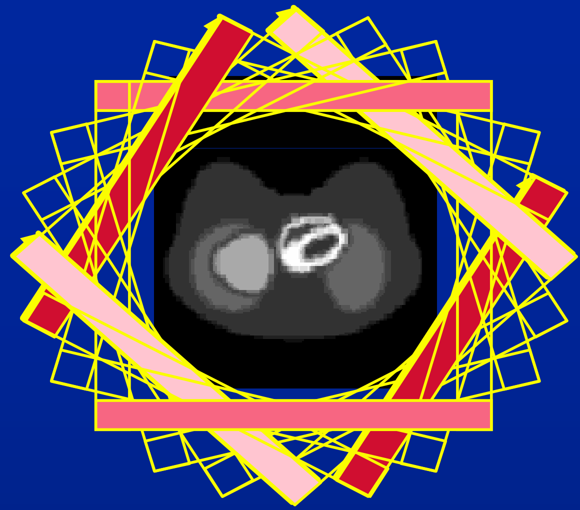
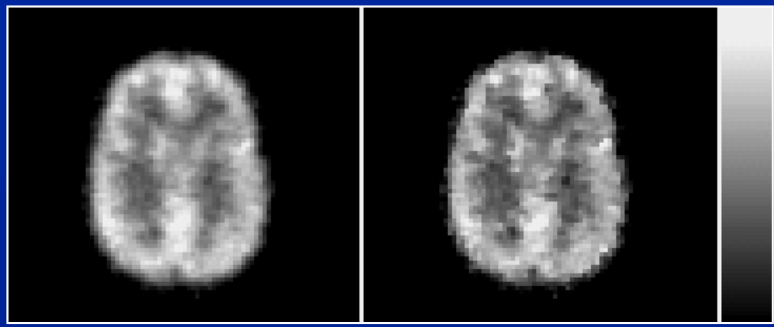


Review of image reconstruction methods (as applied to PET)

Brian F Hutton
Department of Medical Physics and
Department of Nuclear Medicine & Ultrasound
Westmead Hospital, Sydney, Australia

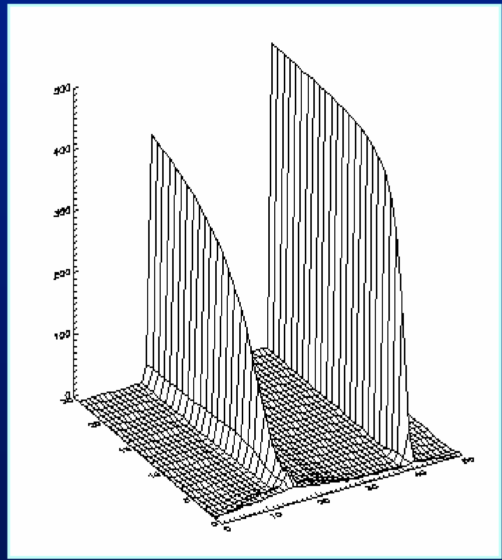
Outline



Why iterative reconstruction?

- limitations of FBP
- appropriate physical and statistical models

Maximum likelihood reconstruction

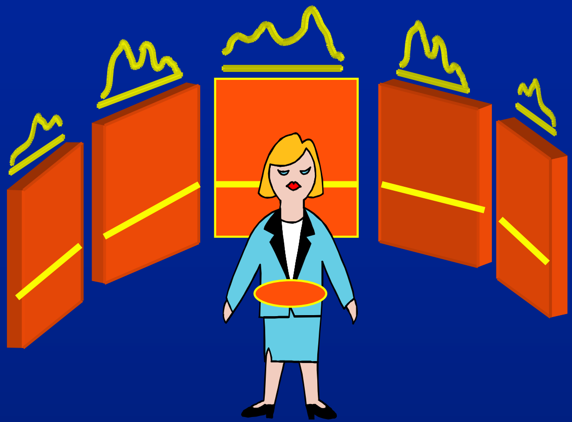


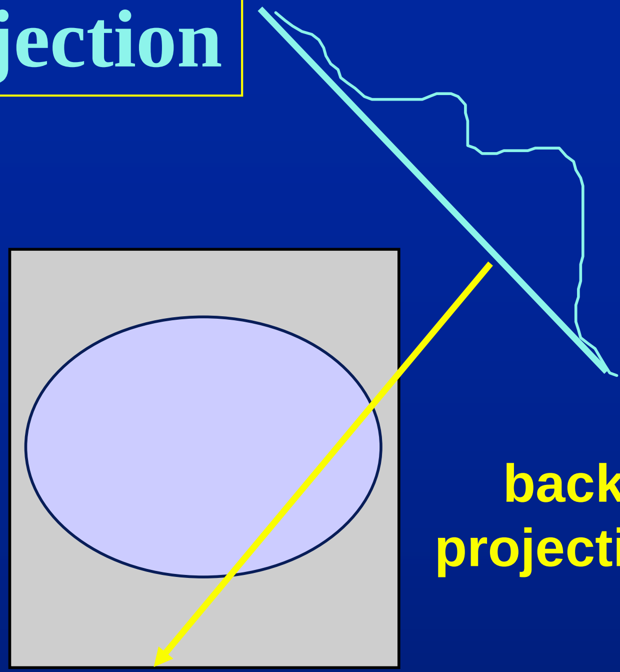
- basic algorithm
- improving speed
- controlling noise

PET considerations

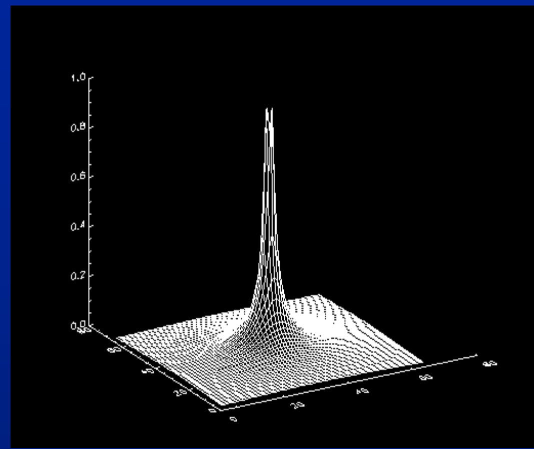
- non-Poisson data e.g. transmission data
- alternative approaches for noise control
- 3d reconstruction

Filtered back projection

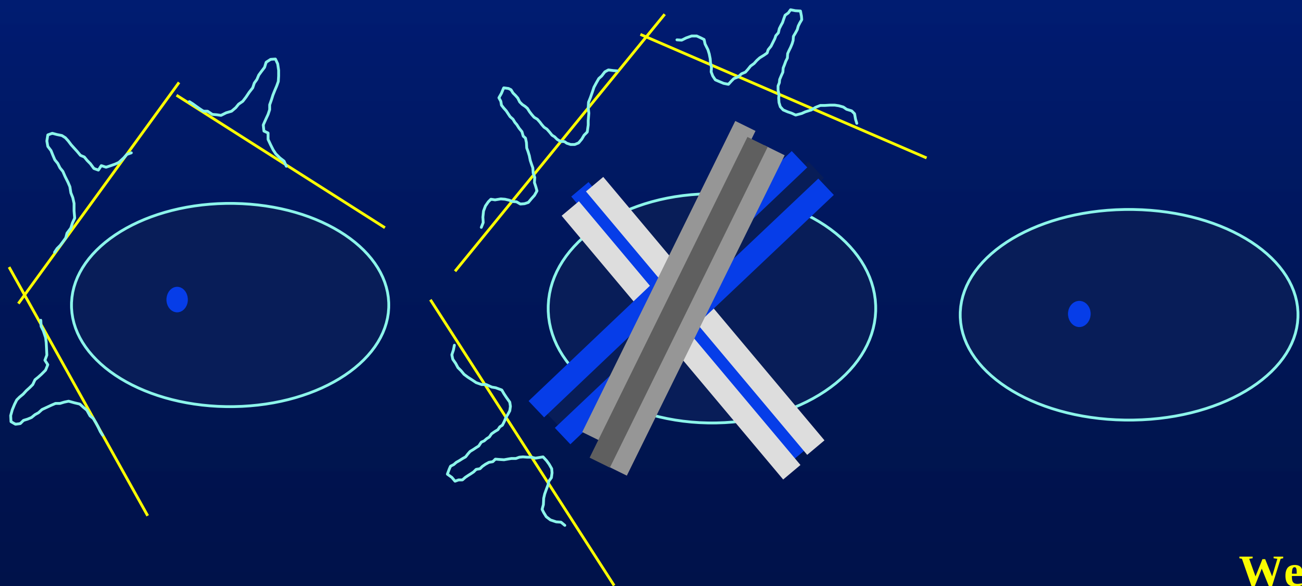


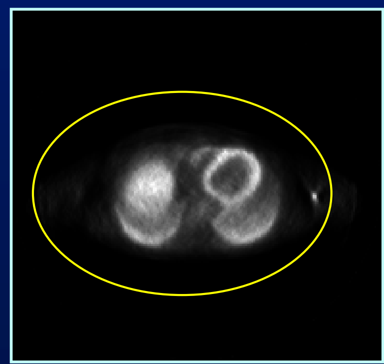


back projection



blurred by 1/r





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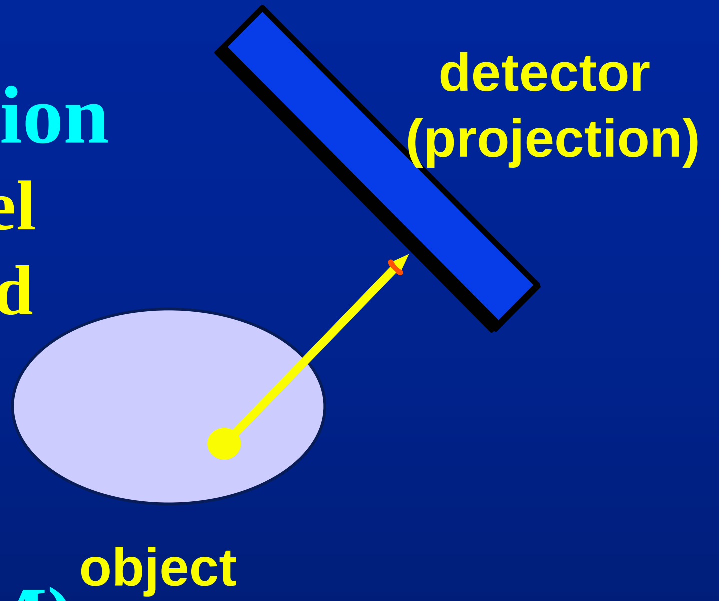
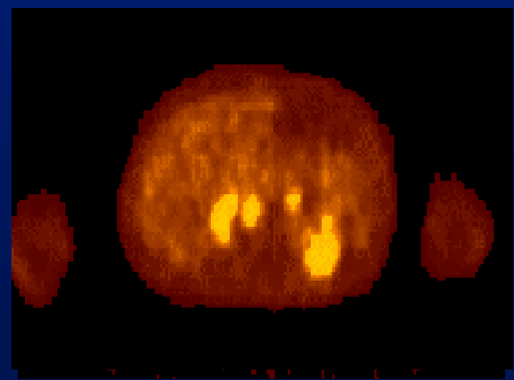
Problems with filtered back projection

- assumes very simple projection model
- non-uniform attenuation not included
- streak artifacts
- noise amplification



Iterative reconstruction (e.g. MLEM)

- more exact projection model
- includes variable attenuation
- flexible detector geometry
- reduces streak artifacts
- handles missing data
- improved noise characteristics



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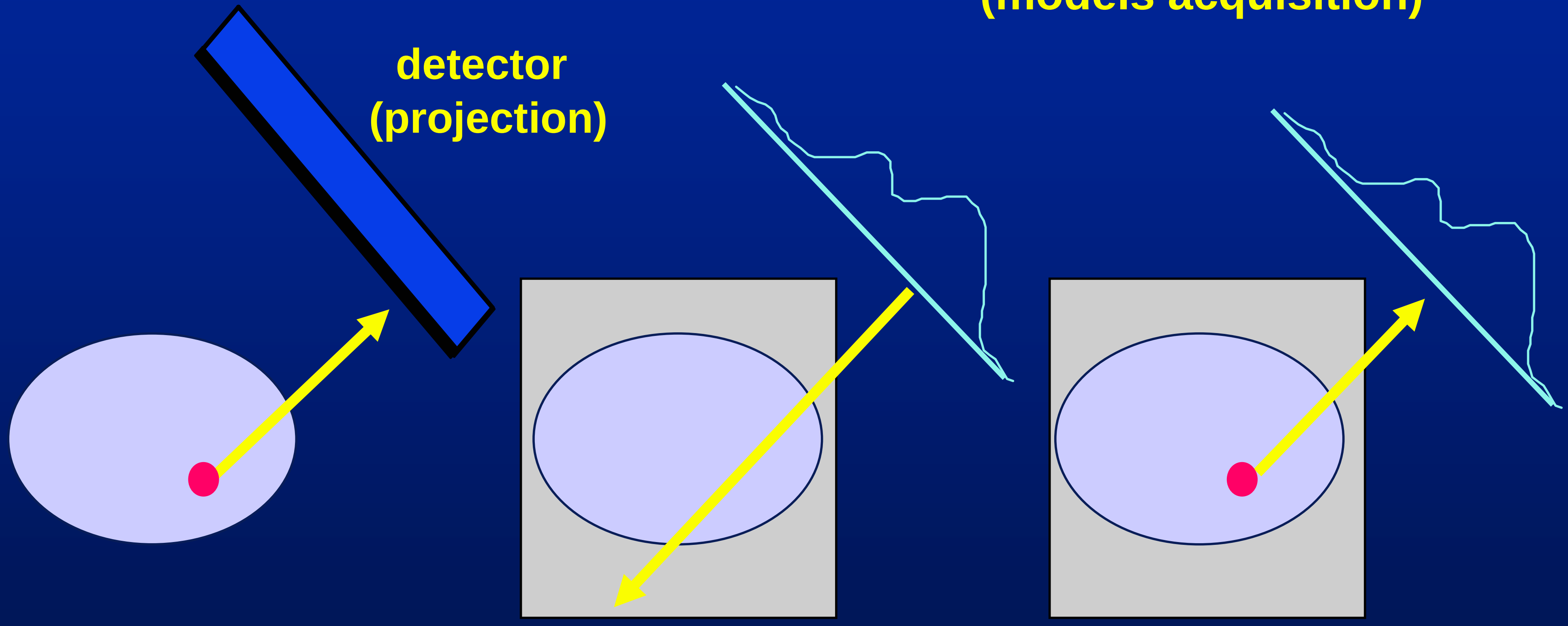
Forward projection (re-projection)

estimate projections from reconstructed estimate (models acquisition)

