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Teaching Tutorial 3: Teaching Mathematics Problem Solving Using Schema-based Strategy Instruction

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About the Author

Dr. Asha Jitendra

Dr. Asha Jitendra is a professor and coordinator of the special education program at Lehigh University. She earned her doctorate at the University of Oregon. Dr. Jitendra's research and teaching interests are in the instruction of mathematics skills and intermediate reading comprehension skills for students with learning disabilities. In addition, her research has focused on curriculum analyses, dynamic assessment, and design of effective instruction for students with learning disabilities. Dr. Jitendra has presented nationally and internationally on effective instructional strategies for enhancing the academic performance of children with learning disabilities.

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1. What is schema-based strategy instruction in mathematics problem solving?

Schema-based strategy instruction teaches students to represent quantitative relationships graphically in order to solve problems. Teaching students to represent quantitative relationships graphically helps them to solve most arithmetic (and, ultimately, algebraic) problems easily. Schema-based strategy instruction involves the use of specific schematic diagrams that facilitate translation and solution of mathematical problems. As with most other effective methods of instruction, one explicitly teaches students strategies for independently solving academic tasks.

Schema-based strategy instruction teaches students to represent problems graphically.

The essential elements of the strategy are:

1. **Problem identification:** Students learn to recognize the type of problem or the problem schema (i.e., is this problem a change, group, or compare problem?).
2. **Problem representation:** Students learn to translate a problem from words into a meaningful, systematic graphic representation (diagram). For these concepts, teachers must teach students to identify the unique features of each type of problem and how to represent the relevant information in the problem situation using the diagrams.
3. **Problem solution:** Students learn to select and apply appropriate mathematical operations based on the diagram and problem type. The key teaching points for problem solution are to
   (a) plan to solve the problem by identifying the action procedure (e.g., counting, adding, subtracting) and sequence of necessary steps and
   (b) complete the operation identified to solve the problem.

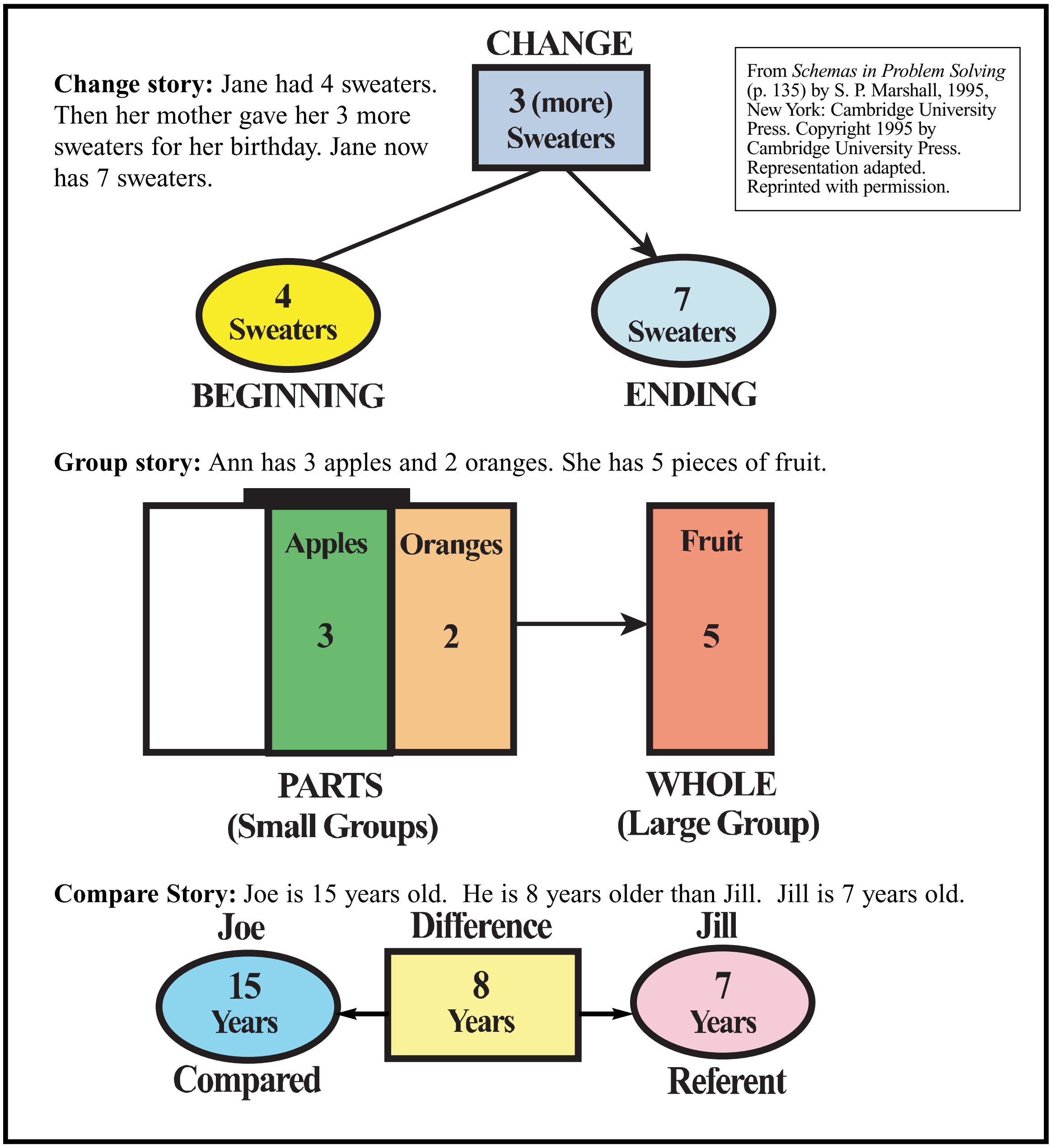
Change problems involve different beginning and end amounts. Group problems involve at least 2 groups combined to form a large group. A compare problem involves a referent and comparison value.

