INTEGRALES POR PARTES

Resolverlas siguientes integrales

1. $\int x\cdot \sec^2 x\cdot dx = \left\{\begin{matrix} u = x \Rightarrow du = dx \\ dv = \sec^2 x\cdot dx \Rightarrow v = tg\, x\end{matrix}\right\} = x\cdot tg\, x - \int tg\, x\cdot dx = x\cdot tg\, x - \int \frac{sen\, x}{\cos x}\cdot dx =$

$$= x\cdot tg\, x - (-1)\int \frac{-sen\, x}{\cos x}\cdot dx = x\cdot tg\, x + Ln|\cos x| + C$$

2. $\int e^{ax}\cos bx\cdot dx = \left\{\begin{matrix} u = \cos bx \Rightarrow du = -b\cdot sen\, bx\cdot dx \\ dv = e^{ax} \Rightarrow v = \frac{1}{a}e^{ax}\end{matrix}\right\} = \cos(bx)\cdot\frac{1}{a}e^{ax} - \int \frac{1}{a}e^{ax}\cdot(-b)\cdot sen\, bx\cdot dx =$

$$= \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a}\cdot\int e^{ax} sen(bx)\cdot dx = \left\{\begin{matrix} u = sen(bx) \Rightarrow du = b\cdot\cos(bx)\cdot dx \\ dv = e^{ax} \Rightarrow v = \frac{1}{a}e^{ax}\end{matrix}\right\} =$$

$$= \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a}\cdot\left[sen(bx)\frac{1}{a}e^{ax} - \int \frac{1}{a}e^{ax} b\cdot\cos(bx)\cdot dx\right] =$$

$$= \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a^2}\cdot sen(bx)\cdot e^{ax} - \frac{b^2}{a^2}\int e^{ax}\cos(bx)\cdot dx$$

igualando con la integral inicial

$$\int e^{ax}\cos(bx)\cdot dx = \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a^2}\cdot sen(bx)\cdot e^{ax} - \frac{b^2}{a^2}\int e^{ax}\cos(bx)\cdot dx$$

ecuación de la que se puede despejar el valor de la integral

$$\int e^{ax}\cos(bx)\cdot dx + \frac{b^2}{a^2}\int e^{ax}\cos(bx)\cdot dx = \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a^2}\cdot sen(bx)\cdot e^{ax}$$

$$\left(1+\frac{b^2}{a^2}\right)\int e^{ax}\cos(bx)\cdot dx = \frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a^2}\cdot sen(bx)\cdot e^{ax}$$

$$\int e^{ax}\cos(bx)\cdot dx = \frac{\frac{1}{a}\cos(bx)\cdot e^{ax} + \frac{b}{a^2}\cdot sen(bx)\cdot e^{ax}}{\left(1+\frac{b^2}{a^2}\right)}$$

operando y ordenando

$$\int e^{ax}\cos(bx)\cdot dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos(bx) + b\cdot sen(bx)\right) + C$$

3. $\int (x^2+1)\cdot 2^x dx = \left\{\begin{matrix} u = x^2+1 \Rightarrow du = 2x\cdot dx \\ dv = 2^x dx \Rightarrow v = v = \frac{2^x}{Ln2}\end{matrix}\right\} = (x^2+1)\cdot\frac{2^x}{Ln2} - \int \frac{2^x}{Ln2}\cdot 2x\cdot dx =$

$$= \frac{(x^2+1)\cdot 2^x}{Ln2} - \frac{2}{Ln2}\cdot\int x\cdot 2^x\cdot dx = \left\{\begin{matrix} u = x \Rightarrow du = dx \\ dv = 2^x dx \Rightarrow v = \frac{2^x}{Ln2}\end{matrix}\right\} =$$

$$= \frac{(x^2+1)\cdot 2^x}{Ln2} - \frac{2}{Ln2}\cdot\left(x\cdot\frac{2^x}{Ln2} - \int\frac{2^x}{Ln2}\cdot dx\right) = \frac{(x^2+1)\cdot 2^x}{Ln2} - \frac{2x\cdot 2^x}{Ln^2 2} + \frac{2}{Ln^2 2}\cdot\int 2^x\cdot dx =$$

$$= \frac{(x^2+1)\cdot 2^x}{Ln2} - \frac{2x\cdot 2^x}{Ln^2 2} + \frac{2}{Ln^2 2}\cdot\frac{2^x}{Ln2} + C = \frac{(x^2+1)\cdot 2^x}{Ln2} - \frac{2x\cdot 2^x}{Ln^2 2} + \frac{2\cdot 2^x}{Ln^3 2} + C =$$

$$= 2^x \cdot \left(\frac{(x^2+1)}{Ln2} - \frac{2x}{Ln^2 2} + \frac{2}{Ln^3 2} \right) + C$$