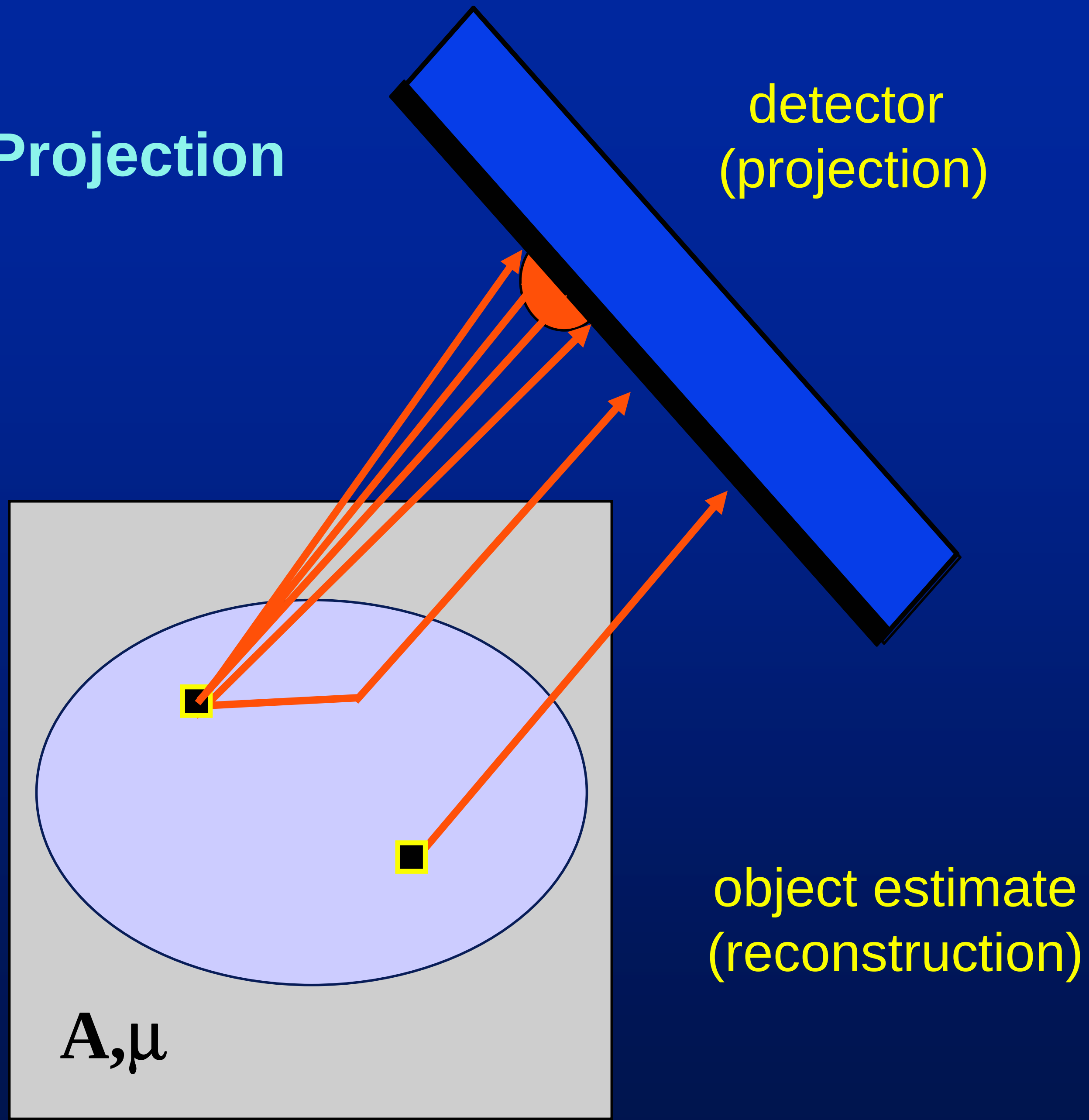


object

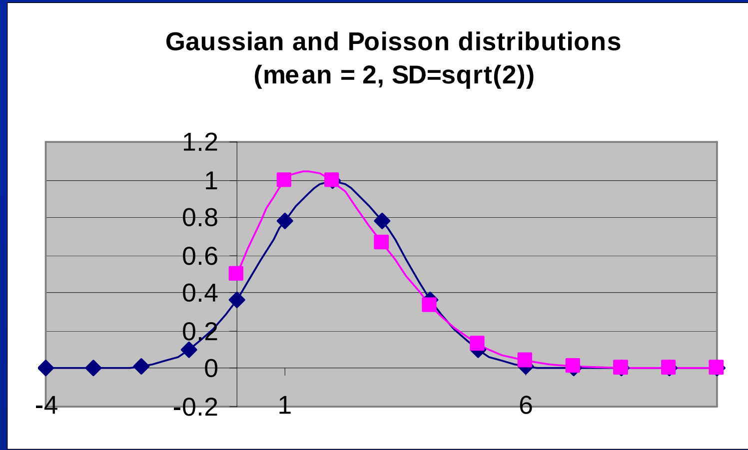
reconstruction back projection

forward projection

Forward Projection



Iterative reconstruction

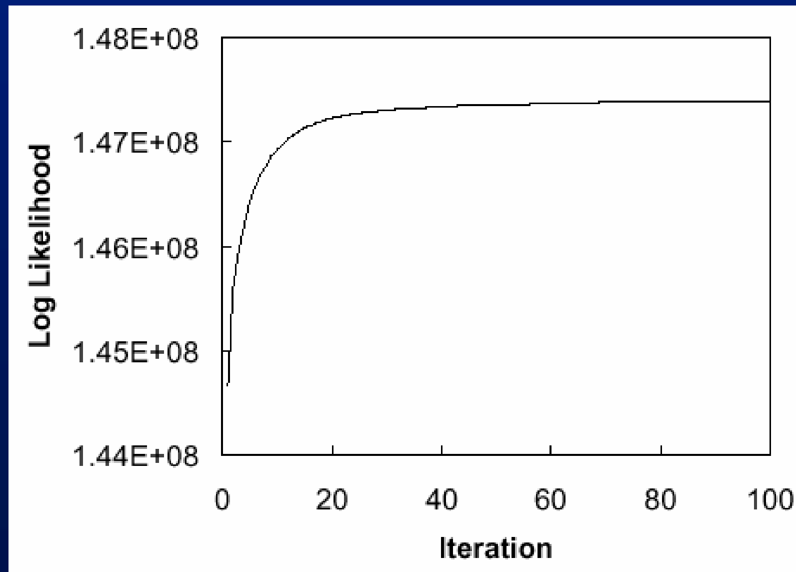


Modelling noise

algebraic reconstruction	ART	no statistical noise model
weighted least squares	WLS	Gaussian noise model
maximum likelihood	MLEM	Poisson noise model

properties of MLEM

- likelihood always increases with iteration
- non-negativity of solution guaranteed



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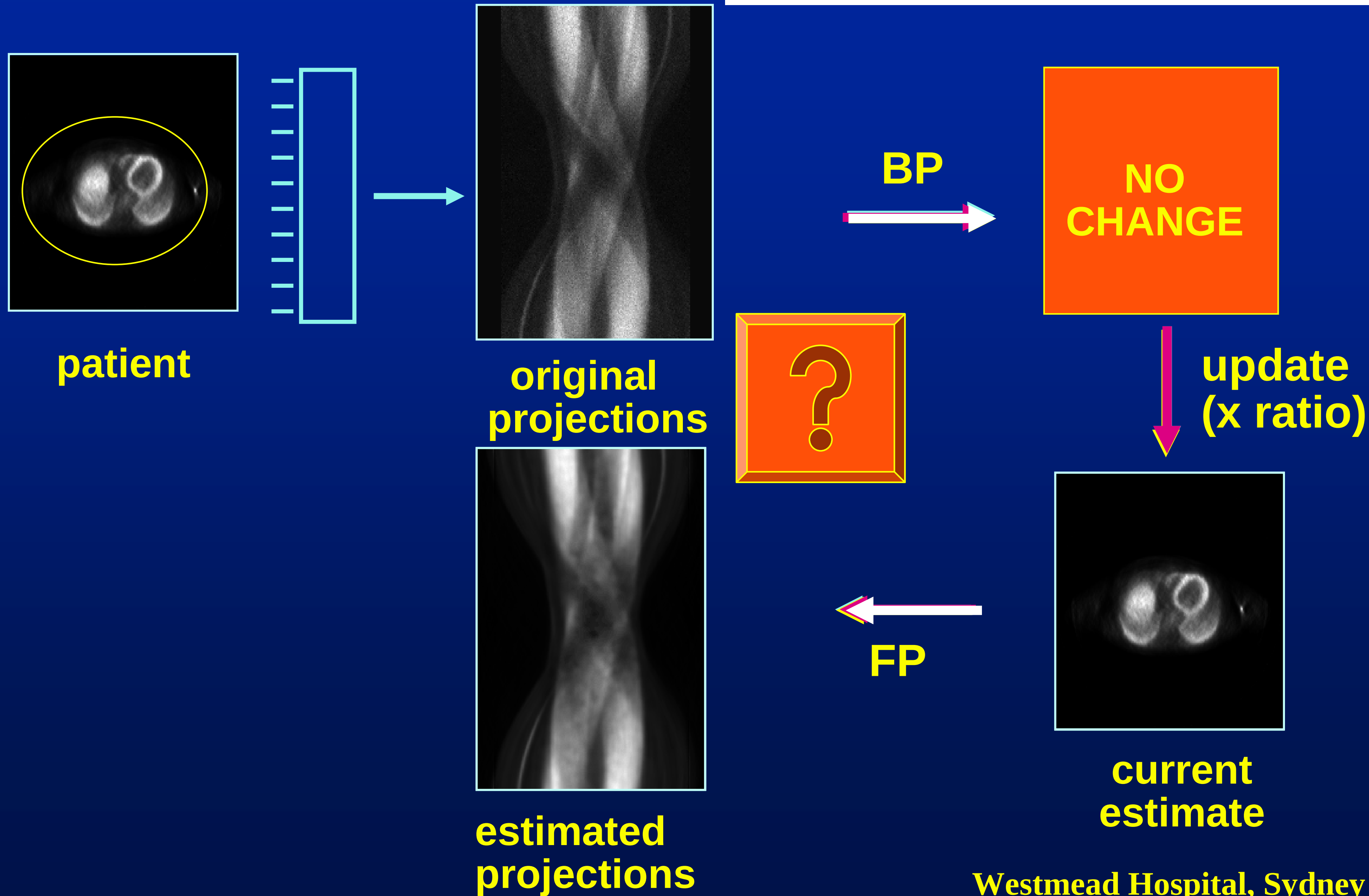
ML-EM reconstruction

$$x_v^{n+1} = \frac{x_v^n}{\sum_p A_{pv}} \cdot \sum_p \left[A_{pv} \cdot \frac{y_p}{\sum_{v'} \left[A_{pv'} \cdot x_{v'}^n \right]} \right]$$



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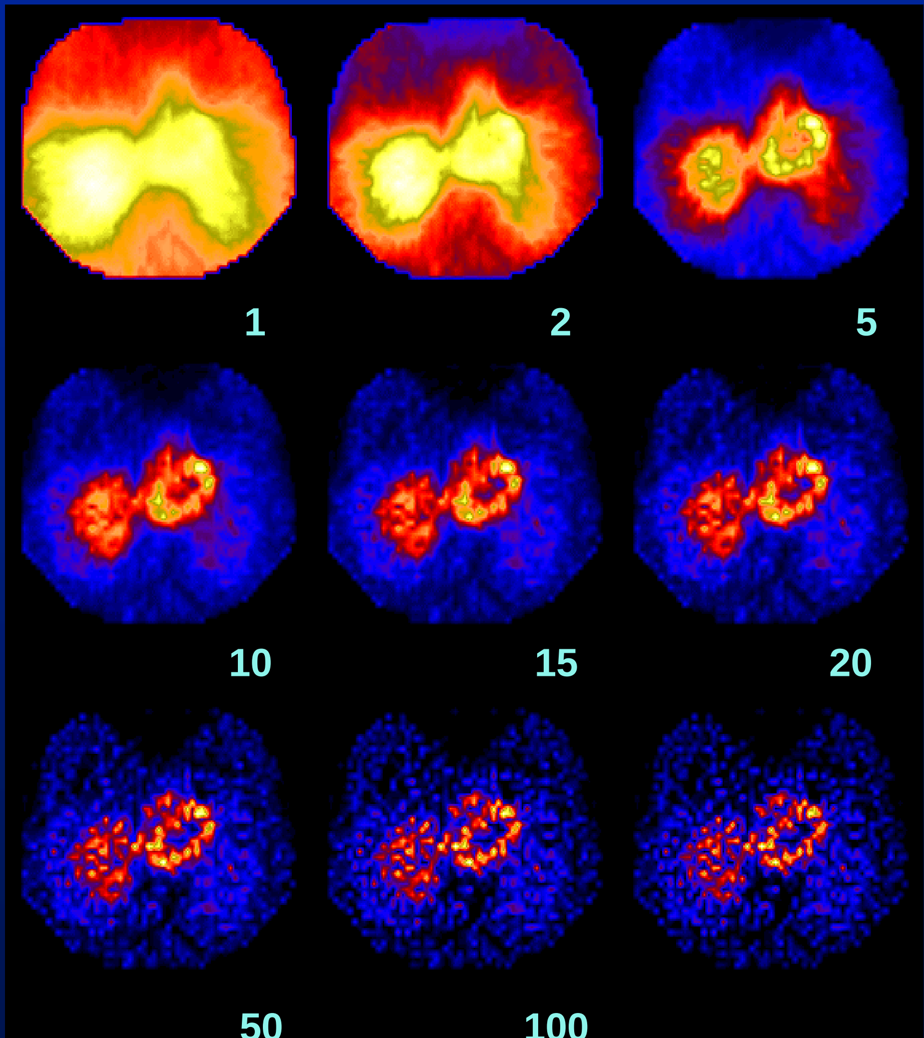
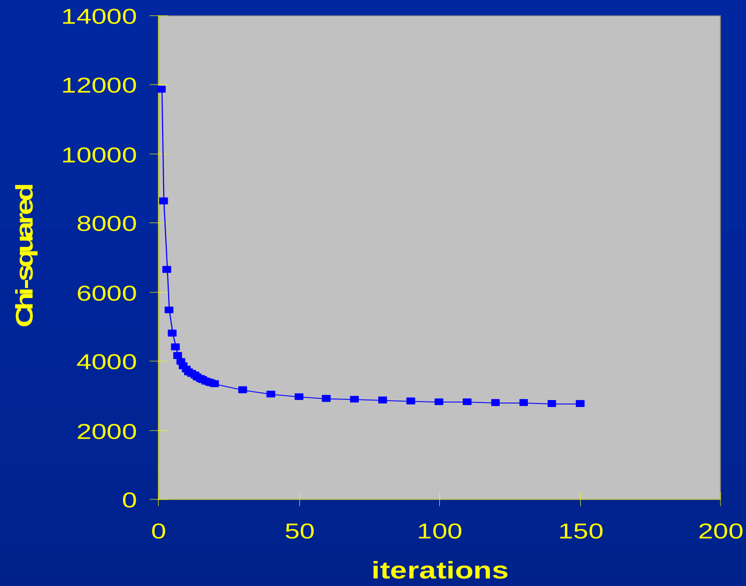
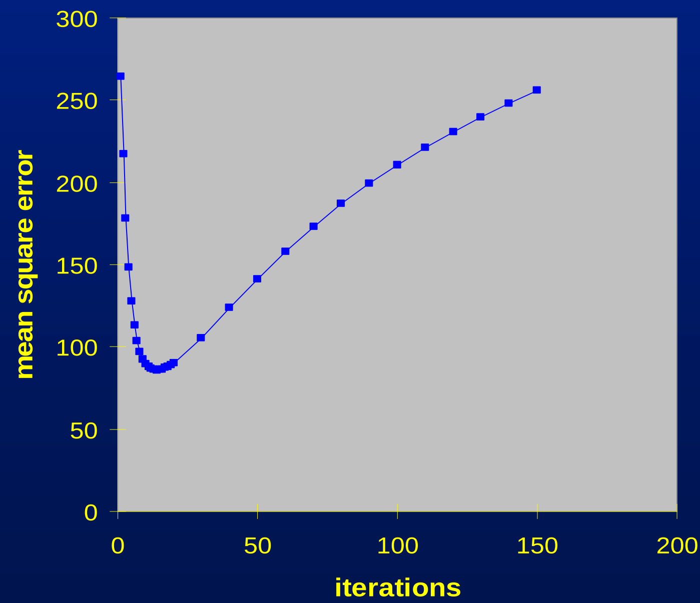
EM reconstruction



150



comparison with projections



comparison with actual object

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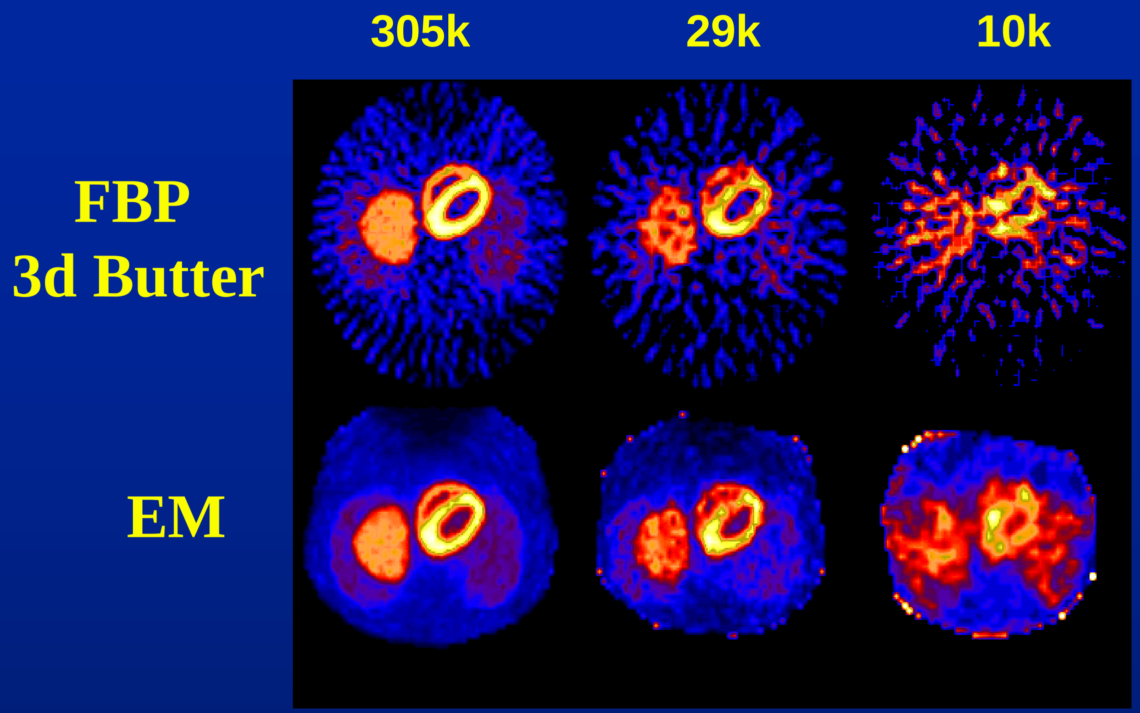
ML-EM: pros and cons



Pros:
- easy to incorporate underlying physics (e.g. attenuation)
- good noise properties
- reduces streak artifacts, effects of missing data

Cons:
- slow: requires multiple iterations
- noise control: unsure when to stop
- hotspots when projections truncated (zero values)

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MLEM versus OSEM (ordered subsets)

$$x_v^{n+1} = \frac{x_v^n}{\sum_{p_s} A_{p_s v}} \cdot \sum_{p_s} \left[A_{p_s v} \cdot \frac{y_{p_s}}{\sum_{v'} \left[A_{p_s v'} \cdot x_{v'}^n \right]} \right]$$