For students to use their knowledge to solve problems, the problem must be translated and organized into an understandable mental representation (i.e., schema or diagram). The diagram, in turn, helps the student access the relevant information to solve the problem. The diagrams serve to reduce a learner's cognitive processing load and make available mental resources for engaging in problem analysis and solution.

Teaching students to use diagrams that include a consistent structure and predictable placement of information for each problem type is different from asking students to draw a picture to represent a problem. Although teaching students to represent problems graphically requires that students have mastery of fundamental arithmetic operations, it goes beyond repetitive use of skills in adding, subtracting, multiplying, and dividing; it helps students to understand when to use those operations by providing them a system for organizing and identifying the arithmetic problem. In sum, schema-based strategy instruction is a viable approach for enhancing students' conceptual knowledge about problem solving, because it emphasizes instruction that focuses more on the understanding of problem structure.

This tutorial addresses the use of specific diagrams for solving change, group, and compare problems. Figure 1 illustrates these three different types of problems.

Figure 1: Sample story situations and schemata diagrams for change, group, and compare problem types.



2. How do we know that schema-based strategy instruction is effective?

The effectiveness of schema-based strategy instruction has been demonstrated in several research studies. Researchers have found instruction to be effective in improving the performance of normally-achieving students, students with learning disabilities, serious emotional disturbances, mild mental retardation, attention deficit hyperactive disorders, and those who are at-risk for mathematics failure.

Table I summarizes some of the research documenting the effectiveness of using schema-based strategy instruction.

Table 1: Selected Studies of Schema-based Strategy Instruction

Study	Subjects (gender, age, grade level, disability, race)	Setting/grouping arrangement/task/ intervention duration	Findings
Jitendra, DiPipi, & Grasso (2001)	5 males, 7 females; 9 to11 years; grade 4; LD/ND; white	General education classroom; small group; one-step and two-step multiplication and division problems.	GR instruction improved pretest to posttest problem solving performance for 9 of the 12 students. Generalization effects were seen for 7 of 11 students.
Jitendra, DiPipi, & Perron-Jones (2002)	3 males, 1 female;13-5 to 13-10 years; grade 8; LD/ADHD; white/African American	Resource room; individual; one-step multiplication and division problems.	All students' word problem solving performance increased substantially with GR instruction. The improved performance was maintained 10, 5-1/2, and 2-1/2 weeks following the termination of instruction for Sara, Tony, and Percy, respectively. In addition, the effects of instruction generalized to novel word problems, including multistep problems, for all 4 students. Student and teacher interviews indicated that the strategy was beneficial in solving problems.
Jitendra & Hoff (1996)	1 male, 2 females; 8-10 to 10-10 years; grades 3 and 4; LD: white	Room adjacent to the students' classrooms; individual; one-step addition and subtraction problems.	GR instruction led to word problem solving gains for all three students. In addition, maintenance of word problem solving was seen two to three weeks following the termination of the intervention. Student interviews indicated that the strategy was beneficial in solving problems.
Jitendra, Hoff, & Beck (1999)	2 males, 2 females; 12-6 to 14-0 years; grades 6 and 7; LD; white/African American	Library; individual; one-step and two-steps addition and subtraction problems.	GR instruction led to an increase in one- and two-step problem solving performance for all four students. Further, these results were maintained at a 2 and 4 week follow-up, and all four students' performance on two-step word problems (M = 86% correct) at the end of the study surpassed that of the normative sample (M = 54% correct). Student treatment acceptability ratings revealed that the strategy was helpful in solving word problems.
Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)	17 males, 17 females; 10-4 mean years; grades 2, 3, 4, and 5; LD/EMR/SED/ND at-risk; white/African American/Hispanic	Various school rooms; small group; one-step addition and subtraction problems.	Although students in both instructional groups showed an increase in problem solving performance from the pretest to posttests, maintained the positive effects, and generalized to novel problems, the differences between groups on the posttest (ES = 0.65), delayed posttest (ES = 0.81), and generalization test (ES = 0.74) were significant, favoring the GR instruction group. In addition, scores on the immediate posttest (77% correct) and delayed posttest (81% correct) for the schema group approached those of a normative sample of third graders (mean = 82% correct).
Xin & Jitendra (2002)	11 males, 11 females, 12-8 mean years; grades 6, 7, and 8; LD/SED/ND at-risk; white/African American/Hispanic	Various school and non-school rooms; small group; one-step multiplication and division problems.	Students receiving GR instruction significantly outperformed students in the traditional instruction (TI) group on the posttest (ES = 1.67), maintenance test (1 to 2 weeks later) (ES = 2.53), and follow-up test (3 weeks to 3 months later) (ES = 2.72). GR instructed students also significantly outperformed the TI group on transfer problems (i.e., structurally similar, but more complex) (ES = 0.89). In addition, the performance of students receiving GR instruction surpassed that of a normative sample of nondisabled sixth graders on the posttest (ES = 1.47), maintenance test (ES = 1.36), and follow-up test (ES = 2.06).