4. $$\int x^2 \cdot e^{4x} \cdot dx = \left\{ \begin{matrix} u = x^2 \Rightarrow du = 2x \cdot dx \\ dv = e^{4x} \cdot dx \Rightarrow v = \frac{1}{4} e^{4x} \end{matrix} \right\} = x^2 \cdot \frac{1}{4} e^{4x} - \int \frac{1}{4} e^{4x} \cdot 2x \cdot dx =$$

$$\frac{x^2}{4} e^{4x} - \frac{2}{4} \int x \cdot e^{4x} dx = \left\{ \begin{matrix} u = x \Rightarrow du = dx \\ dv = e^{4x} \cdot dx \Rightarrow v = \frac{1}{4} e^{4x} \end{matrix} \right\} = \frac{x^2}{4} e^{4x} - \frac{1}{2} \cdot \left(x \cdot \frac{1}{4} e^{4x} - \int \frac{1}{4} e^{4x} \cdot dx \right) =$$

$$= \frac{x^2}{4} e^{4x} - \frac{x}{8} \cdot e^{4x} + \frac{1}{8} \int e^{4x} \cdot dx = \frac{x^2}{4} e^{4x} - \frac{x}{8} \cdot e^{4x} + \frac{1}{8} \cdot \frac{1}{4} e^{4x} + C =$$

$$= \frac{x^2}{4} e^{4x} - \frac{x}{8} \cdot e^{4x} + \frac{1}{32} \cdot e^{4x} + C = \frac{e^{4x}}{32} \cdot (8x^2 - 4x + 1) + C$$

5. $$\int (Ln\, x)^2 dx = \left\{ \begin{matrix} u = Ln^2 x \Rightarrow du = 2Ln\, x \cdot \frac{1}{x} \\ dv = dx \Rightarrow v = x \end{matrix} \right\} = Ln^2 x \cdot x - \int x \cdot 2Ln\, x \cdot \frac{1}{x} \cdot dx =$$

$$= x \cdot Ln^2 x - 2 \cdot \int Ln\, x \cdot dx = \left\{ \begin{matrix} u = Ln\, x \Rightarrow du = \frac{1}{x} \cdot dx \\ dv = dx \Rightarrow v = x \end{matrix} \right\} = x \cdot Ln^2 x - 2 \cdot \left(Ln\, x \cdot x - \int x \cdot \frac{1}{x} \cdot dx \right) =$$

$$= x \cdot Ln^2 x - 2x \cdot Ln\, x + 2 \int dx = x \cdot Ln^2 x - 2x \cdot Ln\, x + 2x + C$$

6. $$\int x (\log_3 x)^2 dx = \left\{ \begin{matrix} u = (\log_3 x)^2 \Rightarrow du = 2 \cdot \log_3 x \cdot \frac{1}{x \cdot Ln3} \cdot dx \\ dv = x \cdot dx \Rightarrow v = \frac{x^2}{2} \end{matrix} \right\} =$$

$$= (\log_3 x)^2 \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot 2 \cdot \log_3 x \cdot \frac{1}{x \cdot Ln3} \cdot dx =$$

$$= \frac{x^2 \cdot (\log_3 x)^2}{2} - \frac{1}{Ln3} \int x \cdot \log_3 x \cdot dx = \left\{ \begin{matrix} u = \log_3 x \Rightarrow du = \frac{1}{x \cdot Ln3} \cdot dx \\ dv = x \cdot dx \Rightarrow v = \frac{x^2}{2} \end{matrix} \right\} =$$

$$= \frac{x^2 \cdot (\log_3 x)^2}{2} - \frac{1}{Ln3} \left[\log_3 x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x \cdot Ln3} \cdot dx \right] = \frac{x^2 \cdot (\log_3 x)^2}{2} - \frac{x^2 \cdot \log_3 x}{2 \cdot Ln3} + \frac{1}{2Ln^2 3} \int x \cdot dx =$$

$$= \frac{x^2 \cdot (\log_3 x)^2}{2} - \frac{x^2 \cdot \log_3 x}{2 \cdot Ln3} + \frac{1}{2Ln^2 3} \cdot \frac{x^2}{2} + C = \frac{x^2}{2} \cdot \left[\log_3^2 x - \frac{\log_3 x}{Ln3} + \frac{1}{2 \cdot Ln^2 3} \right] + C$$

7. $$\int \frac{x \cdot arcsen\, x}{\sqrt{1-x^2}} \cdot dx = \left\{ \begin{matrix} u = arcsen\, x \Rightarrow du = \frac{dx}{\sqrt{1-x^2}} \\ dv = \frac{x}{\sqrt{1-x^2}} \cdot dx \Rightarrow v = -\sqrt{1-x^2} \end{matrix} \right\} = arcsen\, x \cdot \left(-\sqrt{1-x^2} \right) - \int -\sqrt{1-x^2} \cdot \frac{dx}{\sqrt{1-x^2}} =$$

$$= -arcsen\, x \cdot \sqrt{1-x^2} + \int dx = -arcsen\, x \cdot \sqrt{1-x^2} + x + C$$