End of first iteration

End of second iteration

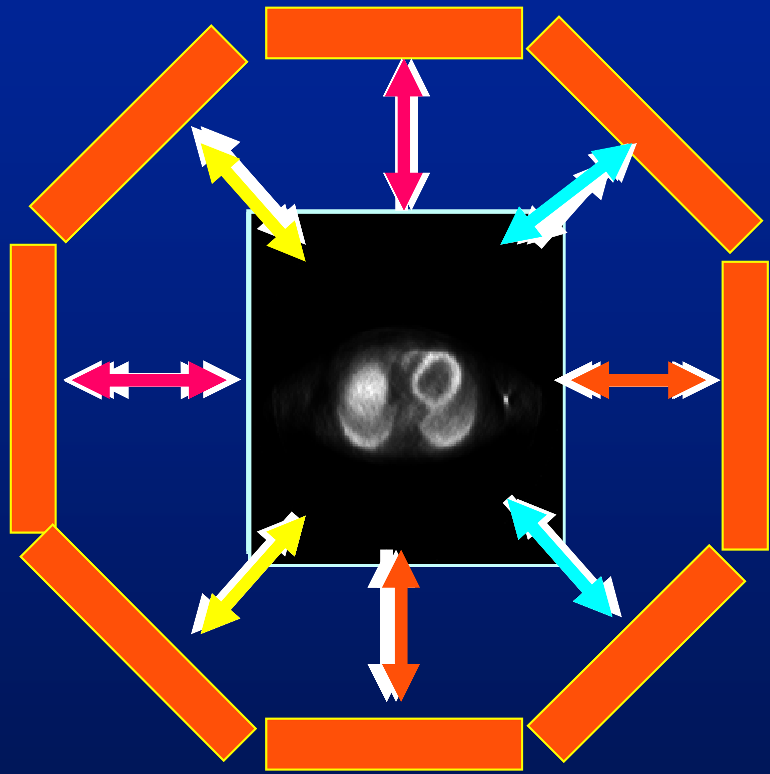
END OF
SINGLE ITERATION

-

-

-

End of n iterations



Ordered subsets (OS) EM reconstruction
conventional EM reconstruction

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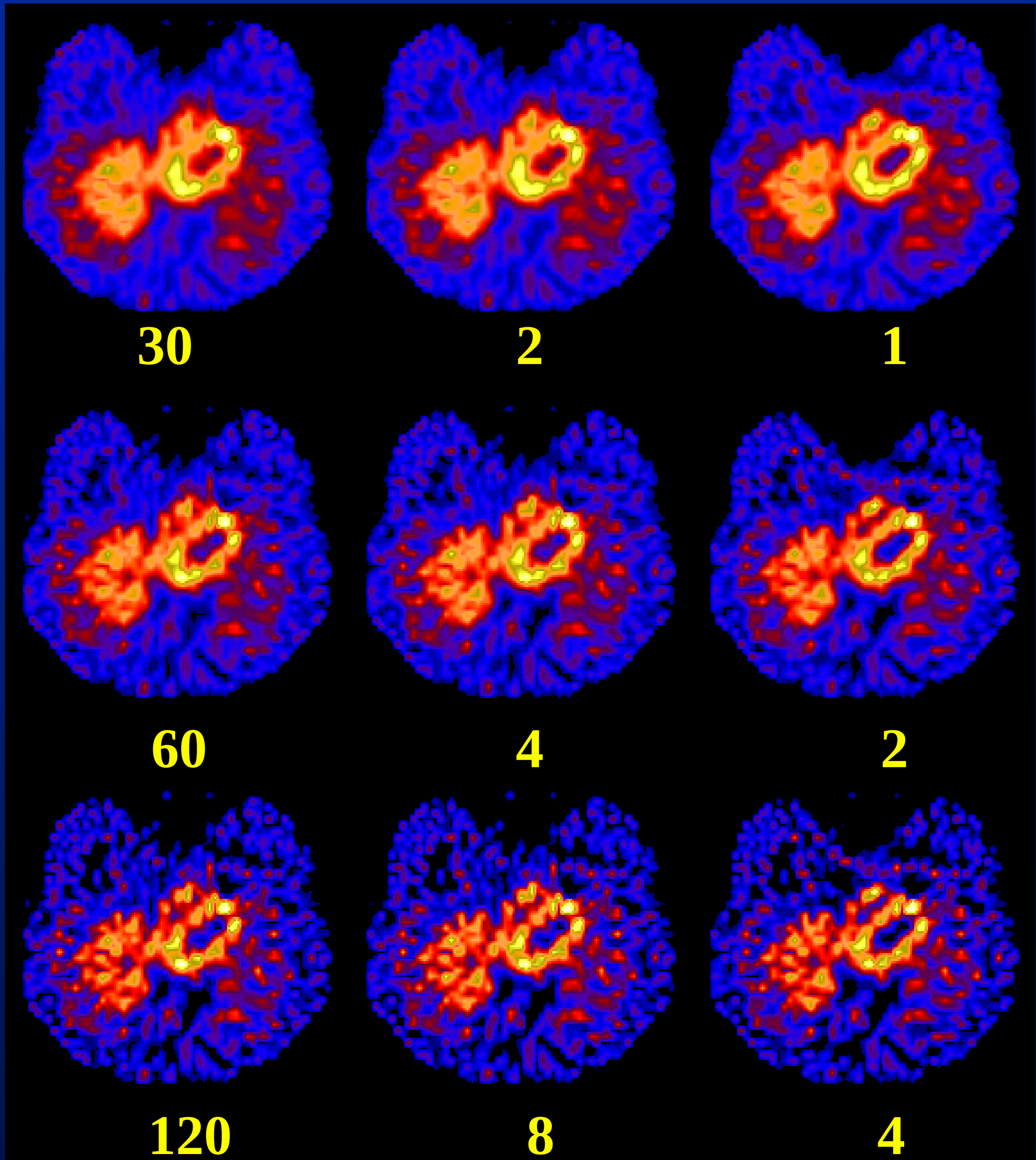
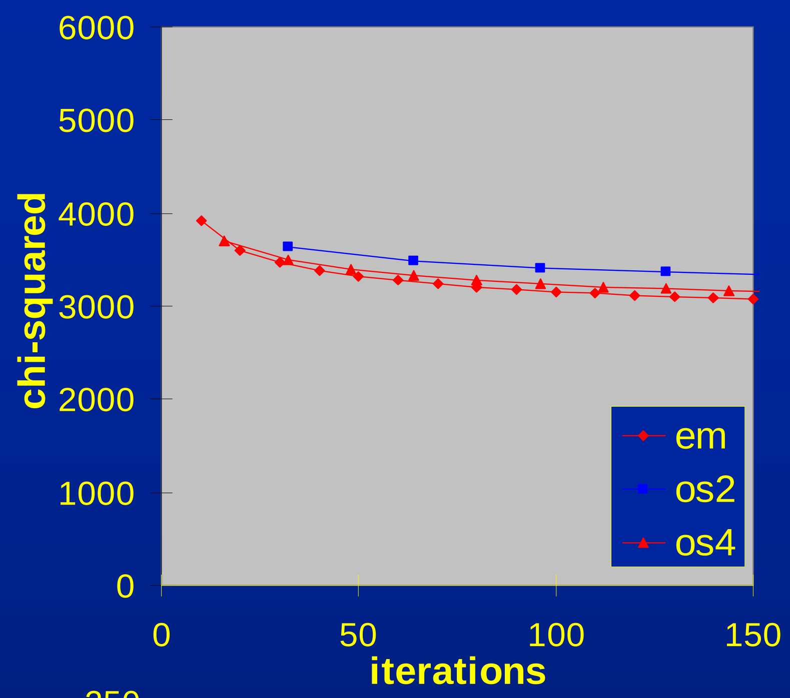
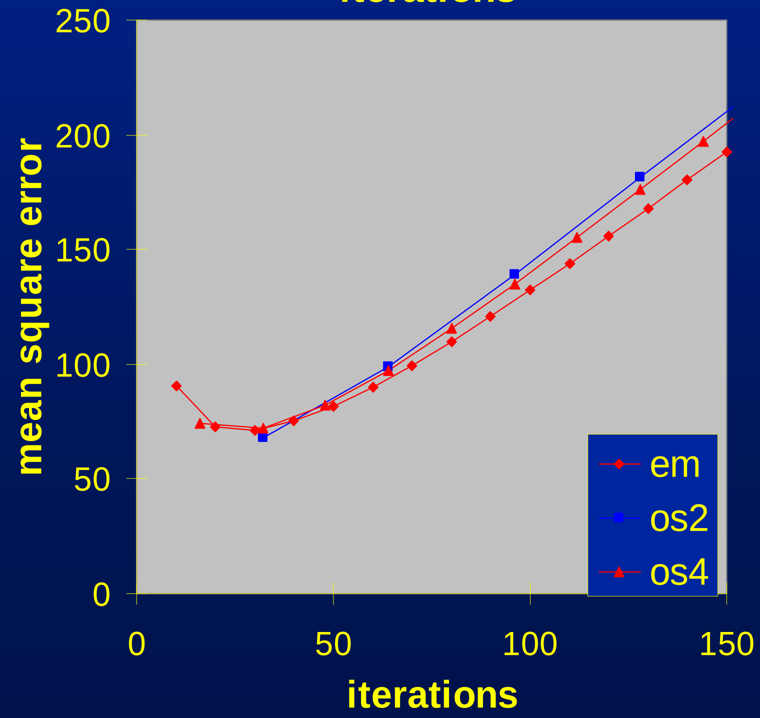
EM OSEM 4 OSEM 2







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Clinical assessment

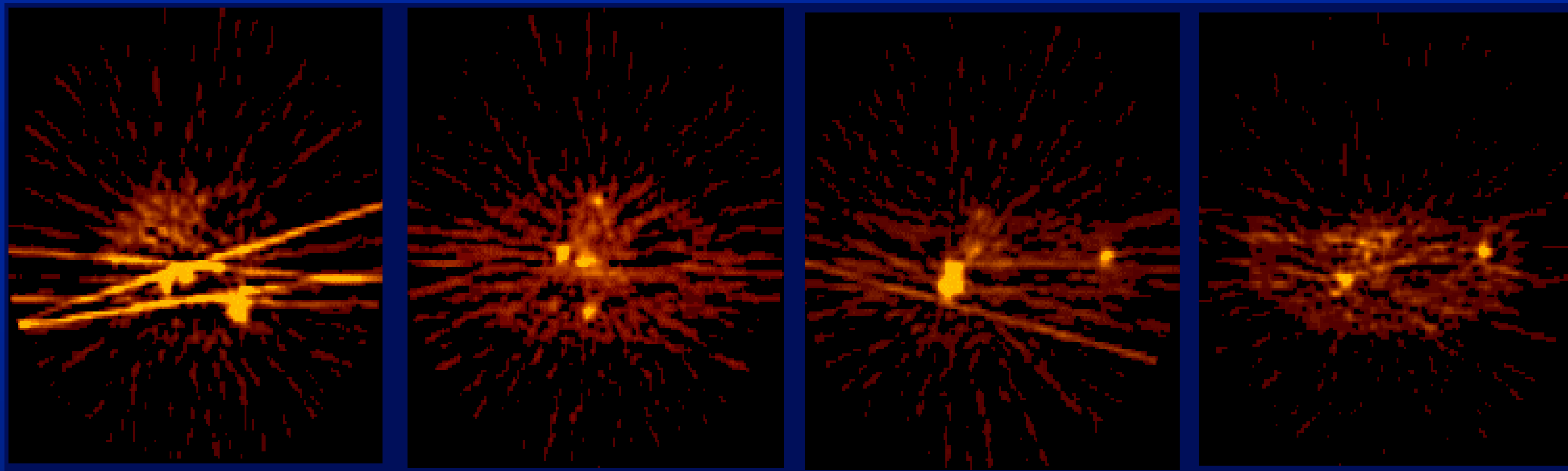
Published literature

- **many papers on OSEM versus FBP**
- **iterative algorithms outperform FBP**
- **quantitative accuracy usually equal or better**
- **better results for SPM, SUV, kinetic studies**

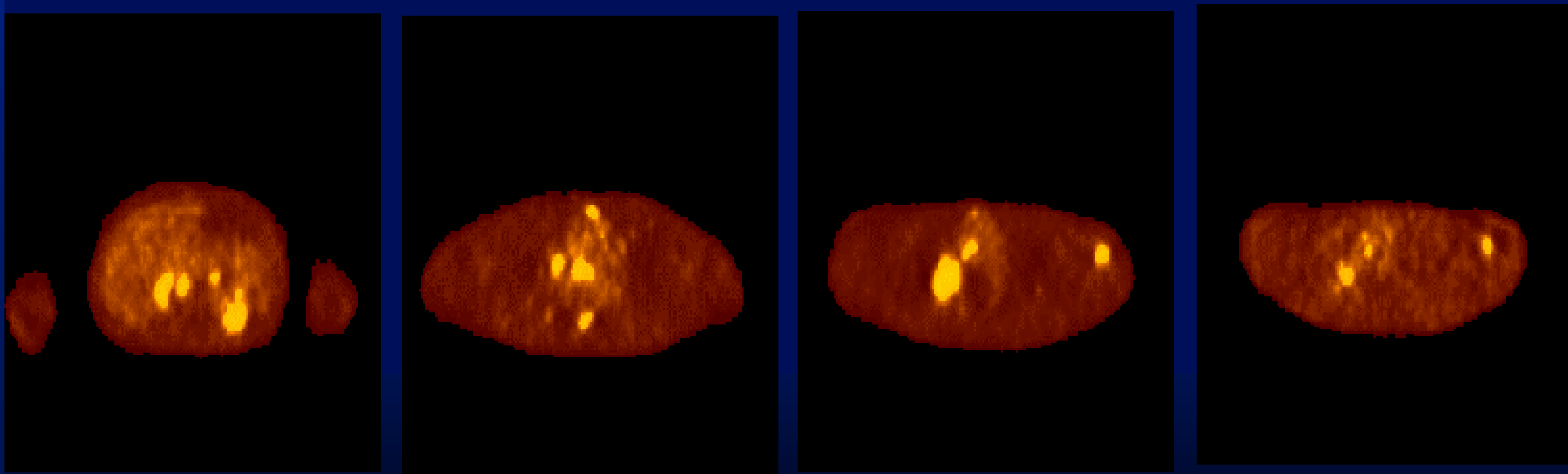
Limitations in some publications

- **often comparison of OSEM *with* AC versus FBP *without* AC**
- **comparison of 2D versus 3D algorithms**
- **not always compared for similar noise conditions**
- **problem in number of available algorithms to compare**

2000: Lahorte (Neuroimage), Gifford (Med Phys), Wells (J Nucl Med)
2001: Bai (Ann Nucl Med), Riddel (J Nucl Med), Boellard (J Nucl med), Delbeke (Radiol), Visvikis (Eur J Nucl Med)
2002: Hsu (Comp Med Imag Graph), Bettinardi (Eur J Nucl Med),



Analytical Reconstruction : FBP



Iterative Reconstruction : OSEM

Image courtesy of Bettinardi et al, Milan

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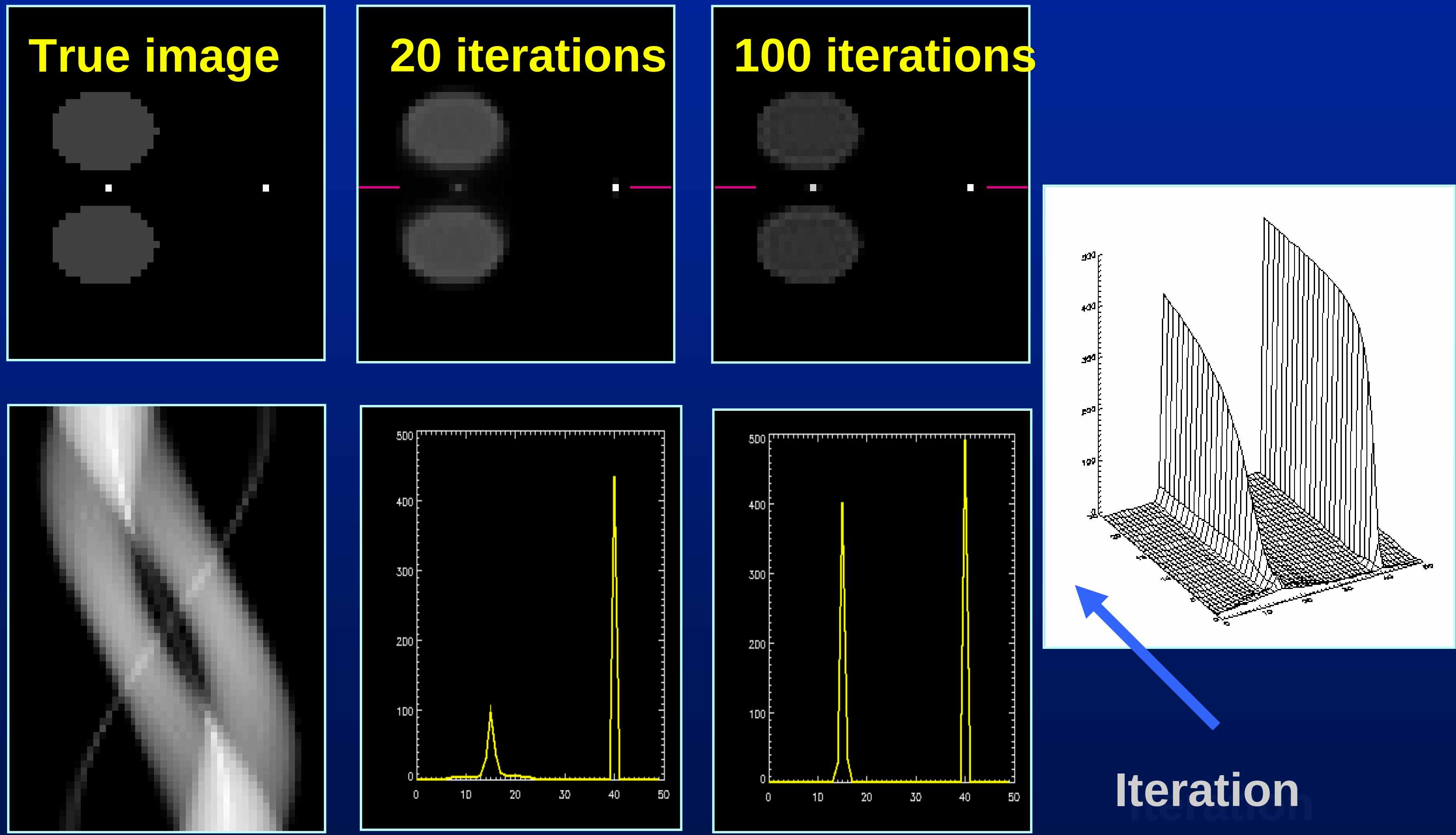
MLEM: non-uniform convergence



sinogram

Courtesy Johan Nuyts, KU Leuven, Belgium

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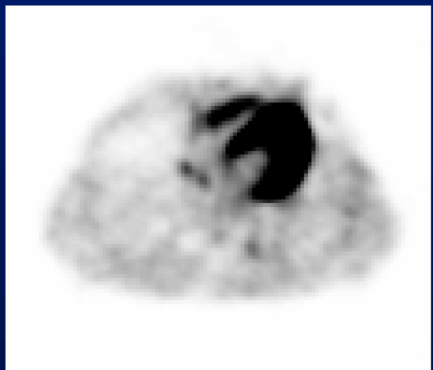
Accelerated ML

properties of OSEM

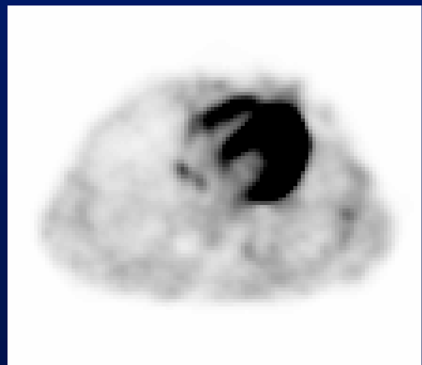
- subset balance assumed: i.e. total counts in each subset the same
- limit cycle possible: solution can oscillate for later subsets
- no proof of convergence to ML: unless subset size varied
- close to MLEM in practice, provided subset size not too small

Alternative algorithms

rescaled block iterative	RBI	subset balance not reqd
row action	RAMLA	subset size varied



OSEM



RAMLA

See editorial by Leahy & Byrne, Trans Med Imag 2000; pp257-260.