Note: LD=learning disabilities, EMR=educable mental retardation; SED=serious emotional disturbance; ND=nondisabled; GR=graphic representational. Additional references are included at the end of this tutorial.

3. When should one use schema-based strategy instruction?

A teacher can use schema-based strategy instruction whenever students are to solve arithmetic story problems that involve one-step or multistep operations. Schema-based strategy instruction is most appropriate for students who are at-risk for mathematics failure or may have learning, attention, organization, and memory difficulties. Research indicates that it is effective for students as young as second grade and as old as college. It can be implemented with individual students or during small- and whole-group instruction. However, consider the following conditions when implementing schema-based strategy instruction:

- **Are students exposed to several problem-solving strategies (e.g., working backwards, using a model, guess and check) at the same time?** If so, the benefits of the schema-based strategy may be compromised for students with disabilities, who may experience cognitive information overload (see Jitendra, DiPipi, & Grasso, 2001). In other words, students will benefit from learning a single strategy at a time and learning that strategy to mastery.
- **How difficult should the word problems be when students are first learning the schema-based technique?** For students to use diagrams in problem solving, they must first master the strategy. Therefore, they must initially be taught the strategy using problems they can read and understand. If the problems are too difficult for students, their ability to understand and map the information onto the diagram could be undermined.
- **Are cooperative learning groups appropriate for schema-based strategy instruction?** Every student in the group must have sufficient practice to master the strategy. Students with learning disabilities will need more practice opportunities. This is important to prevent instances of students with disabilities assuming a passive role in the group (see Jitendra et al., 2001).
- **When implementing schema-based strategy instruction in general education classrooms, do some students need more intense and systematic instruction than others?** Yes. Some students, particularly those with disabilities, may need more practice opportunities and more explicit instruction in using diagrams to problem solve. Teachers should consider the importance of appropriately mediating instruction (e.g., providing extended practice) for students with disabilities to be successful problem solvers (see Jitendra et al., 2001).

4. What does one need to prepare to teach a schema-based strategy?

Materials needed:

- Scripted lessons,
- Change, group, and compare problems,
- Posters or note cards with the problem-solving strategy steps for change, group, and compare problems, and
- Diagrams for change, group, and compare problems.

Scripted Lessons

It is useful to script initial lessons so that teachers can emphasize specific features of the process behaviors (e.g., pointing to diagrams, presenting information, asking questions) instead of planning as they instruct. Teaching procedures must be carefully designed to include clear and consistent wording. In other words, when problem identification, translation, and solution are taught, teachers must use the same words, clues, and questions to guide students through the process each time. The explanations must be explicit and emphasize key vocabulary (e.g., beginning, change, ending, small groups, larger group, compared, referent, difference) as a means to ensure that students easily understand the information presented. The objective is to provide students with the words and the process to solve problems. This process must become automatic for them and will not if the procedures or vocabulary change from problem to problem. Appendix E contains examples of scripted lessons for each stage.

When problem identification, translation, and solution are taught, teachers must use the same words, clues, and questions to guide students through the process each time.