8.

$$\int (x^2+1)\cdot \text{sen } 2x\cdot dx = \left\{\begin{matrix} u = x^2+1 \Rightarrow du = 2x\cdot dx \\ dv = \text{sen } 2x\cdot dx \Rightarrow v = -\frac{1}{2}\cos 2x \end{matrix}\right\} = (x^2+1)\cdot\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x\cdot 2x\cdot dx =$$

$$= -\frac{(x^2+1)\cos 2x}{2} + \int x\cdot\cos 2x\cdot dx = \left\{\begin{matrix} u = x \Rightarrow du = dx \\ dv = \cos 2x\cdot dx \Rightarrow v = \frac{1}{2}\text{sen } 2x \end{matrix}\right\} =$$

$$= -\frac{(x^2+1)\cos 2x}{2} + x\cdot\frac{1}{2}\text{sen } 2x - \int \frac{1}{2}\text{sen } 2x\cdot dx = -\frac{(x^2+1)\cos 2x}{2} + \frac{x\cdot\text{sen } 2x}{2} + \frac{\cos 2x}{4} + C$$

9. $\int (x^2\cdot \text{Ln}x - x\cdot \text{Ln}^2 x)\cdot dx = \int x^2\cdot\text{Ln}x\cdot dx - \int x\cdot\text{Ln}^2 x\cdot dx$

$$\int x^2\cdot\text{Ln } x\cdot dx = \left\{\begin{matrix} u = \text{Ln } x \Rightarrow du = \frac{1}{x}\cdot dx \\ dv = x^2\cdot dx \Rightarrow v = \frac{x^3}{3} \end{matrix}\right\} = \text{Ln } x\cdot\frac{x^3}{3} - \int \frac{x^3}{3}\cdot\frac{1}{x}\cdot dx = \frac{x^3\cdot\text{Ln } x}{3} - \frac{1}{3}\int x^2\cdot dx =$$

$$= \frac{x^3\cdot\text{Ln } x}{3} - \frac{1}{3}\cdot\frac{x^3}{3} + C = \frac{x^3\cdot\text{Ln } x}{3} - \frac{x^3}{9} + C_1$$

$$\int x\cdot\text{Ln}^2 x\cdot dx = \left\{\begin{matrix} u = \text{Ln}^2 x \Rightarrow du = 2\cdot\text{Ln } x\cdot\frac{1}{x}\cdot dx \\ dv = x\cdot dx \Rightarrow v = \frac{x^2}{2} \end{matrix}\right\} = \text{Ln}^2 x\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot 2\cdot\text{Ln } x\cdot\frac{1}{x}\cdot dx =$$

$$= \frac{x^2\cdot\text{Ln}^2 x}{2} - \int x\cdot\text{Ln } x\cdot dx = \left\{\begin{matrix} u = \text{Ln } x \Rightarrow du = \frac{1}{x}\cdot dx \\ dv = x\cdot dx \Rightarrow v = \frac{x^2}{2} \end{matrix}\right\} = \frac{x^2\cdot\text{Ln}^2 x}{2} - \left[\text{Ln } x\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}\cdot dx\right] =$$

$$\frac{x^2\cdot\text{Ln}^2 x}{2} - \frac{x^2\cdot\text{Ln } x}{2} + \frac{1}{2}\int x\cdot dx = \frac{x^2\cdot\text{Ln}^2 x}{2} - \frac{x^2\cdot\text{Ln } x}{2} + \frac{x^2}{4} + C_2$$

sustituyendo en la integral inicial:

$$\int (x^2\cdot\text{Ln}x - x\cdot\text{Ln}^2 x)\cdot dx = \frac{x^3\cdot\text{Ln } x}{3} - \frac{x^3}{9} - \frac{x^2\cdot\text{Ln}^2 x}{2} + \frac{x^2\cdot\text{Ln } x}{2} - \frac{x^2}{4} + C$$