- **OSEM (OSGP)** ***(Hudson, Larkin 1994)***

$$x_v^{(n,s+1)} = x_v^{(n,s)} + \frac{1}{\sum_{p_s} A_{p_s v}} x_v^{(n,s)} \cdot \sum_{p_s} \left[A_{p_s v} \left(\frac{y_{p_s}}{\sum_{v'} \left[A_{p_s v'} \cdot x_{v'}^{(n,s)} \right]} - 1 \right) \right]$$

- **RBI-EMML (-MAP)** ***(Byrne 1996)***

$$x_v^{(n,s+1)} = x_v^{(n,s)} + \frac{1}{\max \sum_{p_s} A_{p_s v}} x_v^{(n,s)} \cdot \sum_{p_s} \left[A_{p_s v} \left(\frac{y_{p_s}}{\sum_{v'} \left[A_{p_s v'} \cdot x_{v'}^{(n,s)} \right]} - 1 \right) \right]$$

- **RAMLA (BSREM)** ***(Browne, De Pierro 1996)***

$$x_v^{(n,s+1)} = x_v^{(n,s)} + \lambda_n x_v^{(n,s)} \cdot \sum_{p_s} \left[A_{p_s v} \left(\frac{y_{p_s}}{\sum_{v'} \left[A_{p_s v'} \cdot x_{v'}^{(n,s)} \right]} - 1 \right) \right]$$

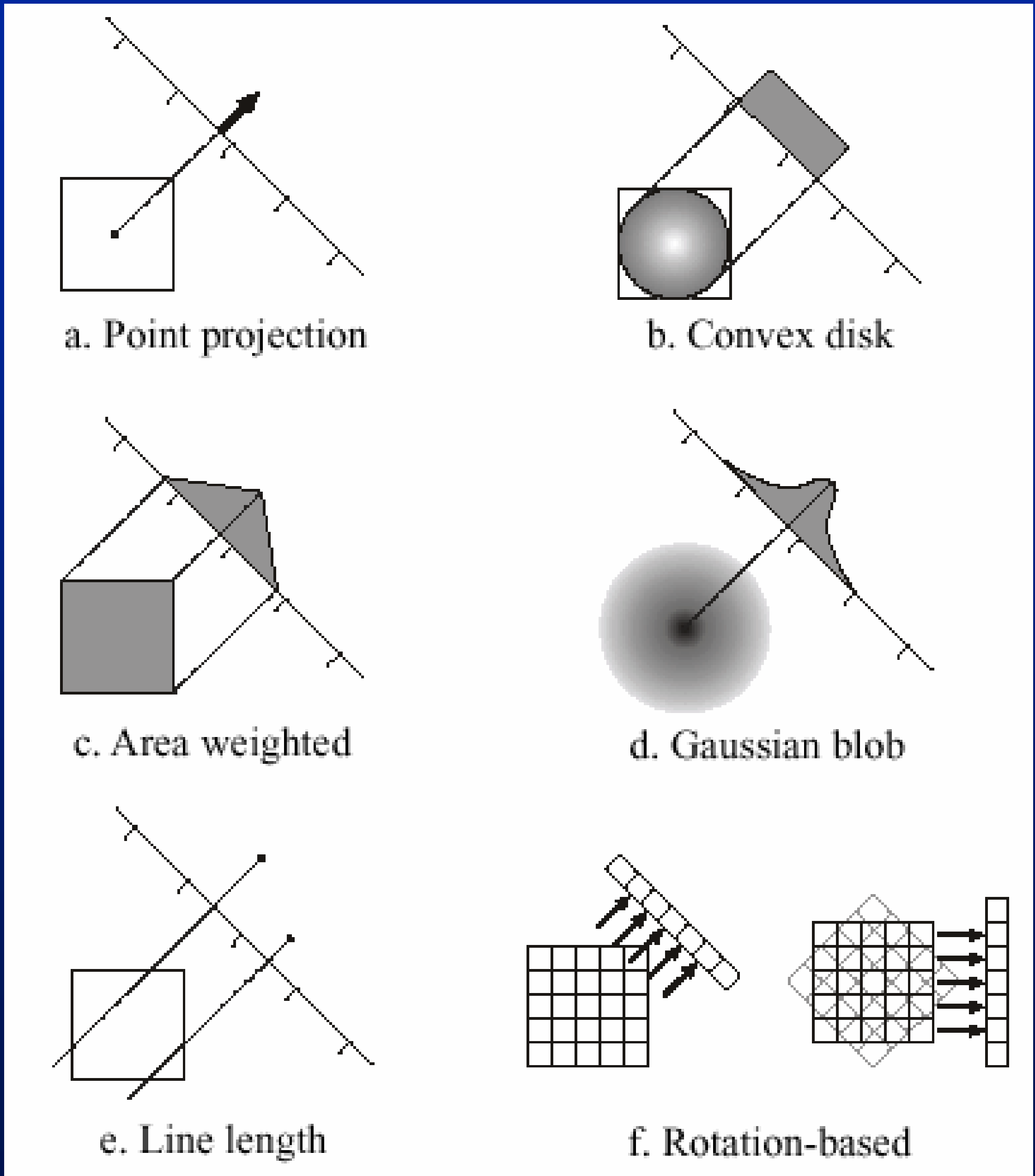
- **DRAMA, DOSEM** ***(Tanaka, Kudo 2003 [under review])***

$$x_v^{(n,s+1)} = x_v^{(n,s)} + \frac{\lambda_n(s)}{\max \sum_{p_s} A_{p_s v}} x_v^{(n,s)} \cdot \sum_{p_s} \left[A_{p_s v} \left(\frac{y_{p_s}}{\sum_{v'} \left[A_{p_s v'} \cdot x_{v'}^{(n,s)} \right]} - 1 \right) \right]$$

Forward projection

considerations

- accuracy
- interpolation
- noise control
- speed



Courtesy David Lalush, Duke University, Durham

Noise control

- stop at an early iteration
- use of a 'sieve': smoothing between iterations
- post-reconstruction smoothing
- penalise 'rough' solutions: regularisation

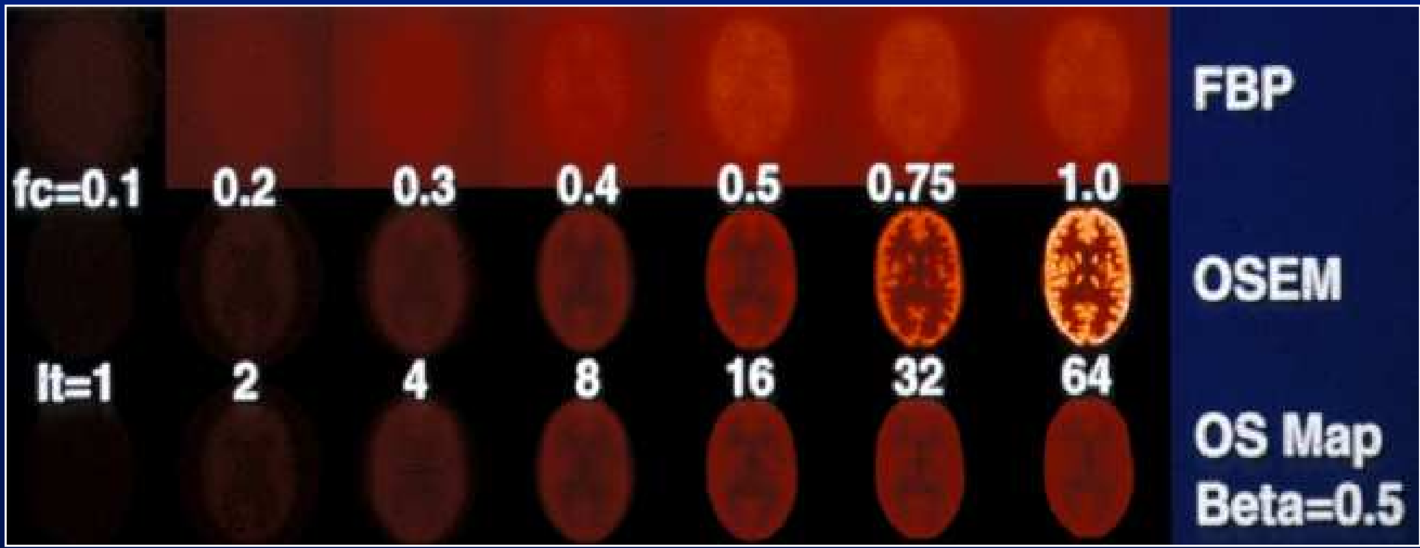


Image courtesy of Stefan Eberl, RPAH

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Maximum *a-posteriori* (MAP)

$$p(\lambda|y) = \frac{p(y|\lambda)\, p(\lambda)}{p(y)}$$

Bayes theorem

$$\ln p(\lambda \mid y) \;=\; \ln p(y \mid \lambda) \;+\; \ln p(\lambda) \quad + \text{const}$$

log (posterior) = log (likelihood) - β. Gibbs prior

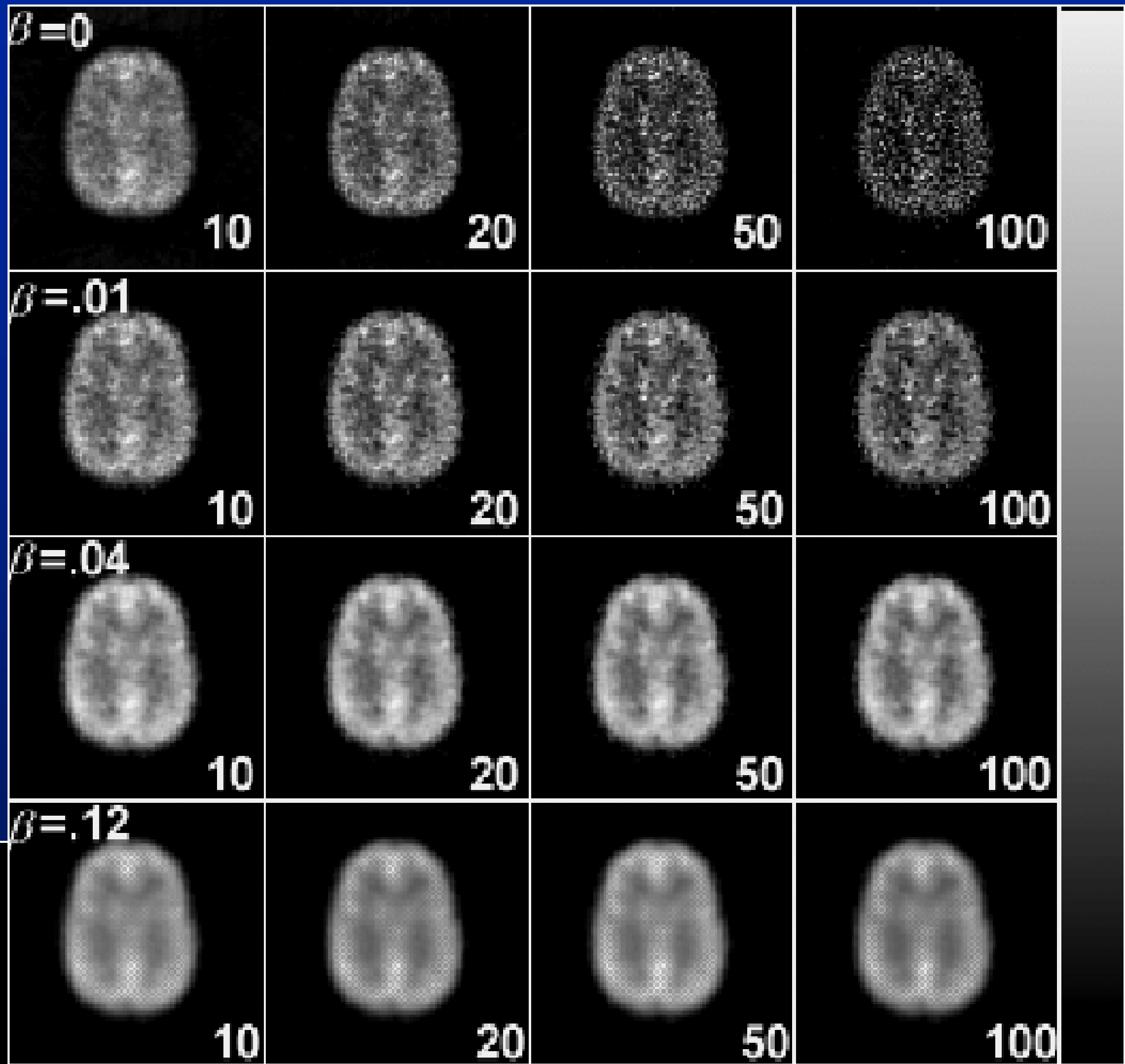
penalizes 'roughness'

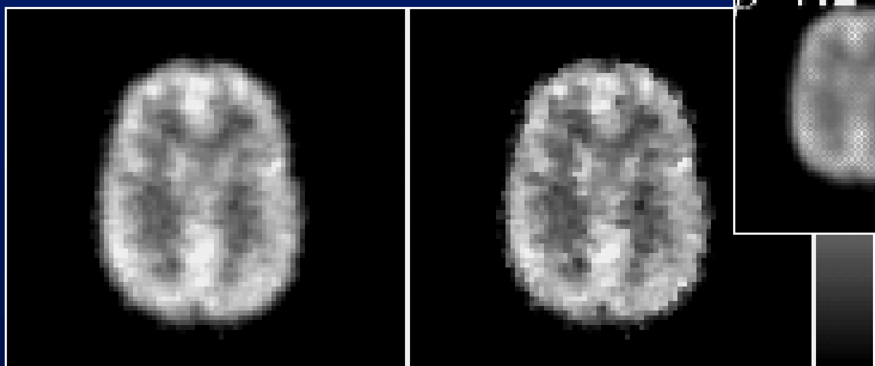
Prior model

- **encourage similar neighbouring values: quadratic**
- **preserve significant 'edges': Huber**
- **incorporate anatomical information: line sites**

MAP

- ‘strength’ of prior defined by β (hyperparameter)
- degree of smoothing versus edge preserving defined by type of prior





Quadratic Huber

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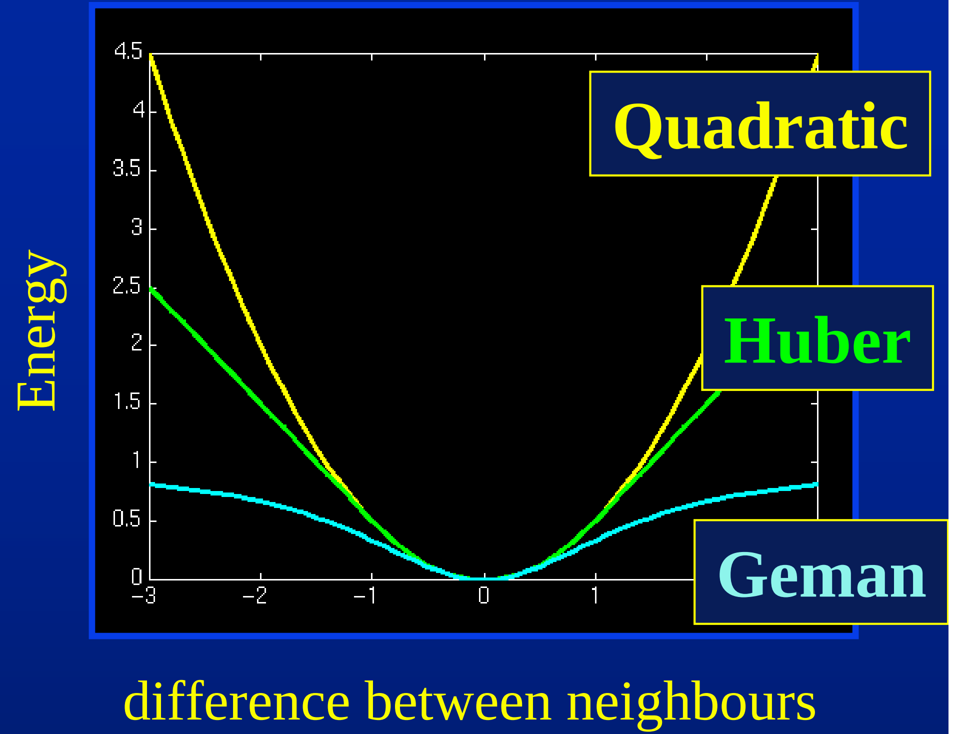
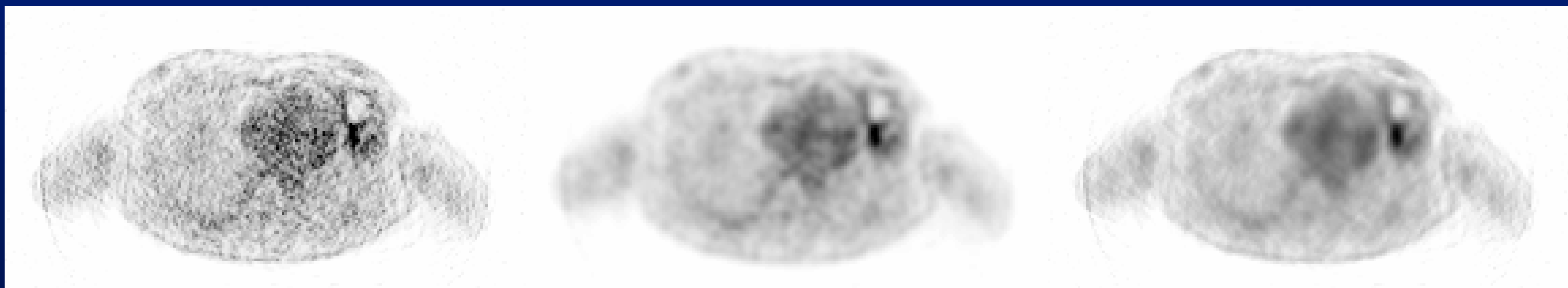
MAP
(Huber prior)