Word Problem Tasks

The first step before instruction is to create or select a variety of story problem situations from appropriate grade level mathematics curricula that involve change, group, and compare problem types. *Until students master the strategy for each problem type, use story problem situations that include NO UNKNOWN information. Once students have mastered the strategy, begin to include story problem situations with UNKNOWN information.*

Initial story situations and word-problem worksheet sets should include <u>only</u> the specific problem type being taught.

The second step before instruction is to create teaching examples and worksheet practice sets. Initial story situations and word-problem worksheet sets should include **only** the specific problem type (e.g., *change, group, or compare*) being taught. Only once students have mastered completing diagrams for a certain problem type can they move on to the next and to discriminating between types. For additional worksheet sets, include all problem types previously taught so that students can discriminate the different problem types and apply the schema-based strategy to each type. Appendix A includes examples of story situations for initial instruction. Appendix B includes example story situations for use after initial mastery.

Posters or Note Cards

When students are first learning the schema-based strategy, posters or note cards with the strategy steps (e.g., rules for identifying the operation) are critical for success. The poster or note card can also serve as a self-instructional sheet during independent learning. Teachers may want to provide individual note cards as a scaffold while students are completing problems during practice trials until they can independently verbalize the rules. Appendix C includes examples of change, group, and compare posters.

Schematic Representations (Diagrams)

Initially, when students are learning to represent information from story situations or problems, they need diagrams for each problem type provided for them. We have designed diagrams that teachers can use to place on student worksheets and to use as models during the teacher-modeling phase of instruction. Teachers can make overhead transparencies or large models of these diagrams on poster board and laminate them. Remember that a teacher should introduce only one type of problem at first, then introduce a second type and stick with those two until students can reliably discriminate between the two types and solve both types consistently. Once students can work successfully with the first two types of problems, then the teacher can introduce the third type and integrate it with the first two types. Appendix D contains blank diagrams for use.

5. How long might it take to prepare to teach a schema-based strategy?

Once all these materials are in place, the amount of preparation and planning time is minimal (no more than routine classroom planning time). The important factor for teachers is to be familiar with the scripts and be consistent in delivery of instruction. It takes approximately 6 to 9 40-minute sessions for each problem type in order for students to learn to apply the strategy independently.

6. How does one implement schema-based strategy instruction?

The research on schema-based problem solving instruction indicates the importance of using scripted lessons, appropriate word problem tasks, posters or note cards, and schematic diagrams. Initial instruction should be explicit, with teachers describing and modeling all the steps of the strategy and providing maximum supports. Instruction should begin by teaching problem identification and translation of all problem types and move to problem solution only after identification and translation are mastered. Later, after all problem types and strategy steps are mastered, students can begin to apply the strategy steps more independently with fewer supports. Eventually, teachers can remove note cards and diagrams as students become more proficient with the procedure.

> ***Instruction should begin by teaching problem identification and translation of all problem types and move to problem solution only after identification and translation are mastered.***

Steps for Introducing a Schema-based Strategy

(Appendix E contains scripted lessons for each step.)

1. Provide an anticipatory set in which you present the goal of the lesson and use a focus statement to gain student's attention as well as explain the relevance of the lesson and its link to previous learning.
   "Today, you will learn a type of addition-subtraction problem called <u>change</u>. You will learn to organize information in <u>change</u> stories using diagrams. Later, you will learn to solve real word problems. Learning to solve word problems can help you use math in every day life."

2. Introduce the strategy steps on the poster, note card, or transparency and model application of the steps using several examples. Initially (Phase 1), introduce only the first two steps in the context of story situations; use examples that have no unknown information. For Step 1, the teacher reads the problem, retells it in his or her own words, and describes the features of the problem.
   "This is a change problem, because there is a change to the beginning amount that increases or decreases the ending amount."

   Next, using an overhead projector or other means of display, the teacher can show the diagram for the specific problem. For Step 2, the teacher underlines important information and maps it onto the diagram. Present this step by asking questions and modeling by emphasizing key information.
   "What does this story talk about?"
   "had 4 sweaters" tells that Jane started with 4 sweaters, *"gave her 3 more"* tells that there was a change to the beginning amount, and *"now has 7 sweaters,"* talks about the ending amount.

   Finally, summarize the key features of the story using the completed diagram.

3. Next introduce the 6 strategy steps using word problems (Phase 2) in which the unknown may involve different information (e.g., beginning, change, ending, smaller group(s), larger group, compared, referent, or difference amounts) to be solved. Instruction to apply steps 1 and 2 are similar to Phase 1, with the exception that during Step 2 students learn to flag the unknown information using a question mark (?). For Step 3, present the strategy to determine whether to add or subtract to solve the problem. For all three problem types, explain that you first need to determine the total amount in the problem. Using the instructions on the poster or note card, model finding the total in the problem and write T under the appropriate set (e.g., large group). Model application of the rule.
   "If the total is given, we subtract to find the part. If the total is not given, we add the parts to find it."