10. $\int \text{arc sen } x\cdot dx = \left\{\begin{matrix} u = \text{arcsen } x \to du = \frac{dx}{\sqrt{1-x^2}} \\ dv = dx \to v = x \end{matrix}\right\} = x\cdot\text{arcsen } x - \int x\cdot\frac{dx}{\sqrt{1-x^2}} =$

$$= x\cdot\text{arcsen } x - \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\cdot dx = x\cdot\text{arcsen } x - \frac{-1}{2}\int (1-x^2)^{-1/2}(-2x)\cdot dx = x\cdot\text{arcsen } x - \frac{-1}{2}\frac{(1-x^2)^{1/2}}{1/2} + C =$$

$$= x\cdot\text{arcsen } x + \sqrt{1-x^2} + C$$

11. $\int 2x\cdot\text{actg } x\cdot dx = \left\{\begin{matrix} u = \text{arctg } x \to du = \frac{dx}{1+x^2} \\ dv = 2x\cdot dx \to v = x^2 \end{matrix}\right\} = x^2\text{arctg } x - \int x^2\cdot\frac{dx}{1+x^2} = x^2\text{arctg } x - \int \frac{x^2}{1+x^2}dx =$

$$= x^2\text{arctg } x - \int\left(1 - \frac{1}{1+x^2}\right)dx = x^2\text{arctg } x - x + \text{arctg } x + C = \text{arctg } x\cdot(x^2+1) - x + C$$

12. $\int \left(x-x^2\right)e^{x/3}\cdot dx = \begin{Bmatrix} u = x-x^2 \to du = (1-2x)\cdot dx \\ dv = e^{x/3}\cdot dx \to v = 3e^{x/3} \end{Bmatrix} = \left(x-x^2\right)\cdot 3e^{x/3} - \int 3e^{x/3}\cdot(1-2x)\cdot dx =$

$= 3e^{x/3}\left(x-x^2\right) - 3\int (1-2x)\cdot e^{x/3}\cdot dx = \begin{Bmatrix} u = 1-2x \to du = -2dx \\ dv = e^{x/3}\cdot dx \to v = 3e^{x/3} \end{Bmatrix} = 3e^{x/3}\left(x-x^2\right) - 3\left[(1-2x)\cdot 3e^{x/3} - \int 3e^{x/3}(-2)\cdot dx\right] =$

$= 3e^{x/3}\left(x-x^2\right) - 9e^{x/3}(1-2x) - 18\int e^{x/3}\cdot dx = 3e^{x/3}\left(x-x^2\right) - 9e^{x/3}(1-2x) - 18\cdot 3e^{x/3} + C =$

$= -3e^{x/3}\left(x^2-7x+21\right) + C$

13. $\int \frac{\operatorname{Ln} x}{\sqrt{x}}\cdot dx = \begin{Bmatrix} u = \operatorname{Ln} x \to du = \frac{dx}{x} \\ dv = \frac{dx}{\sqrt{x}} = x^{-1/2}\cdot dx \to v = \frac{x^{1/2}}{1/2} = 2\sqrt{x} \end{Bmatrix} = \operatorname{Ln} x\cdot 2\sqrt{x} - \int 2\sqrt{x}\cdot\frac{dx}{x} = 2\sqrt{x}\cdot\operatorname{Ln} x - \int\frac{2dx}{\sqrt{x}} =$

$= 2\sqrt{x}\cdot\operatorname{Ln} x - 2\int x^{-1/2}dx = 2\sqrt{x}\cdot\operatorname{Ln} x - 2\frac{x^{1/2}}{1/2} + C = 2\sqrt{x}\cdot\operatorname{Ln} x - 4\sqrt{x} + C = 2\sqrt{x}\cdot(\operatorname{Ln} x - 2) + C =$

14. $\int x\cdot\cos^2 x\cdot dx = \begin{Bmatrix} u = x \to du = dx \\ dv = \cos^2 x\cdot dx \to v = \frac{x}{2} + \frac{\operatorname{sen}2x}{4} \end{Bmatrix} = x\left(\frac{x}{2}+\frac{\operatorname{sen}2x}{4}\right) - \int\left(\frac{x}{2}+\frac{\operatorname{sen}2x}{4}\right)\cdot dx =$

$= \frac{x^2}{2} + \frac{x\cdot\operatorname{sen}2x}{4} - \frac{x^2}{4} + \frac{\cos 2x}{8} + C = \frac{x^2}{4} + \frac{x\cdot\cos 2x}{4} - \frac{\operatorname{sen}2x}{8} + C$

15. $\int e^{\sqrt{x+1}}\cdot dx = \begin{Bmatrix} x+1 = t^2 \\ ds = 2t\cdot dt \end{Bmatrix} = \int e^{\sqrt{t^2}}\cdot 2t\cdot dt = 2\int t\cdot e^t dt = \begin{Bmatrix} u = t \to du = dt \\ dv = e^t dt \to v = e^t \end{Bmatrix} =$

$= 2t\cdot e^t - 2\int e^t dt = 2t\cdot e^t - 2e^t + C = \begin{Bmatrix} x+1 = t^2 \\ t = \sqrt{x+1} \end{Bmatrix} = 2\sqrt{x+1}\cdot e^{\sqrt{x+1}} - 2e^{\sqrt{x+1}} + C =$

$= 2e^{\sqrt{x+1}}\left(\sqrt{x+1}-1\right) + C$