- **penalty defined by derivative of energy function**
- **prior penalises small differences (noise) rather than large differences (edges)**





MLEM

MLEM postsmoothed

MAP (Huber prior)

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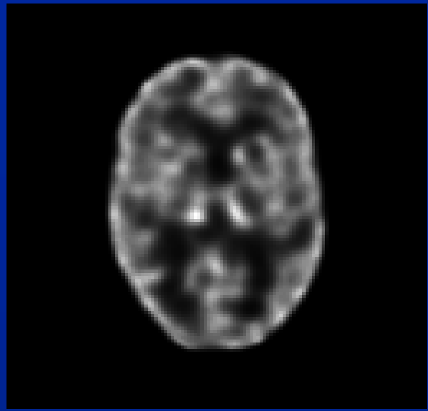
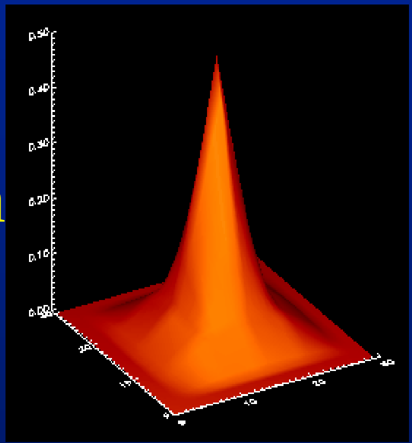
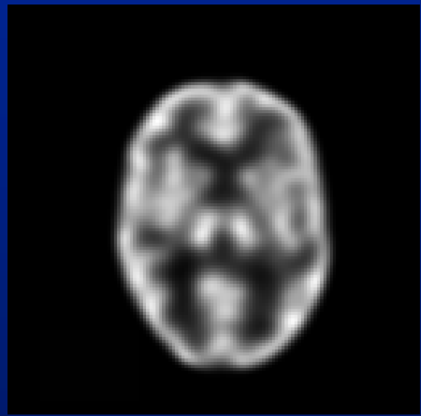
Reconstruction based on anatomical prior (minimum cross entropy)

EM

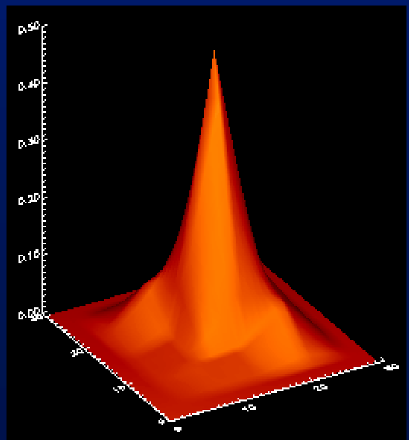


EM + smooth prior





EM + MRI based prior



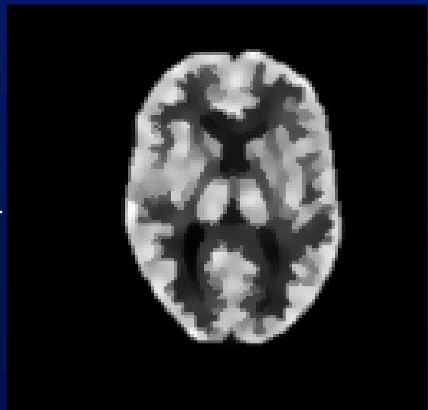


Image courtesy of Seu Som, POWH

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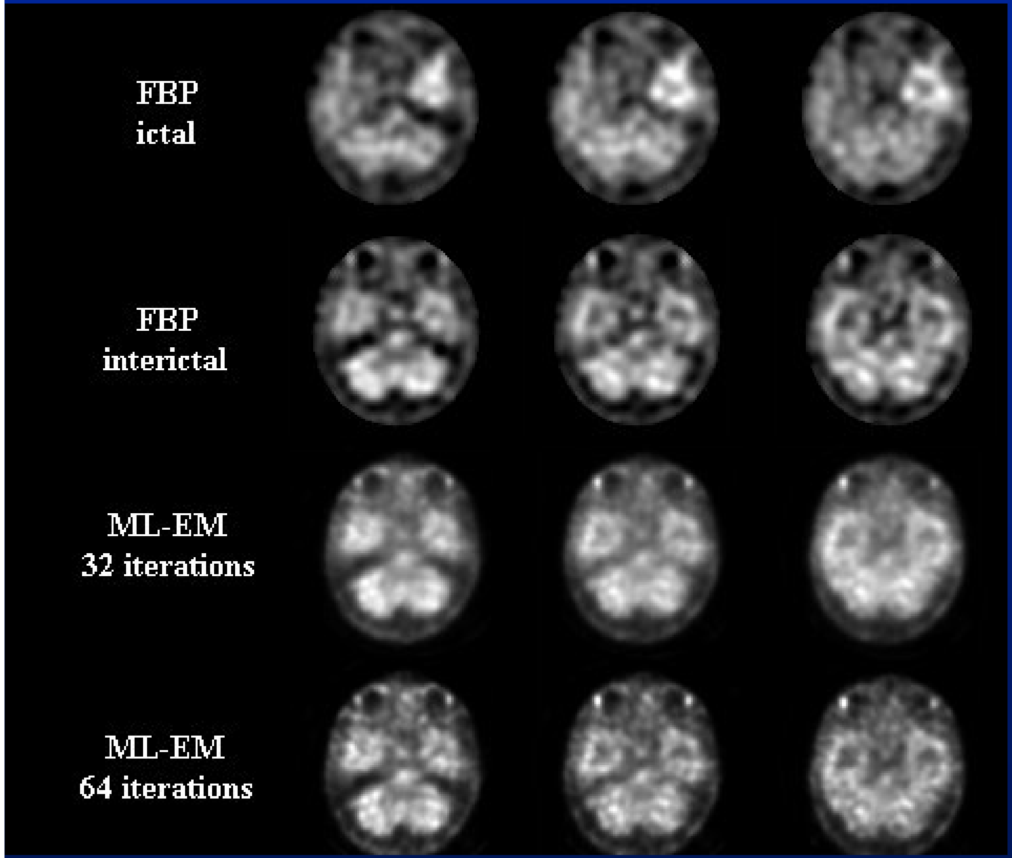
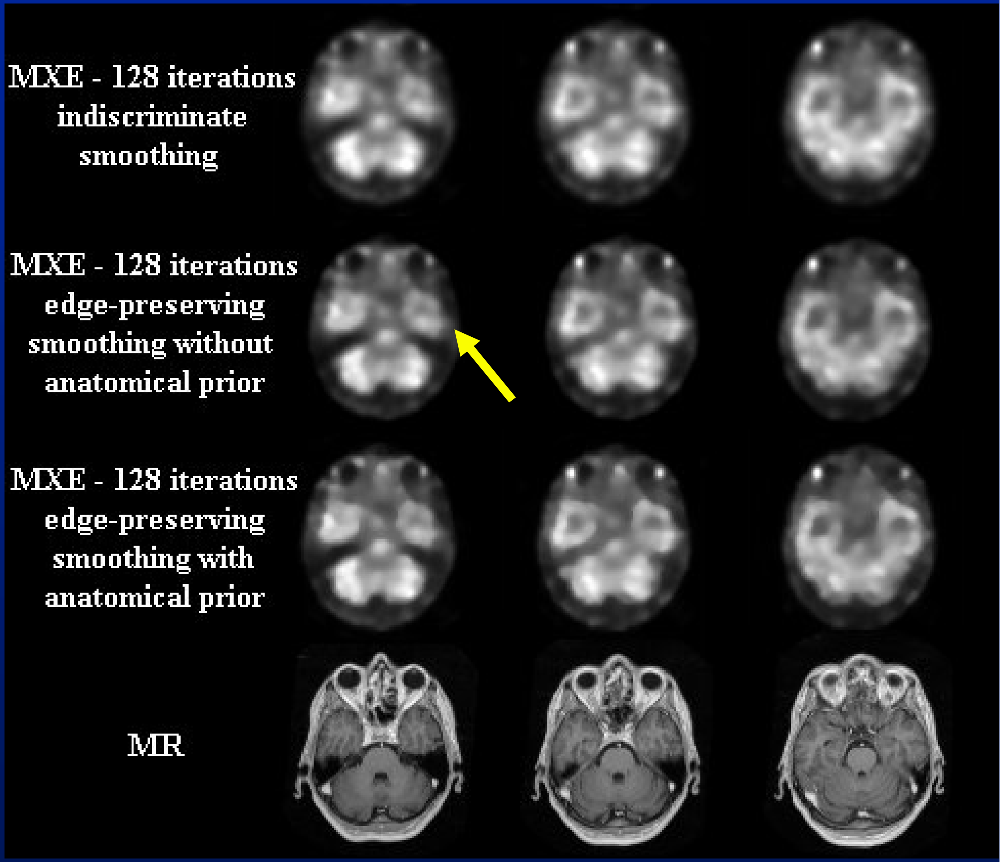
MXE with prior





Interictal SPECT comparison

Image courtesy of Seu Som, POWH

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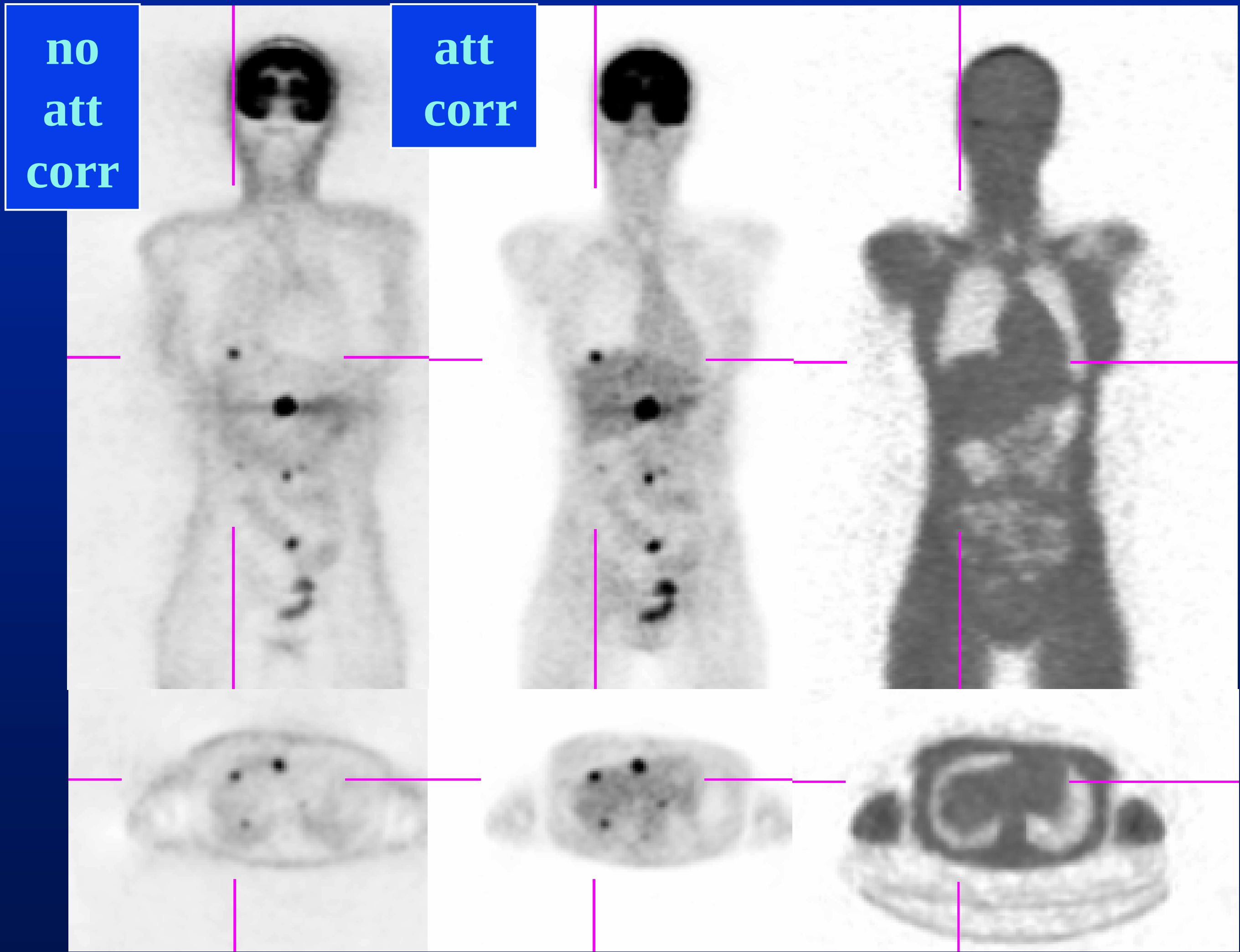
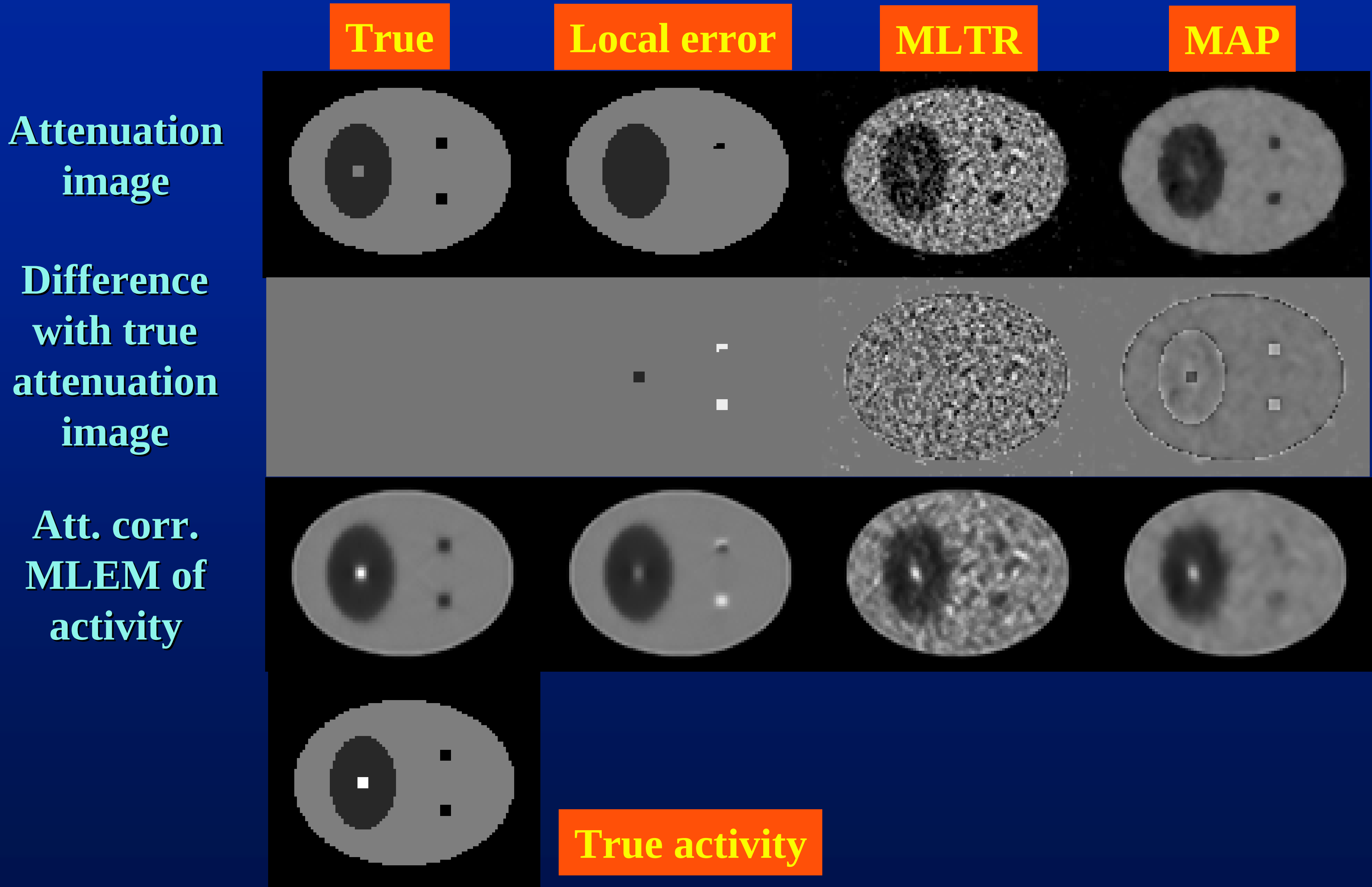
PET whole body



Courtesy Johan Nuyts, KU Leuven, Belgium

Attenuation errors in PET



Courtesy Johan Nuyts, KU Leuven, Belgium

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Non-Poisson data:
NEC weighted OSEM

transmission $x_i = ln[b_i / y_i]$ with b_i noise-free

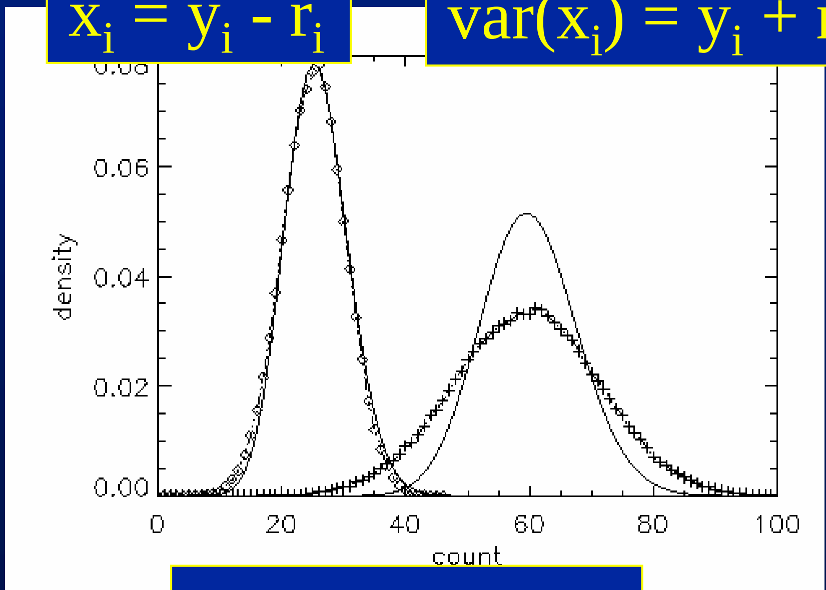
correction for sensitivity, attenuation $x_i = y_i / a_i$

correction for scatter $x_i = y_i - s_i$ with s_i noisy or noise-free

correction for randoms $x_i = y_i - r_i$ with r_i noisy

$x_i = y_i - r_i$

$var(x_i) = y_i + r_i$



$var(a_i x_i) = a_i x_i$

scale with a_i to restore Poisson distribution!