   For Step 4, present the math sentence and carry out the operation (addition or subtraction).

   For Step 5, write the complete answer. Finally, model checking the answer to see if it makes sense for Step 6.

4. Have students demonstrate the schema-based strategy when provided with several different examples.

Important Points to Remember When Introducing a Schema-based Strategy

1. Make sure that sufficient time is available to emphasize the key features of the diagram AND strategy and for students to process the information. Sessions can run anywhere from 25 minutes to 45 minutes.

2. Although instruction should be explicit and overt modeling of correct story mapping by the teacher is important, it is equally necessary for the teacher to employ frequent student exchanges to facilitate the identification of critical elements of the story. Check student understanding and provide appropriate feedback on use of each strategy as needed.

   Remember to keep the vocabulary and the process questions consistent.

3. Present the problem-solving strategy using 2-4 modeled examples, 3-5 guided practice examples, and 5-10 independent practice examples. The number of examples will vary based on the learner.

4. Make sure that the language for story situations is simple and straightforward; it is crucial that students can read and understand the stories. For word problems, create problems or select problems from students' texts and model the use of the strategy using a range of easy to difficult problems (indirect language, irrelevant information). Additionally, provide guided practice using a range of problems before students are to solve problems independently.

Steps for Reviewing a Schema-based Strategy

(Appendix E contains scripted lessons for each step.)

1. Once students have learned the steps for using schema-based strategies to solve each type of problem, present a review of the three different problem types. Ask students to read the problem with a partner or independently and to identify the problem type. Present worksheets that include all three problem diagrams, and ask students to select the appropriate diagram to map information from the story situation or word problem.

2. Check student worksheets at the end of each session and provide appropriate feedback on strategy usage as needed.

Important Points to Remember when Reviewing

1. Provide feedback if students have difficulty discriminating between the different problem types or applying the strategy steps. For example, on the first day of review you may have to correct the student if he or she inaccurately identifies the problem type and may have to remind students to check the strategy steps and apply them in the correct sequence.

2. Provide lots of practice solving word problems independently to aid maintenance of the strategy. Present a variety of problems to promote generalization of the problem solving skill.

3. Cue students to use the strategy whenever they have to solve word problems.

Steps for Fading Supports

(Appendix E contains scripted lessons for each step.)

1. Once students have learned the steps of a schema-based strategy to a level where they can independently verbalize them, remove the posters or note cards.

2. Once students meet criterion (e.g., 90% correct on 2 days) in solving word problems on daily worksheets, remove the diagrams from worksheets. Tell students that they have been doing a really good job solving problems using the diagrams and that it is now time to remove them. Encourage the students to generate their own diagram to represent the problem. Tell them that their diagram should not only help them illustrate the problem features, but should also be more efficient-simpler—than actually drawing the diagrams.

Important Points to Remember when Fading Supports

1. Make sure that students are proficient in verbalizing the strategy steps and solving problems using diagrams before removing the posters or note cards and diagrams.

2. If a student has difficulty creating his or her own diagram, the teacher or another student in the classroom can share their diagram.

3. Cue students to use the strategy and to use their diagrams whenever they have to solve word problems.

7. How does one know whether the schema-based strategy instruction is working?

Initially, assess students' performance in solving one problem type. Later, when students complete instruction in the use of the strategy steps for all problem types, present tests that include all problem types. In addition, evaluate students' maintenance of strategy use over time and transfer of problem solving skill to solve novel and complex problems. Finally, assess student satisfaction to determine the benefits of the strategy instruction.

If schema-based instruction is not having the desired effects, ask yourself the following questions:

- Has the student mastered the prerequisite skills to a criterion level (e.g., identifying the different problem diagram)?
- Has sufficient modeling of strategy steps using several examples and explanations been provided?
- Has systematic and varied practice been provided?

Initially, assess students' perform-ance in solving one problem type. Later, when students complete instruction in the use of the strategy steps for all problem types, present tests that include all problem types.

8. Where can one get additional information about schema-based strategy instruction?

Books

Marshall, S. P. (1995). *Schemas in problem solving.* New York: Cambridge University Press.

Journal Articles

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Presentations

Jitendra, A. K., & Griffin, C. (2004, April). *Results of design studies to understand teaching and learning of mathematical problem-solving in inclusive classrooms: Findings from two sites.* Paper to be presented at the 2004 Council for Exceptional Children (CEC) Annual Convention, New Orleans, LA. (Invited presentation).

Appendix A: Examples of Story Situations for Initial Strategy Instruction

No unknown information is included.

CHANGE

The situation begins with a quantity, the quantity is changed, and the situation ends with a new quantity.

