Why Study Mathematics?

Mathematics reveals hidden patterns that help us understand the world around us. Now much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behavior, and social systems.

The process of "doing" mathematics is far more than just calculation or deduction; it involves observation of patterns, testing of conjectures, and estimation of results. As a practical matter, mathematics is a science of pattern and order. Its domain is not molecules or cells, but numbers, chance, form, algorithms, and change. As a science of abstract objects, mathematics relies on logic rather than on observation as its standard of truth, yet employs observation, simulation, and even experimentation as means of discovering truth.

The special role of mathematics in education is a consequence of its universal applicability. The results of mathematics--theorems and theories--are both significant and useful; the best results are also elegant and deep. Through its theorems, mathematics offers science both a foundation of truth and a standard of certainty. In addition to theorems and theories, mathematics offers distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols.

Mathematics, as a major intellectual tradition, is a subject appreciated as much for its beauty as for its power. The enduring qualities of such abstract concepts as symmetry, proof, and change have been developed through 3,000 years of intellectual effort. Like language, religion, and music, mathematics is a universal part of human culture.

What Is Mathematics?

Mathematics is the study of structure and the way it can be applied to solve specific problems. The mathematics one sees in high school and the first year or so of college—techniques for solving equations, trigonometry, analytic geometry and calculus—represents only a small corner of the discipline. Some of the structures discussed in more advanced courses include algebraic systems such as groups and vector spaces, geometric notions such as surfaces, manifolds and topological spaces, and spaces of differentiable or integrable functions. Such structures are used to construct mathematical models that may explain and predict events in a wide variety of disciplines. Mathematics has been with us since antiquity and is a pervasive force in our society; it is a diverse field encompassing many subjects that are, unfortunately, largely unknown outside mathematics. The major branches of mathematics and a few of their applications are briefly described below.

Pure Mathematics

Pure mathematics has as its main purpose the search for a deeper understanding of mathematics itself. As a result pure mathematics seems at first far removed from everyday life. However, many important applications have been the results of advances in pure mathematics. This subject is traditionally divided into four main areas: algebra, analysis, geometry, and logic. However, some of the most exciting developments throughout the history of mathematics have resulted from the interaction between different areas.

Algebra is the study of abstract mathematical systems. These systems, such as groups, rings, and fields, generalize properties of familiar structures such as integers, polynomials, and matrices. The general, abstract approach of algebra has been fruitful in solving many problems in both mathematics and other disciplines. For example, using algebraic techniques, one can show that it is impossible to trisect an angle using only a straight-edge and compass. Group theory has been employed liberally in quantum mechanical physics and physical chemistry. Other recent applications of algebra include cryptography and coding theory.
Analysis is the study of infinite processes. As such, it concerns itself with phenomena that are continuous as opposed to discrete. Starting with the fundamental notions of function and limit, it builds differential and integral calculus, which is the mathematics of continuous change. Analysis in turn gives rise to a deeper and more general study of functions of both real and complex variables. The area is pervasive, and it finds rich and varied applications in almost every field of pure and applied mathematics.

Geometry is the study of curves, surfaces, their higher-dimensional analogues, and the properties they possess under various types of transformations. Geometry and topology frequently make use of techniques and notions from algebra and analysis. Geometry is an important subject for our understanding of the nature and structure of spatial relations.

Logic is at the very foundation of mathematics. In this field one studies the formulation of mathematical statements, the meaning and nature of mathematical truth and proof, and what can possibly be proved in a mathematical system. For example, there is a famous theorem due to Kurt Goedel that says that in any logical system rich enough to contain arithmetic there are true statements that can neither be proved nor disproved. Logic has found many important applications in the study of computability in computer science.

Applied Mathematics

Applied mathematics is the development and use of mathematical concepts and techniques to solve problems in many other disciplines. Unlike pure mathematics, the areas of applied mathematics fall under no simple classification. Nonetheless, the following topics cover many of the important applications.

Applied analysis involves the study of techniques for analyzing continuous processes and phenomena. For example, many methods from real and complex analysis are utilized when looking at problems of a physical or computational nature. Differential equations and numerical analysis are two examples of subjects that come under this heading.

Combinatorics, the study and enumeration of patterns and configurations, is one technique for analyzing phenomena that do not behave in a smooth or continuous fashion. Techniques from algebra and other areas are applied to study a wide variety of problems in such areas as graph theory, scheduling, and game theory. There are also many important applications of combinatorics to computer science.

Probability and statistics are among the most fundamental tools for mathematical modeling. The importance of probability lies in its formulation of chance (or stochastic) processes and its applicability to the analysis of non-deterministic phenomena. Statistics deals with the collection and analysis of data and with the making of decisions in the face of uncertainty. It is frequently used in the social sciences as well as in all areas of experimental science.

Operations research involves applications of mathematical models and the scientific method to help organizations or individuals to make complex decisions. Traditionally, it has focused on mathematical optimization theory. Typical problems in operations research include the development of an optimal flight schedule for an airline and finding the best inventory policy for a bookstore.

Actuarial mathematics is one of the early examples of mathematical modeling. This field uses the methods of probability and statistics, along with a study of economic factors, to estimate the financial risk of future events.