$$a_i = mean(x_i)/var(x_i)$$

e.g. $a_i = (y_i - r_i)/(y_i + r_i)$

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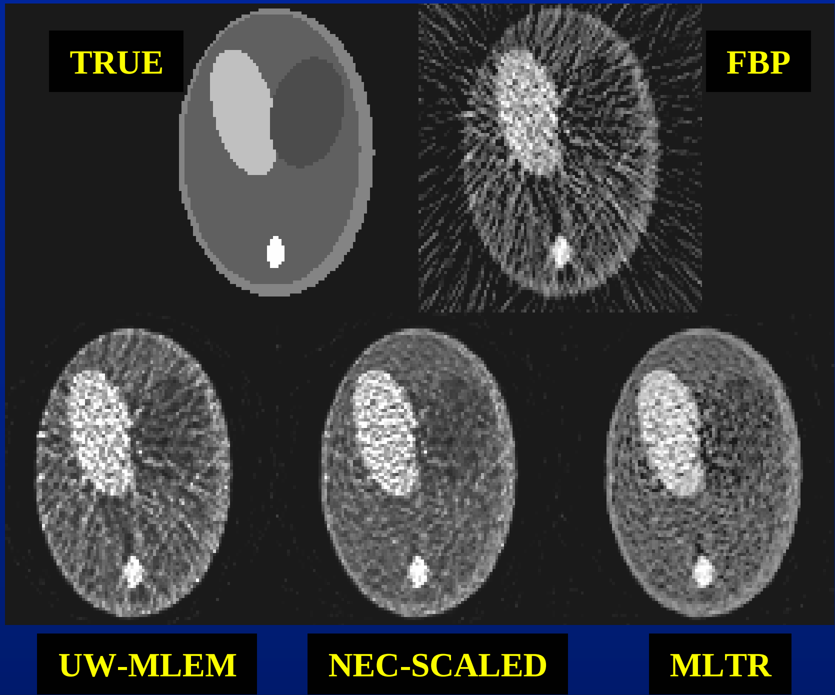
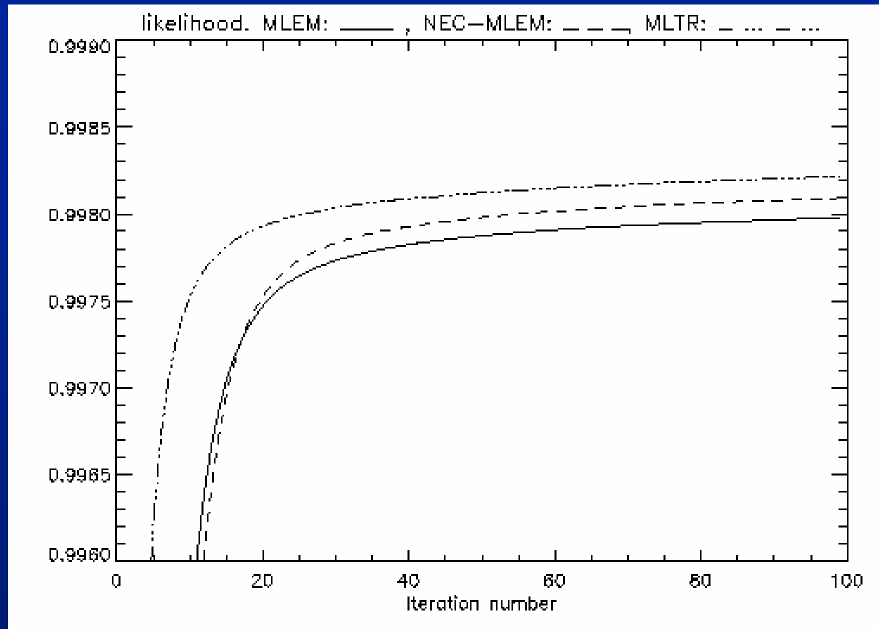
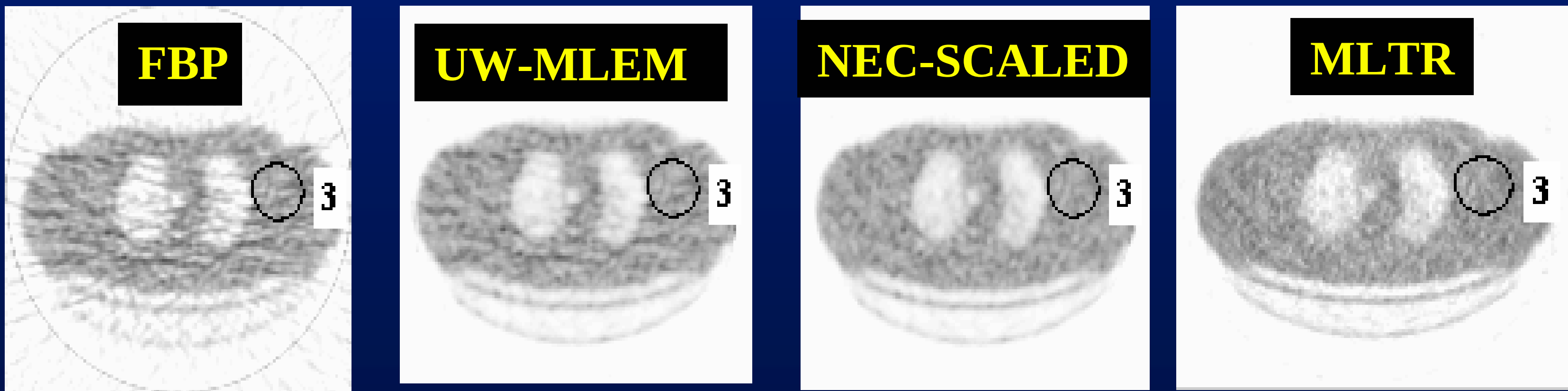
NEC weighting for transmission







Courtesy Johan Nuyts, KU Leuven, Belgium

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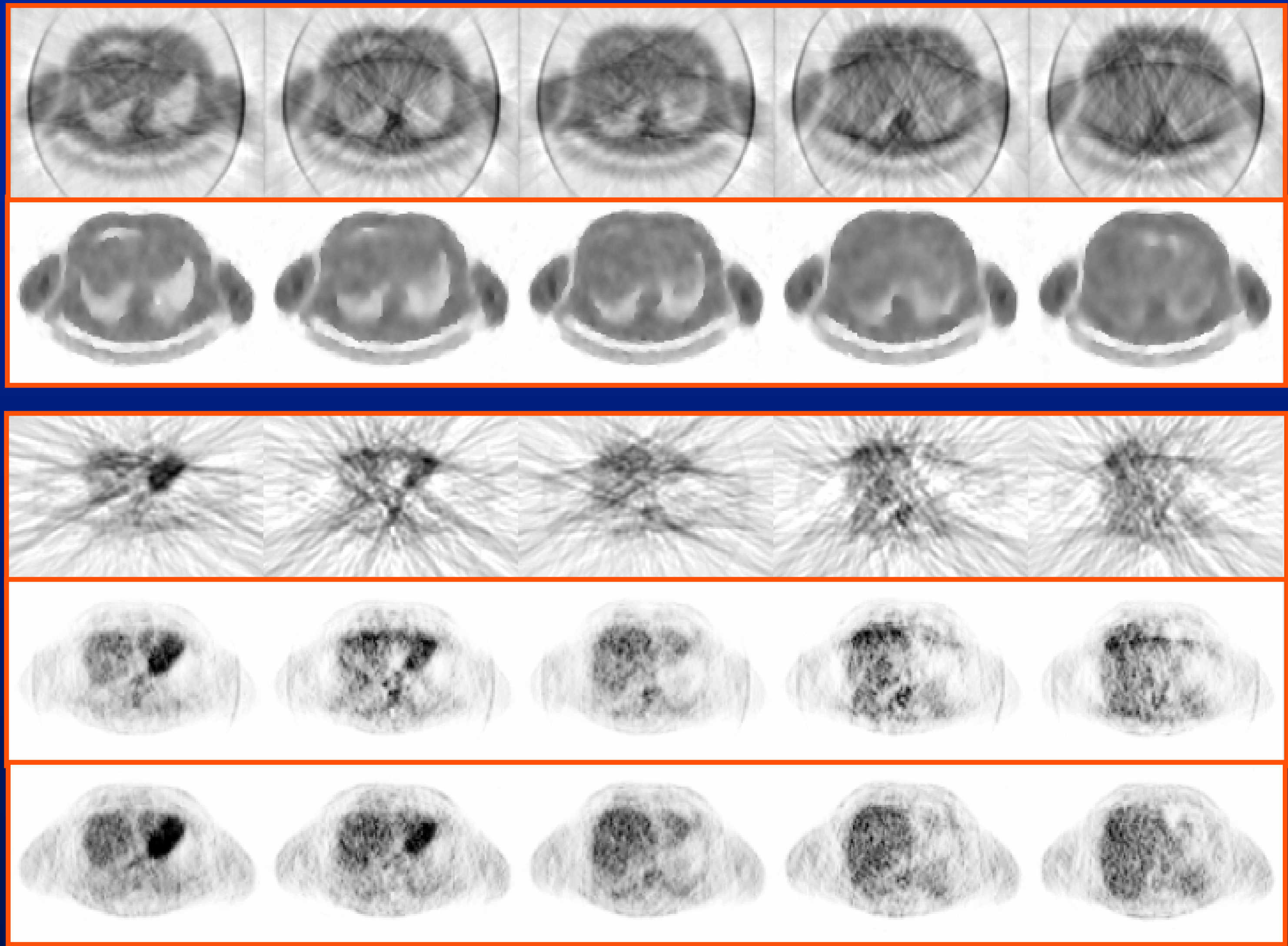
Short transmission scan



Classic
Transm.

M.A.P
Reconstr.

•Classic
Atten cor
•FBP

•Classic
Atten cor
•MLEM

•M.A.P.
Atten cor
•MLEM

Ecat
931

Courtesy Johan Nuyts, KU Leuven, Belgium

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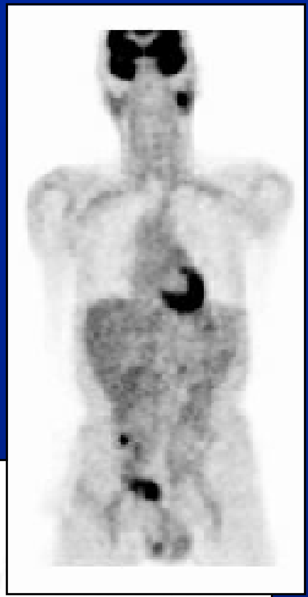
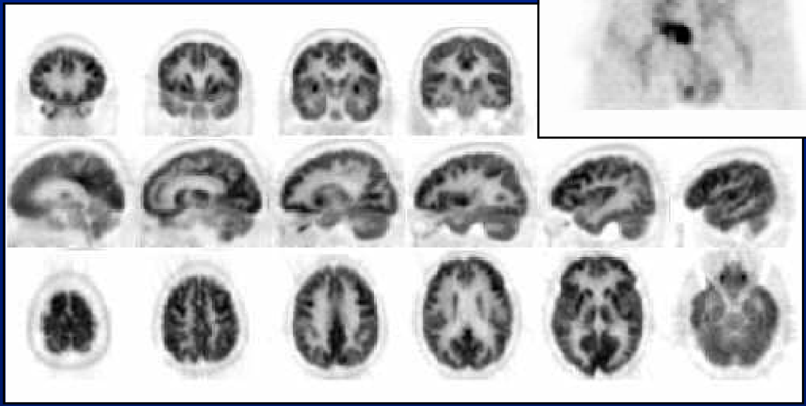
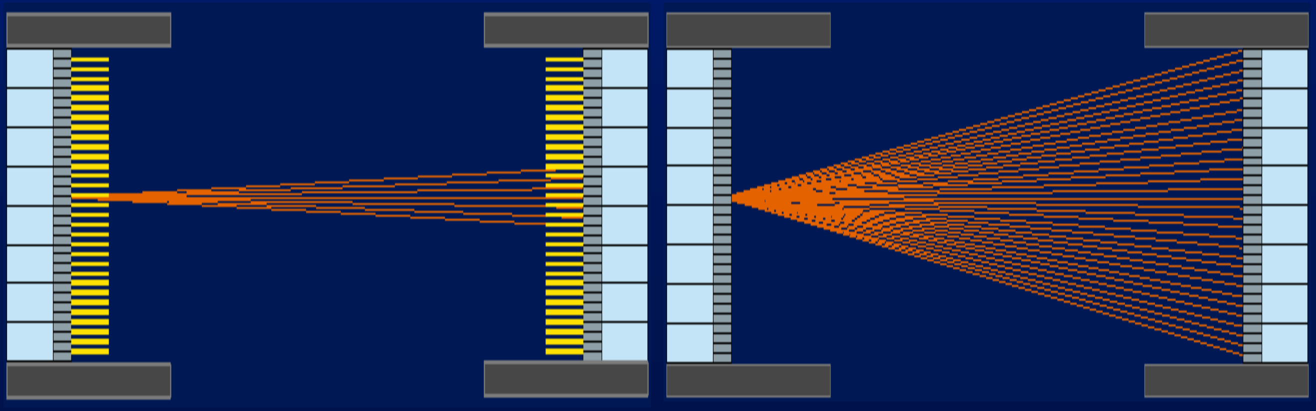
3D reconstruction

Approaches

- rebin data followed by 2D reconstruction
  - single slice rebinning (SSRB)
  - multi-slice rebinning (MSRB)
  - Fourier rebinning (FORE)
- full 3D reconstruction
  - 3D OSEM
  - 3D RAMLA







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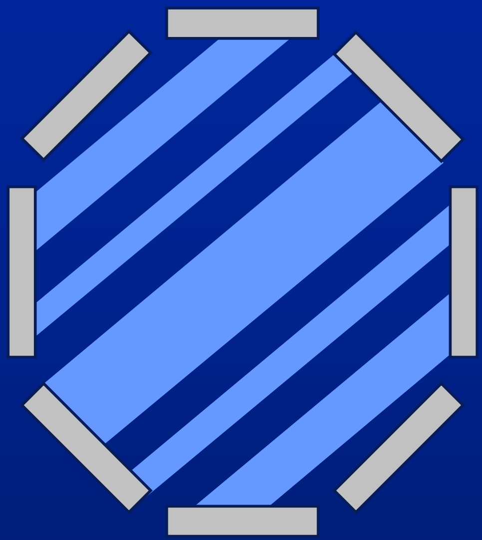
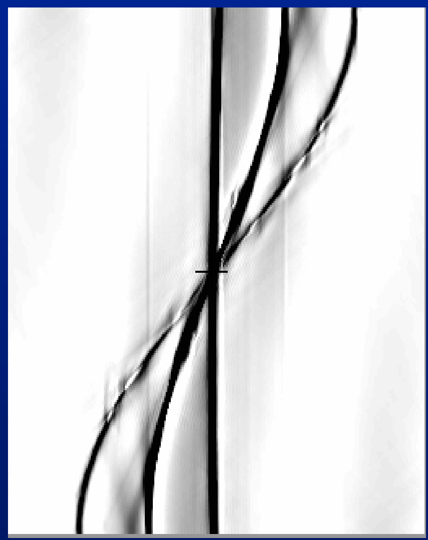
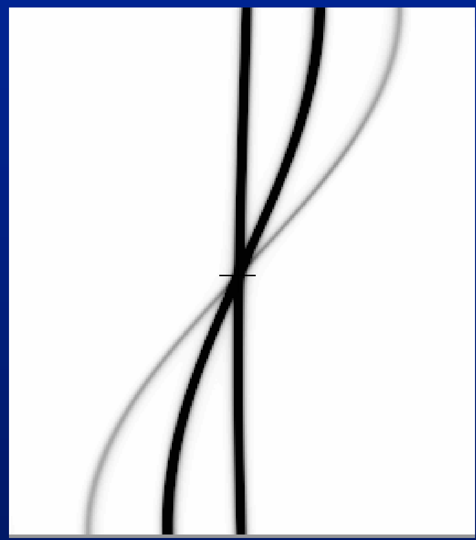
3D OSEM vs FORE