1. Jane had 4 sweaters. Then her mother gave her 3 more sweaters for her birthday. Jane now has 7 sweaters.
2. Before he gave away 14 marbles, James had 36 marbles. Now he has 22 marbles.
3. Tom had 42 baseball cards. He now has 55 baseball cards after he bought 13 more cards.
4. Before Joey threw back 11 fish that were too small, he had caught 23 fish. Now Joey has 12 fish.
5. There were 58 people on the bus when we boarded at Center Street. Now there are 43 people after 15 people got off the bus at Main Street.

GROUP

The situation includes 2 or more quantities that are combined or separated to create a single quantity.

1. Ann has 3 apples and 2 oranges. She has 5 pieces of fruit.
2. At Hillcrest Elementary, 48 students took part in the school play. There were 29 students in the school play who were third graders. The remaining 19 students in the school play were not third graders.
3. Mrs. Smith's class read 28 books in September, 30 books in October, and 35 books in November. The class read 93 books altogether in the three months.
4. On Tuesday, Michelle rode her bike for 12 miles and walked for 5 miles. Michelle traveled 17 miles on Tuesday.
5. Mr. Bradley owns 17 rabbits. Six of the rabbits are long-eared and 11 are short-eared.

COMPARE

The situation includes two quantities and a comparison statement about the quantities.

1. Joe is 15 years old. He is 8 years older than Jill. Jill is 7 years old.
2. Michael has 43 CDs and Melissa has 70 CDs. Melissa has 27 more CDs than Michael.
3. One week Anita's horse ate 9 carrots and 7 apples. He ate 2 more carrots than apples that week.
4. Angie sold 72 magazines for the school fund-raiser. Ed sold 26 fewer magazines than Angie. Ed sold 46 magazines.
5. Kelly scored 21 goals in soccer. She scored 14 less goals than Janet. Janet scored 35 goals.

Appendix B: Examples of Story Situations for Use After Strategy is Mastered

Unknown information included.

CHANGE

1. Iguanas can lose parts of their tails. There are 7 sections on this iguana's tail. If 3 sections break off, how many sections are left?
2. The zoo tour bus is pulling into the Fairview stop. At the stop, 14 people get on the bus. Now there are 35 people on the bus. How many people were on the bus before the Fairview stop?
3. Pedro and his father have sheared 8 pounds of wool from an alpaca. So far some of the wool has been used to make a sweater. Now there are 5 pounds of wool left. How many pounds of wool have been used?
4. Corell likes to paint pictures of lions. She has painted 8 pictures so far. If she paints 3 more pictures, how many will she have?
5. Yesterday Jan put some rocks in her Lizard's terrarium. Today she put in 8 more rocks. Now there are 23 rocks in the terrarium. How many rocks did she put in her Lizard's terrarium yesterday?

GROUP

1. In May the shelter took in 19 cats. In June the shelter took in 22 cats. In July only 9 cats came to the shelter. How many cats did the shelter take in during May, June, and July?
2. Three buses took students on a field trip. One bus carried 45 students, another bus carried 38 students, and the third bus carried 37 students. How many students went on the trip?
3. At a party you break a piñata and win some favors. You have 7 stickers, 5 balloons, and 2 whistles. How many favors do you have in all?
4. Farmer Joe has 55 animals on his farm. He only has cows and pigs. There are 39 cows on the farm. How many pigs are on the farm?
5. You go to the market. You decide to buy milk and bread. Milk costs $2.09 and bread costs $1.78. How much will both items cost?

COMPARE

1. Lin is 5 years older than his cousin. If Lin is 11 years old, how old is his cousin?
2. An adult ticket for a movie about a garden costs $12.75. A child's ticket costs $1.85 less. What is the cost of a child's ticket?
3. The music library on the math mobile has 54 tapes and 8 very, very old records. How many more tapes than records are there?
4. Suppose it takes you 20 minutes to walk to the school. It takes 10 fewer minutes to ride your bike to the school than to walk. How long will it take you to ride your bike to the school?
5. Mega computers on the math mobile have 90 games. Mini computers only have 70 games. How many more games are there on the Mega computer?

Appendix C: Examples of Posters or Note Cards

CHANGE POSTER

Problem Identification and Translation

Step 1. Read the problem and retell the problem using your own words.
Discover the problem type.

Step 2. Underline the important information.
Map the information in the word problem onto the change diagram.

Problem Solution

Step 3. Find out whether to add or subtract to solve the problem.

(a) Decide whether the beginning or ending amount is the total and write T under the total.

- If we end up with more than we started with, then the ending amount is the total.
- If we end up with less than we started with, then the beginning amount is the total.

(b) If the total (whole) is given, we subtract to find the part. If the total (whole) is not given, we add to find it (total).

Step 4. Write the math sentence and solve it.

