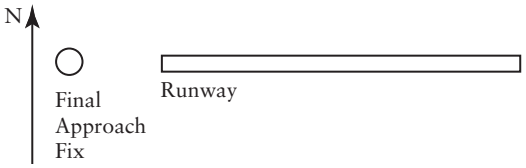
13. Given parallelogram ABCD, what is the relationship between

a) $\overrightarrow{AB}$ and $\overrightarrow{DC}$? **b)** $\overrightarrow{BC}$ and $\overrightarrow{DA}$?



Justify your response.

14. Chapter Problem Air traffic control (ATC) will often assign a pilot a velocity to fly, known as an approach vector, such that the aircraft arrives over a point known as the final approach fix (FAF) at a particular time. From this fix, the pilot turns toward the runway for landing. Suppose that an aircraft is 60 km west and 25 km north of the FAF shown. ATC would like the aircraft to be over the FAF in 10 min. Determine the approach vector to be assigned.



C Extend and Challenge

15. The standard unit of measurement of force is the newton (N). It is the force needed to accelerate a mass of 1 kg at 1 m/s^2.

On Earth's surface, a mass of 1 kg requires a force of 9.8 N to counteract the acceleration due to gravity of 9.8 m/s^2 downward. Multiplying the mass by this acceleration gives the weight. On the Moon, the acceleration due to gravity is 1.63 m/s^2 downward.

a) A person has a mass of 70 kg. What would this person weigh on Earth? on the Moon?

b) A truck has a mass of 2000 kg. What would it weigh on Earth? on the Moon?

c) When a certain object is floating in water on Earth, 75% of it is submerged. If water were found on the Moon, and the same object was floating in it, how much of it would be submerged?

16. Prove or disprove each statement.

a) If $\vec{a} = \vec{b}$, then $|\vec{a}| = |\vec{b}|$.

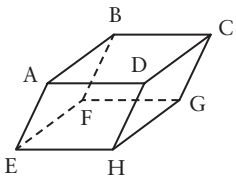
b) If $|\vec{a}| = |\vec{b}|$, then $\vec{a} = \vec{b}$.

17. The diagram below is a parallelepiped.

a) State one equivalent vector and one opposite vector for each of the following.

i) $\overrightarrow{AB}$ **ii)** $\overrightarrow{ED}$

iii) $\overrightarrow{BD}$ **iv)** $\overrightarrow{FB}$



b) Does $\overrightarrow{AG} = \overrightarrow{CE}$? Explain.

CONNECTIONS

A parallelepiped is a solid whose six faces are parallelograms.

18. Math Contest Which expression is equivalent to the zero vector?

a) $\overrightarrow{QB} + \overrightarrow{YW} + \overrightarrow{BY}$ **b)** $\overrightarrow{CK} - \overrightarrow{KJ} - \overrightarrow{JC}$

c) $\overrightarrow{EU} - \overrightarrow{EP} - \overrightarrow{PU}$ **d)** $\overrightarrow{KJ} - \overrightarrow{KC} - \overrightarrow{JC}$

19. Math Contest The centroid of a triangle is where the three medians of a triangle meet. $\triangle$DEF has vertices D(1, 3) and E(6, 1), and centroid at C(3, 4). Determine the coordinates of point F.

20. Math Contest Quadrilateral ABCD has vertices at A(13, 9), B(14, 2), C(7, 1), and D(5, 5). Show that ABCD is cyclic; that is, all four points lie on a circle.