3 points phantom



FORE sinogram



3D OSEM predicted sinogram

Projection

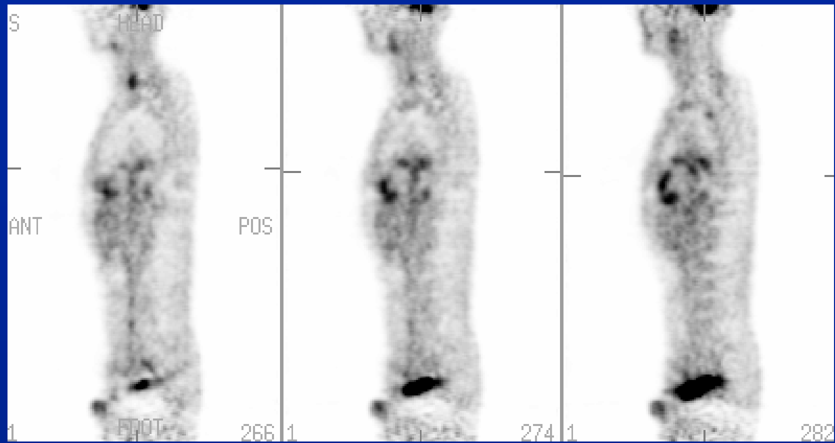
Sinogram

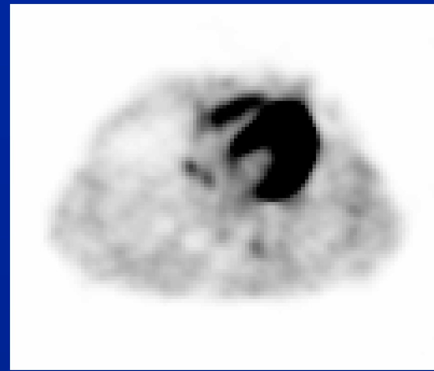
Normalization

Klaus Wienhard, MPI, Köln Christian Michel, CTI

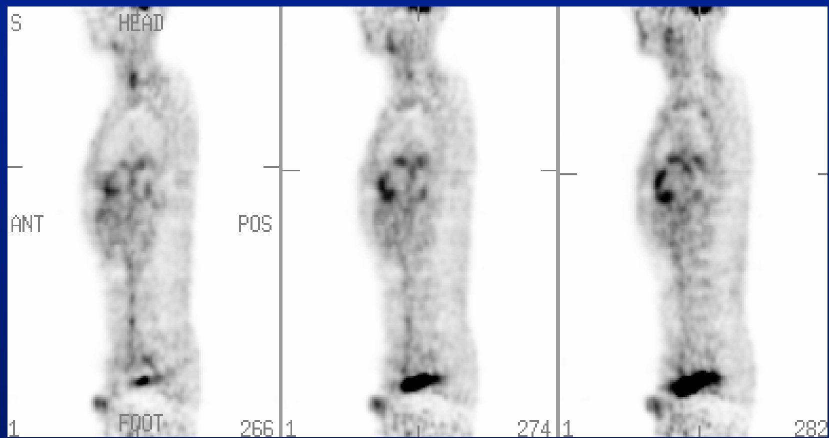
Westmead Hospital, Sydney

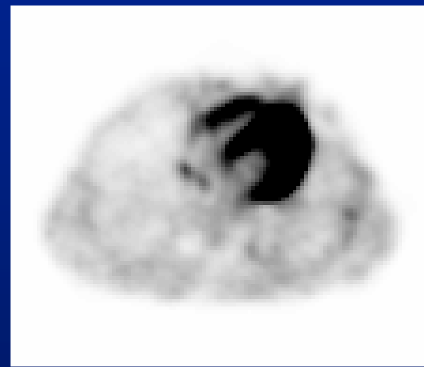
FORE + OSEM



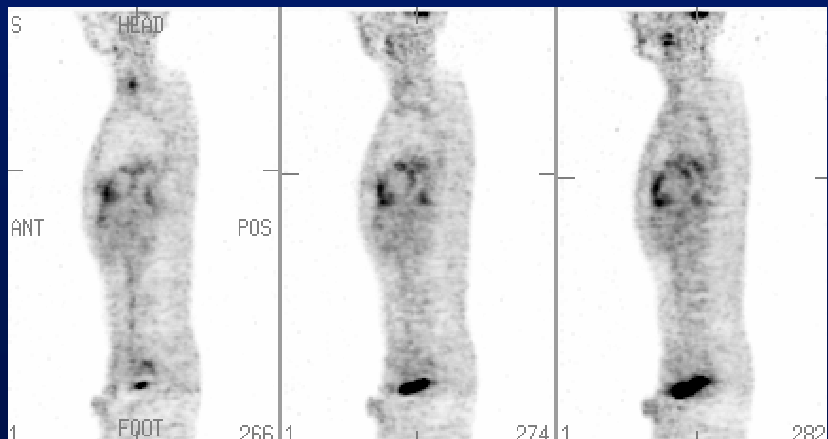


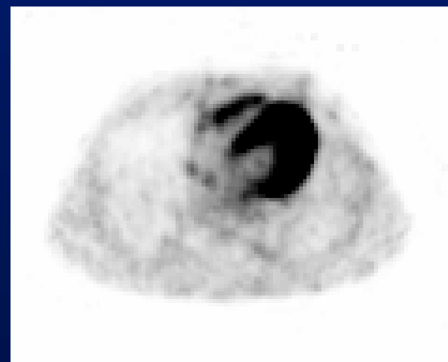
FORE + 2D RAMLA





3D RAMLA





Images courtesy ADAC / HIA Val de Grace

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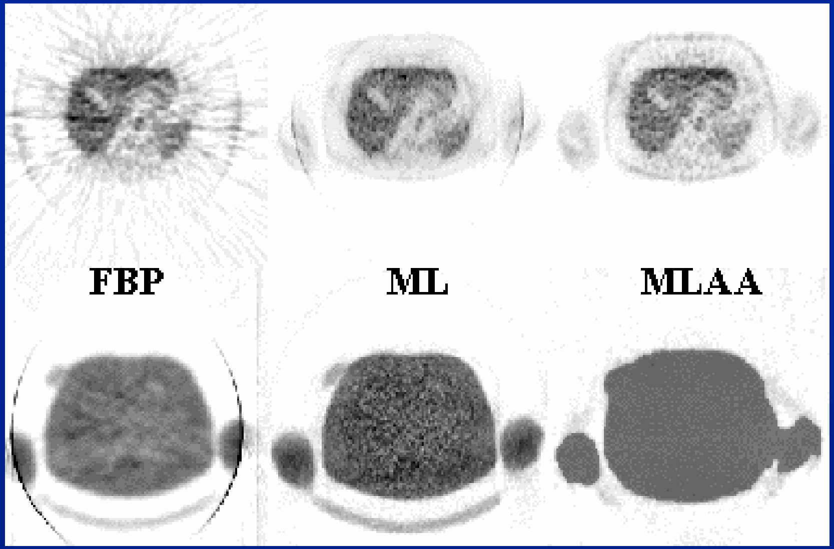
Recent research and future directions



emission only

- 3d reconstruction problems: PET, motion correction
- scatter and randoms correction
- alternative geometry instruments e.g. sparse data
- depth of interaction
- list mode reconstruction (OSEM)
- simultaneous emission / transmission from emission only

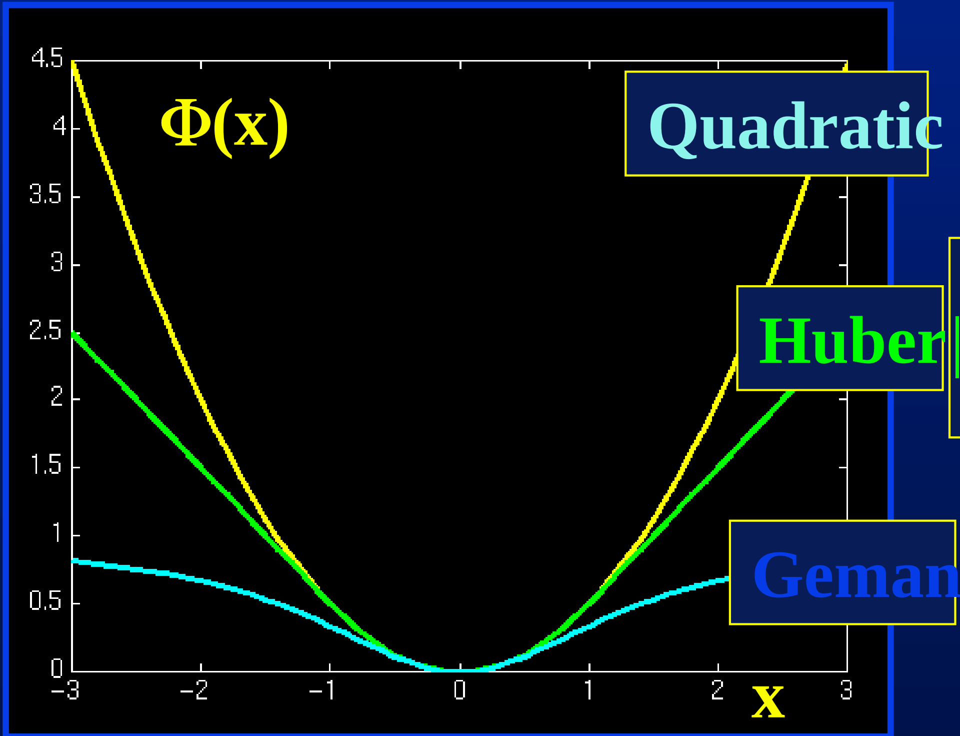
Smoothing prior

Gibbs prior:

$$M_g(\mu) = \frac{1}{Z}\exp\big(U(\mu)\big)$$

Often used: $U(\mu) = -\sum_j \sum_{k \in N_j} \beta_{jk} \Phi(\mu_j - \mu_k)$

U is concave if Φ(x) is convex



Quadratic: $\frac{x^2}{2\sigma^2}$

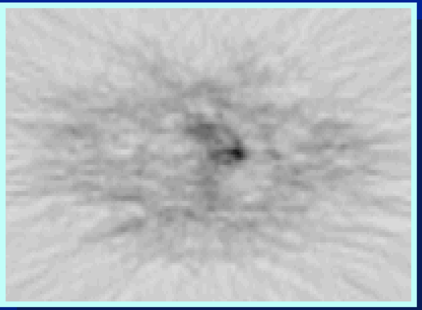
Huber: $|x| < \sigma : \frac{x^2}{2\sigma^2}, \quad \sigma < |x| : \frac{|x| - \sigma/2}{\sigma}$

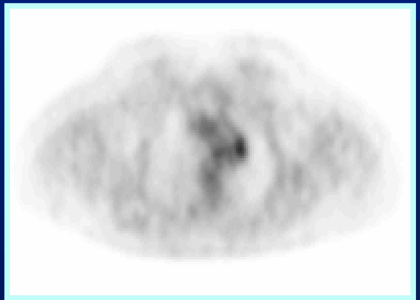
Geman: $\frac{x^2}{2\sigma^2 + x^2}$

Application of OSEM

- attenuation correction
- compensation for distance-dependent resolution
- scatter correction

- motion correction
- incorporation of anatomical information
- whole body PET imaging





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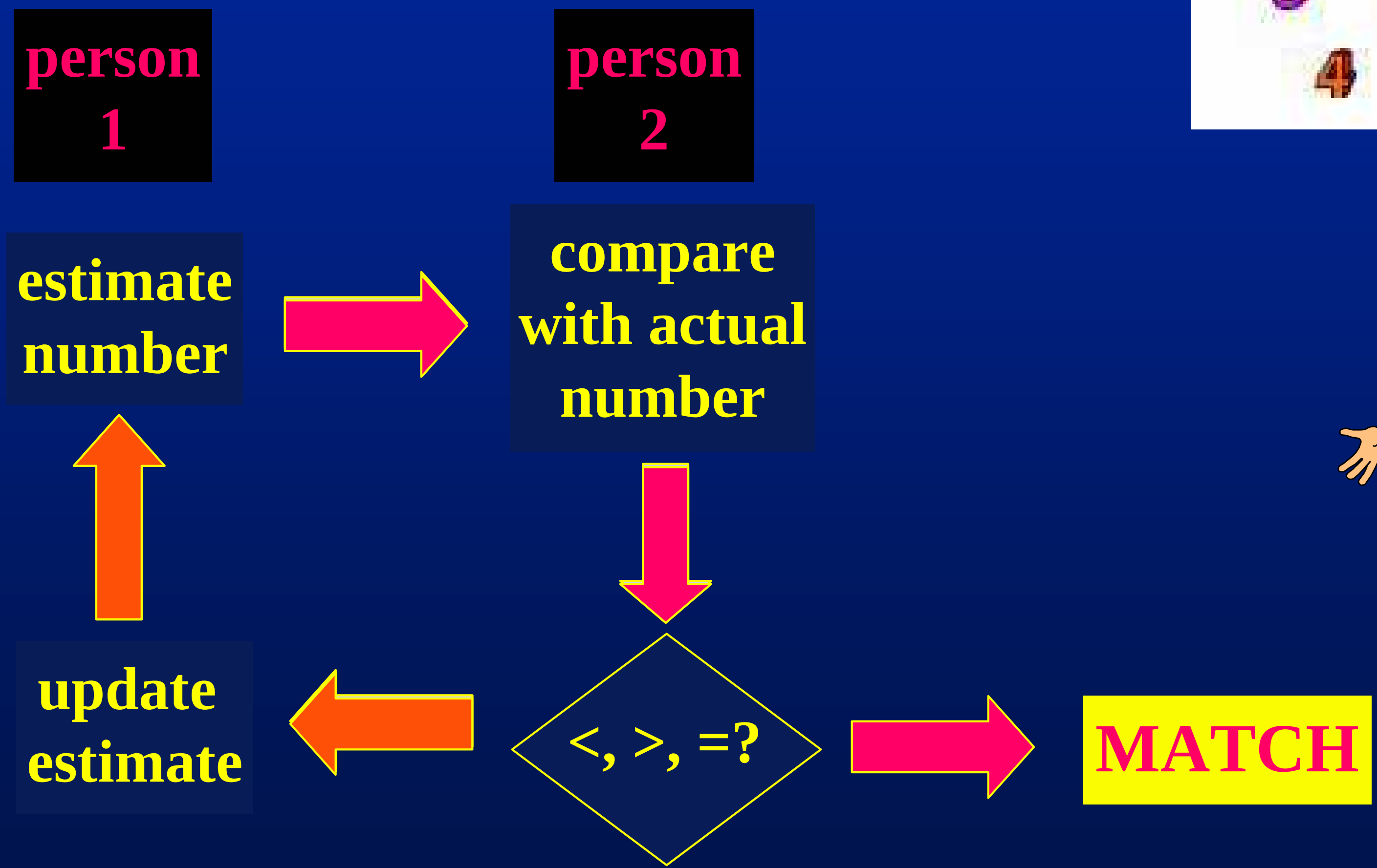
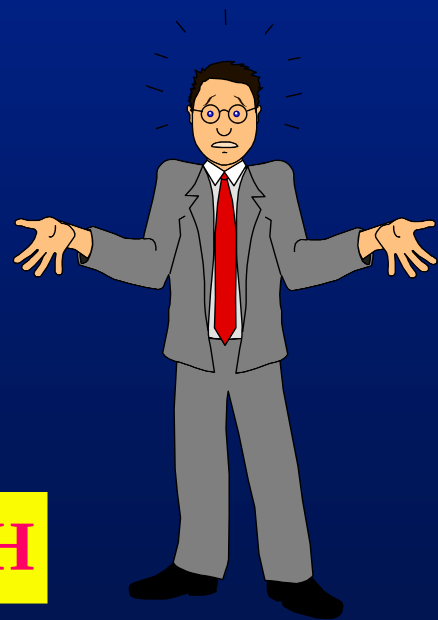
What does iterative mean?

• choose a number between 1 and 20





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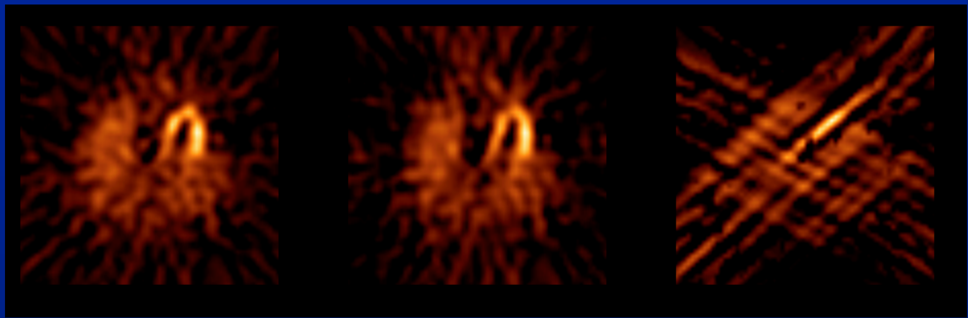
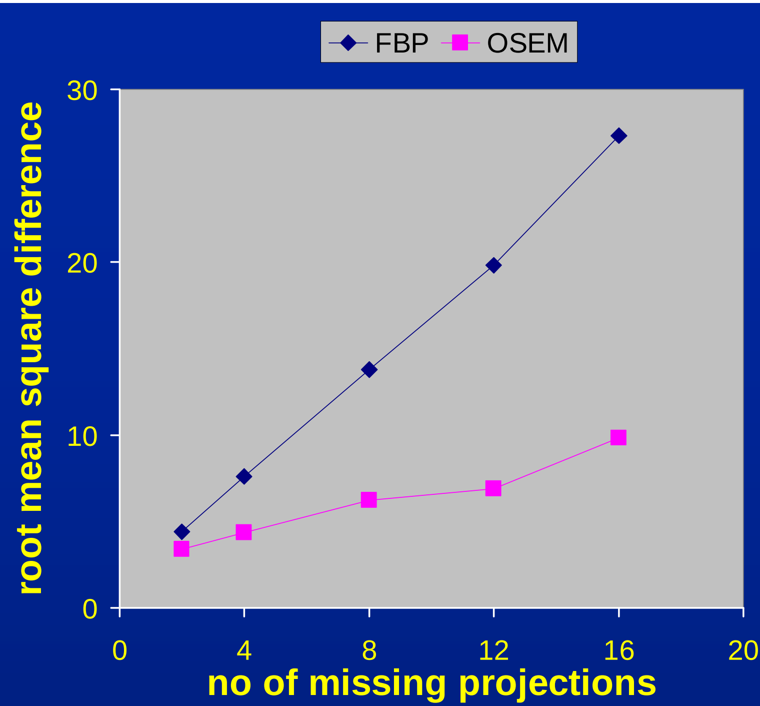
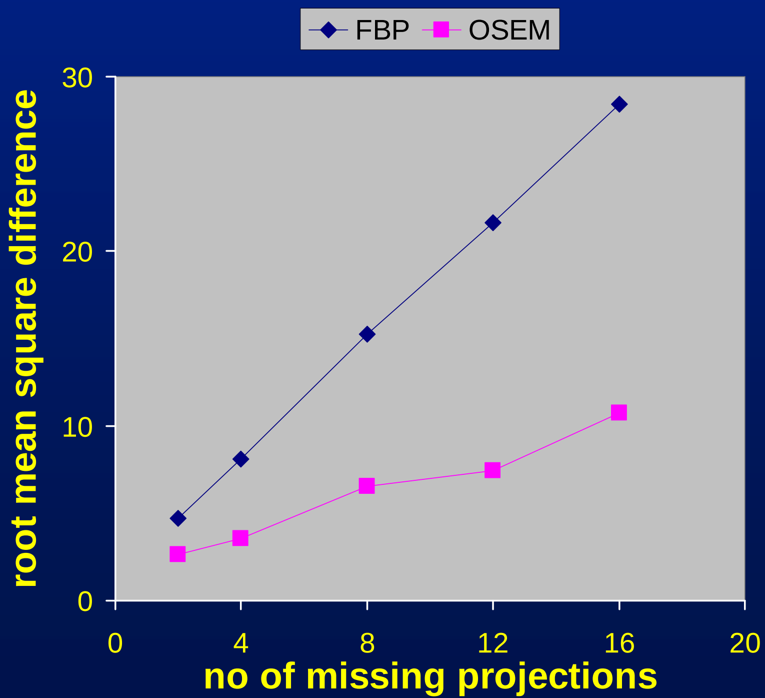
Missing data: clinical studies