Step 5. Write the complete answer.

Step 6. Check the answer.

GROUP POSTER

Problem Identification and Translation

Step 1. Read the problem and retell the problem using your own words.
Discover the problem type.

Step 2. Underline the important information.
Map the information in the word problem onto the group diagram.

Problem Solution

Step 3. Find out whether to add or subtract to solve the problem.

(a) The large group is always the total, so write T under the large group.

(b) If the total (whole) is given, we subtract to find the part. If the total (whole) is not given, we add to find it (total).

Step 4. Write the math sentence and solve it.

Step 5. Write the complete answer.

Step 6. Check the answer.

COMPARE POSTER

Problem Identification and Translation

Step 1. Read the problem and retell the problem using your own words.
Discover the problem type.

Step 2. Underline the important information.
Map the information in the word problem onto the compare diagram.

Problem Solution

Step 3. Find out whether to add or subtract to solve the problem.

(a) Decide whether the compared or referent amount is the total and write T under the total.

- Read the difference sentence to find out whether the compared or the referent is the larger amount.

(b) If the total (whole) is given, we subtract to find the part. If the total (whole) is not given, we add to find it (total).

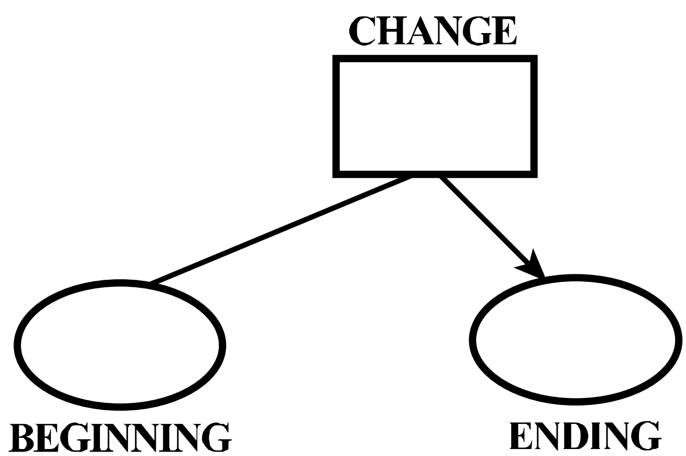
Step 4. Write the math sentence and solve it.

Step 5. Write the complete answer.

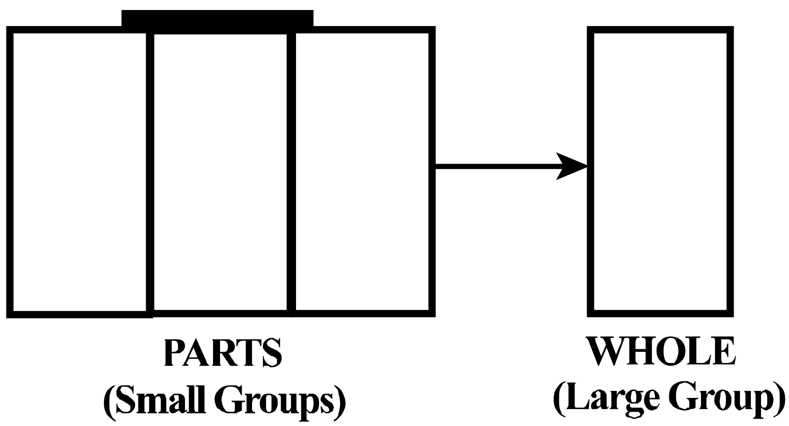
Step 6. Check the answer.

Appendix D: Diagram Examples

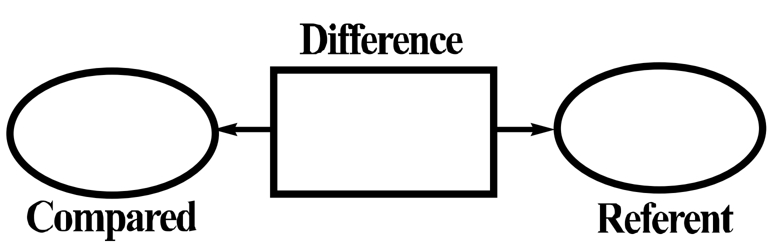
CHANGE DIAGRAM



GROUP DIAGRAM



COMPARE DIAGRAM



Appendix E: Scripted Lesson Examples

Script for Teacher Introduction of Step 2

Strategy for Change Problems

What does this story talk about? (Teacher waits for student response.) Let's underline "sweaters" in the problem and write "sweaters" for the beginning, *change*, and ending in the *change* diagram. Now, let's read the story again and write the given information onto our *change* diagram. The first sentence says, "Jane had 4 sweaters." From this sentence, I know that it is talking about the beginning amount. The word "had" (underline) tells me that Jane started with 4 sweaters. I will write this information in the diagram for the "beginning amount." The second sentence says, "Then her mother gave her 3 more sweaters for her birthday." From this sentence, I know that it is talking about the *change* amount. The words, gave her 3 more (underline), tells me that there was a change to the beginning amount. I will write this information in the diagram for the *change* amount (Teacher writes the word "more" or "+" next to 3 to help students understand whether the story situation ended up with more than or less than the beginning amount.) The third sentence says, "Jane now has 7 sweaters." From this sentence, I know that it is talking about the ending amount. The words, now has (underline), tells me that Jane ended up with 7 sweaters. I will write this information in the diagram for the ending amount. Let's look at the diagram and read what it says. We have a beginning amount of 4 sweaters, a change amount of 3 more sweaters, and an ending amount of 7 sweaters. This seems to make sense, because if Jane received 3 more sweaters from her mother, then she should end up with more sweaters than she started with.

Strategy for Group Problems

What does this story talk about? (Teacher waits for student response.) Typically, the large *group* or whole tells about all the things in the problem. In this story situation, fruit describes the whole or large *group*, because apples and oranges are all fruit. I will underline fruit in the sentence, "She has 5 pieces of fruit," which is the large *group*. Let's write fruit in the diagram for the whole or large group. Now we need to find the small *groups* or parts. The two small *groups* in this story are apples and oranges. I will underline these two words in the sentence, "Ann had 3 apples and 2 oranges." What are the small *groups* in this story? How do you know? (Teacher waits for student response.) Let's write apples and oranges in the diagram. Now, let's read the story again and write the given information onto our *group* diagram. (Teacher models writing it on the overhead or other display.) Let's look at the diagram. The story has (1) two small *groups* (3 apples and 2 oranges) that combine to make a large *group* (5 fruits) and (2) the whole (fruit) is equal to the sum of the parts (apples and oranges). Teacher points out that the whole is equal to the sum of the parts in this story by referring to information in the diagram, 5 (fruit) = 3 (apples) + 2 (oranges).

Strategy for Compare Problems

Before I write all the information from the story onto the diagram, I need to figure out the *compared* and referent in this story. To do that, I must find the difference or *comparison* sentence and underline it. Typically, the *comparison* sentence is the one that contains phrases such as "more than" or "less than." In this story situation, the sentence, "He (Joe) is 8 years older than Jill," is the *comparison* sentence (point to "older than"), because it *compares* the ages of two people (Joe and Jill) and tells us the difference between them with regards to their age. I will underline this sentence to be the *comparison* or difference sentence. This sentence also helps me identify the *compared* and referent objects. Here's a way to figure out the *compared* object or subject. The person or thing before the *comparison* phrase older than (teacher circles the phrase) in the comparison sentence is usually the *compared* person or thing. Joe is the *compared* in this story, because his age is *compared* to Jill's age. What is the *compared* in this sentence? How do you know? (Teacher waits for students to respond). Let's circle Joe in the story and write "Joe" in the diagram for the *compared*. The referent is the subject or object that follows the phrase "older than." Jill is the referent in this problem, because she is the person to whom Joe is compared. Let's circle Jill in the story and write "Jill" in the diagram for the referent.

Now, we need to figure out the measure of comparison used to *compare* the compared and referent objects or subjects. The *comparison* or difference sentence, "He is 8 years older than Jill," tells me that the measure of comparison is years (age). Now I'll read the problem and write the information given for the *compared*, referent, and difference amounts (Teacher models underlining the information for the compared and referent and writing them on the overhead or other display). Let's look at the diagram and read what it says. We have Joe who is 15 years old (the compared amount), Jill who is 7 years old (the referent amount), and a difference of 8 years between them. This seems to make sense, because if Joe is older than Jill, then the difference of 8 years is correct.

Script for Teacher Summarization of Key Features of Story Situations

Strategy for Change Problems

In this *change* story, (1) there is a beginning amount, a *change*, and an ending amount (point to the diagram), (2) we began with sweaters and ended with sweaters. The *change* also involved sweaters (point to the diagram), and (3) both the beginning and ending states cannot occur at the same time. That is, Jane began with 4 sweaters and ended with 7 sweaters to tell us about a *change* in time from past (i.e., Jane **had**) to present (Jane now **has**).

Strategy for Group Problems

In this *group* story, (1) two small groups (3 apples and 2 oranges) combine to make a large group (5 fruits) and (2) the whole (fruit) is equal to the sum of the parts (apples and oranges).

Strategy for Compare Problems

In this *compare* story, (1) there is a compared, a referent, and a difference amount (point to the diagram) and (2) we are comparing Joe' age in years to Jill's age in years.