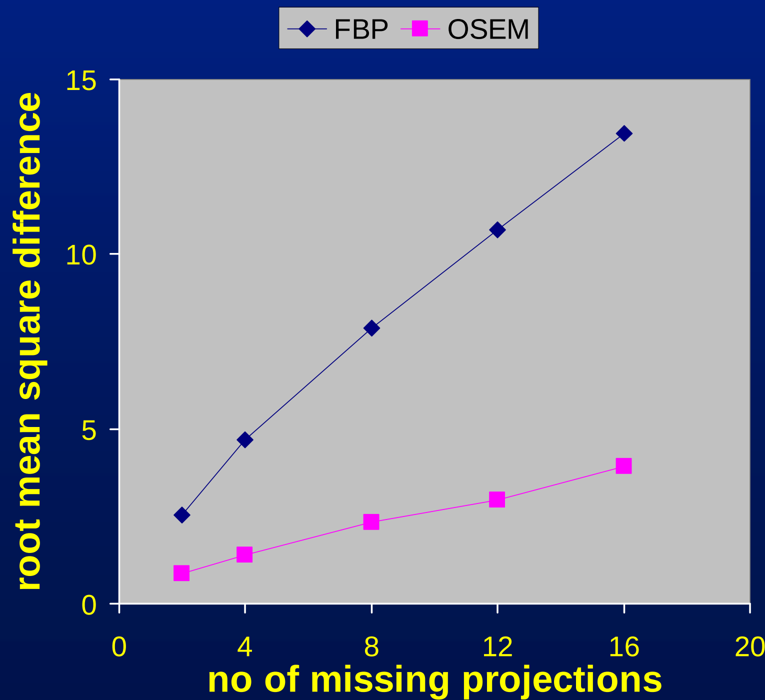
RMSD for heart region



a) 0 missing b) 8 missing c) a-b







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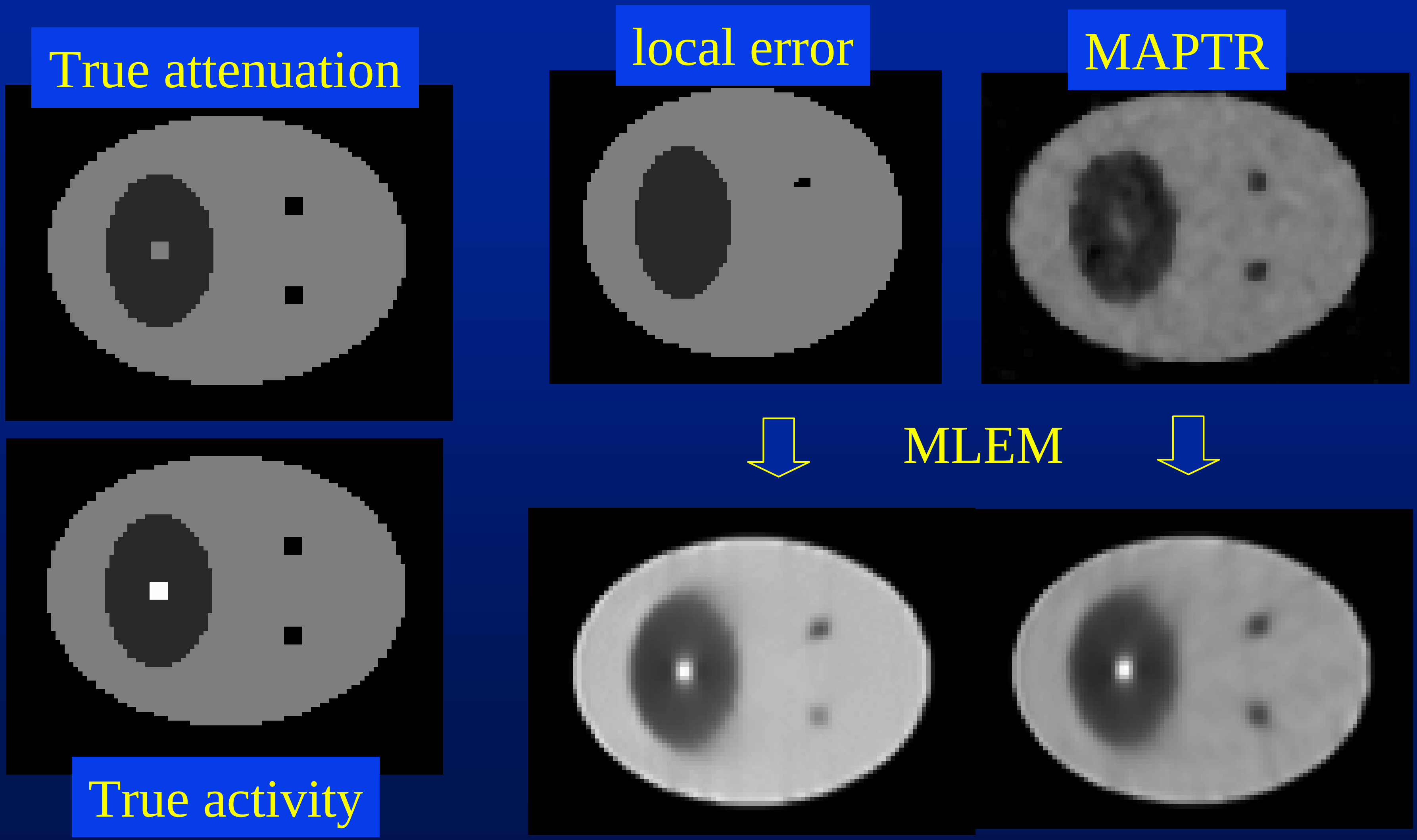
Attenuation image errors in SPECT



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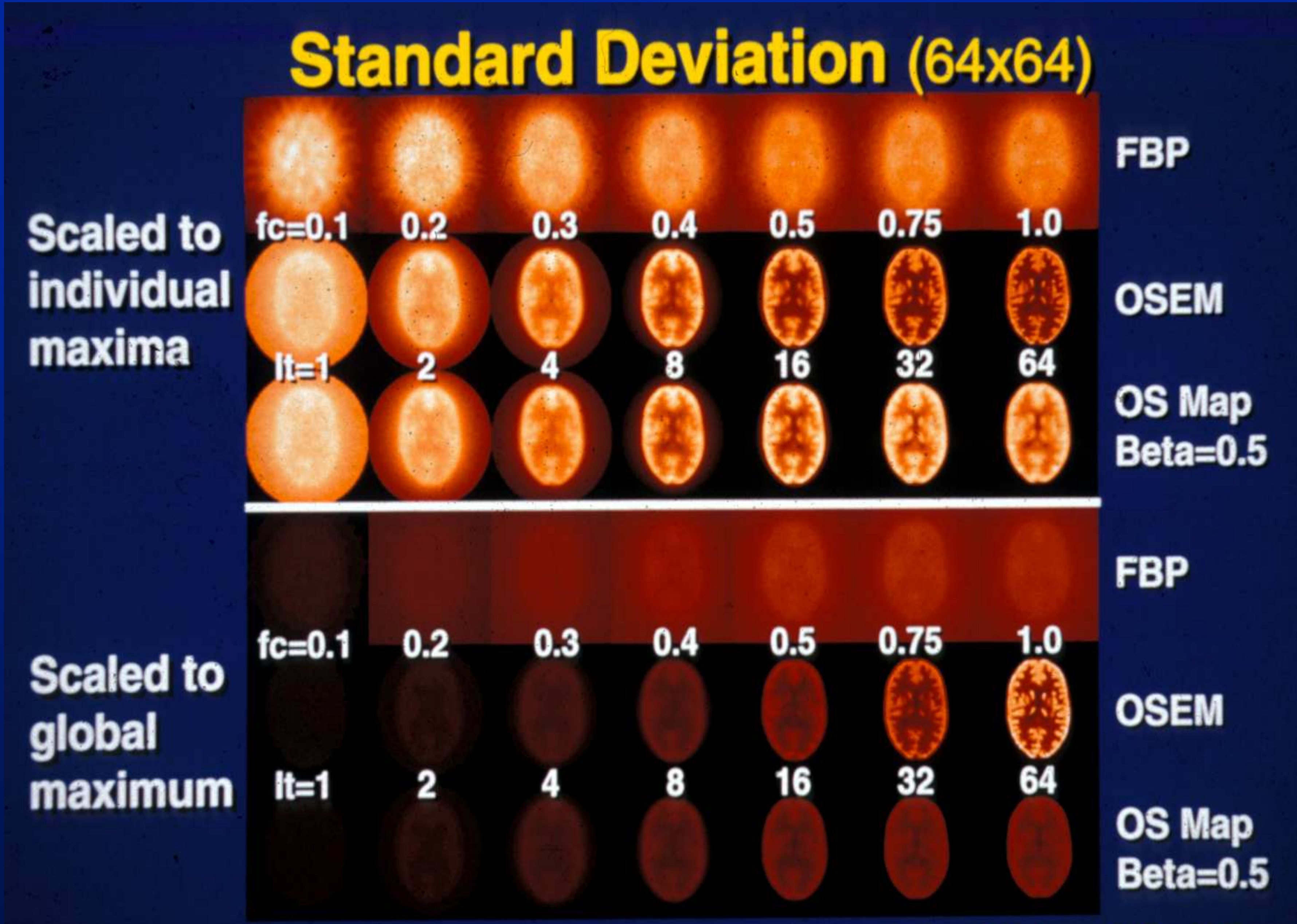

Courtesy Stefan Eberl, RPAH

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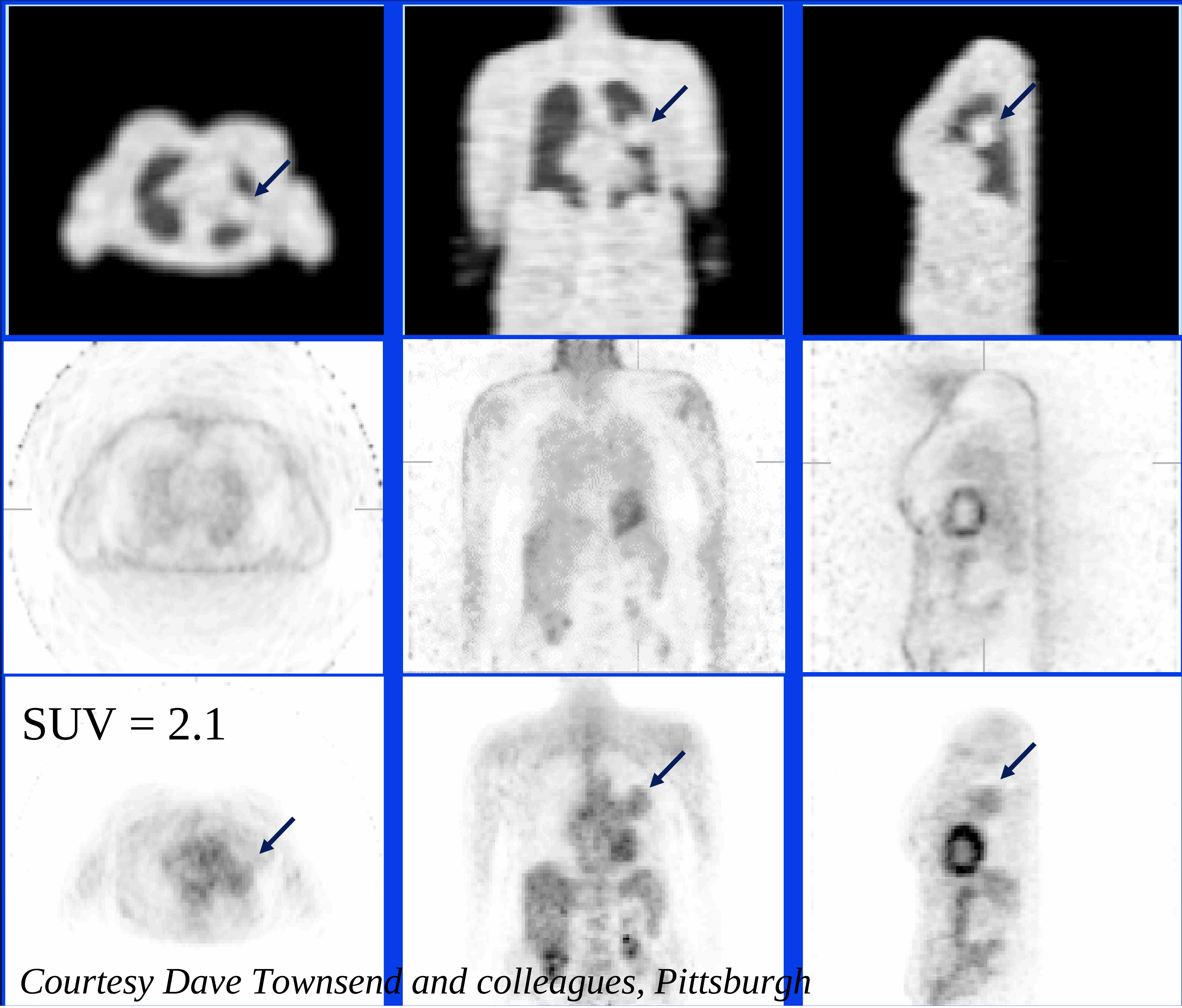
Patient study: 3 cm hamartoma



Transmission

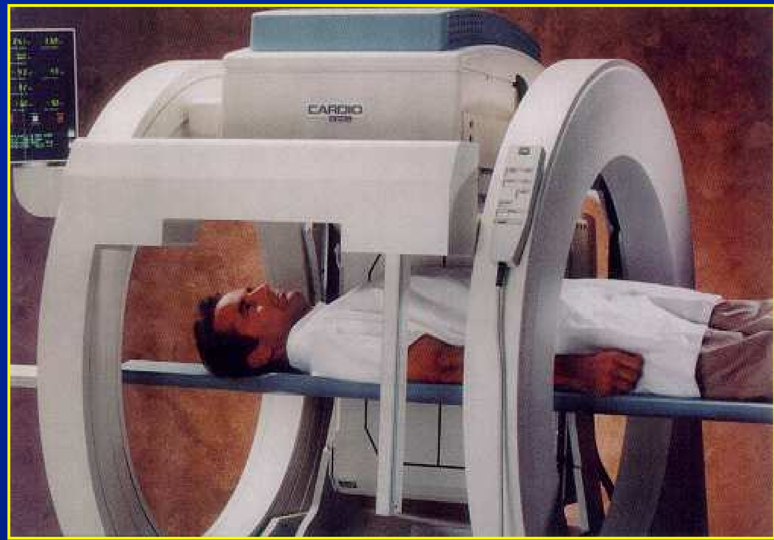
Emission
No attenuation correction

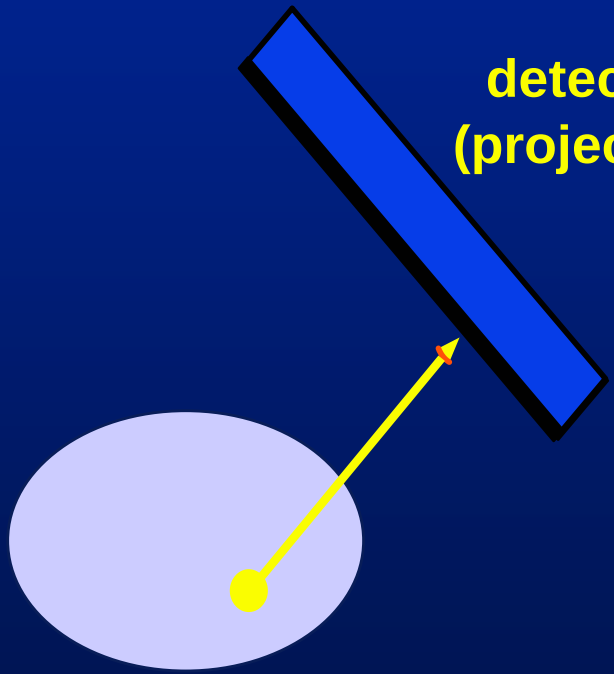
Emission
Attenuation corrected

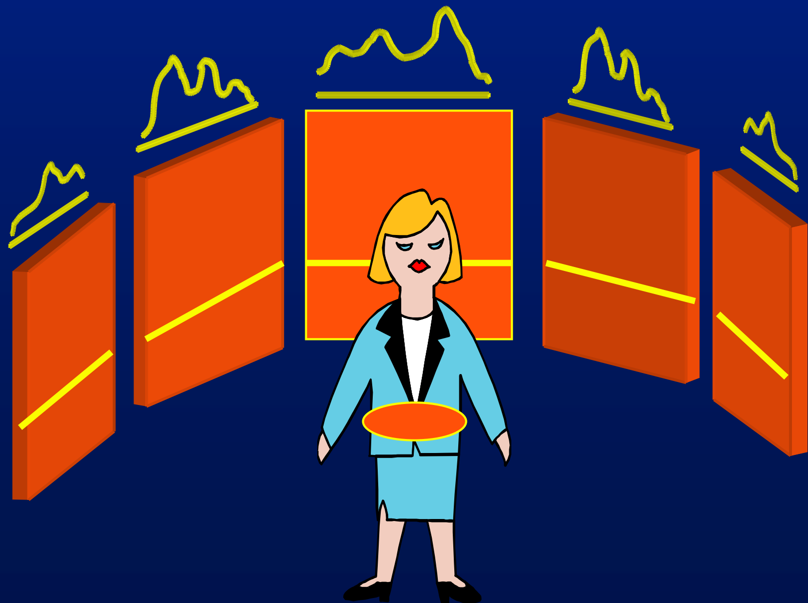
Courtesy Dave Townsend and colleagues, Pittsburgh

Iospital, Sydney

Tomographic acquisition (SPECT)







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