

# **Supply Chain Management Instructional Modules for Mathematics With Assessments**

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## X,Y - Coordinates: Location Planning

**Course:** Algebra I/Algebra II

### **Indiana Standards:**

A1.3.2 – Interpret a graph representing a given situation

MF3.1 – Distribution Content Standards: Students acquire foundational knowledge of distribution to understand its role in marketing.

**Objective:** Students will determine the best location for a distribution center using the Cartesian plane and various maps.

### **Materials:**

- Indiana Map
- U.S. Map
- Calculator

### **Job Connection:**

- Duke Realty – [www.dukerealty.com](http://www.dukerealty.com)
  - Development - Associates in Duke's Development Group are responsible for completing due diligence activities to ensure that each project is developed to its full potential and for managing schedules and controlling site-related costs pertaining to land positions. Though each project is unique, the pre-construction items typically addressed by the Development Group include land acquisition, physical characteristics of the property, zoning, planning, engineering, and permitting.
  - <http://www.dukerealty.com/company/careers/department.aspx#le>

### **Vocabulary:**

Distribution Center: A warehouse or other specialized building which is stocked with products to be re-distributed to retailers, wholesalers, or consumers

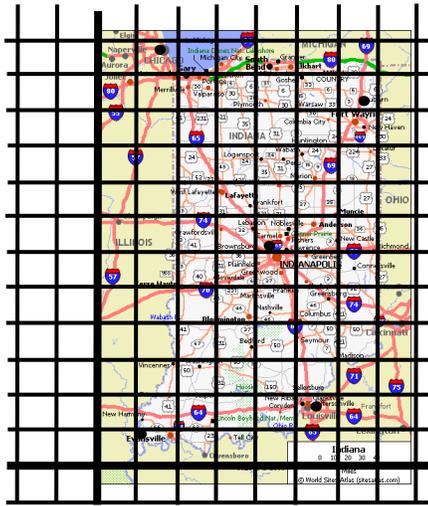
Cartesian plane: Another name for the rectangular coordinate system. The plane contains the intersection of the x and y axis.

### **Introduction:**

- Visit Duke Realty Website
  - What does the company do?
  - What type of careers are can you have at Duke Realty?
  - Why is this type of business important?

### **Input:**

- How would you decide where to locate a distribution center if you knew the number of truckloads per week going to each retail store in the state of Indiana?
- Take a look at this example:



City	Loads	X	Y
Indianapolis	50		
Gary	40		
Fort Wayne	45		
Evansville	25		
Louisville	30		

- Use the Coordinate system to determine which coordinate is best location for the distribution center

City	loads	X	Y
Indianapolis	50	4.2	6.1
Gary	40	1.6	11.8
Fort Wayne	45	6.7	10.2
Evansville	25	1.0	1.0
Louisville	30	5.4	1.8
sum	190		

$$x = \frac{4.2(50) + 1.6(40) + 6.7(45) + 1.0(25) + 5.4(30)}{190}$$

$$y = \frac{6.1(50) + 11.8(40) + 10.2(45) + 1.0(25) + 1.8(30)}{190}$$

$$x=4.0$$

$$y=6.9$$

- What city is the closest to the point (4.0, 6.9) on the graph?

**Assessment:** Students will work in groups to complete two additional examples. One example will be in the state of Indiana; the other will be using the whole United States.

**Additional Documents:**

XYcoordLocationSlides.doc

XYcoordLocationHomework.doc

XYcoordLocationTest.doc

# Location Planning

University of Indianapolis  
Department of Mathematics

Leslie Gardner, Ph.D., Professor

## **Indiana Math Standards:**

A1.3.2 Interpret a graph representing a given situation.

Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.

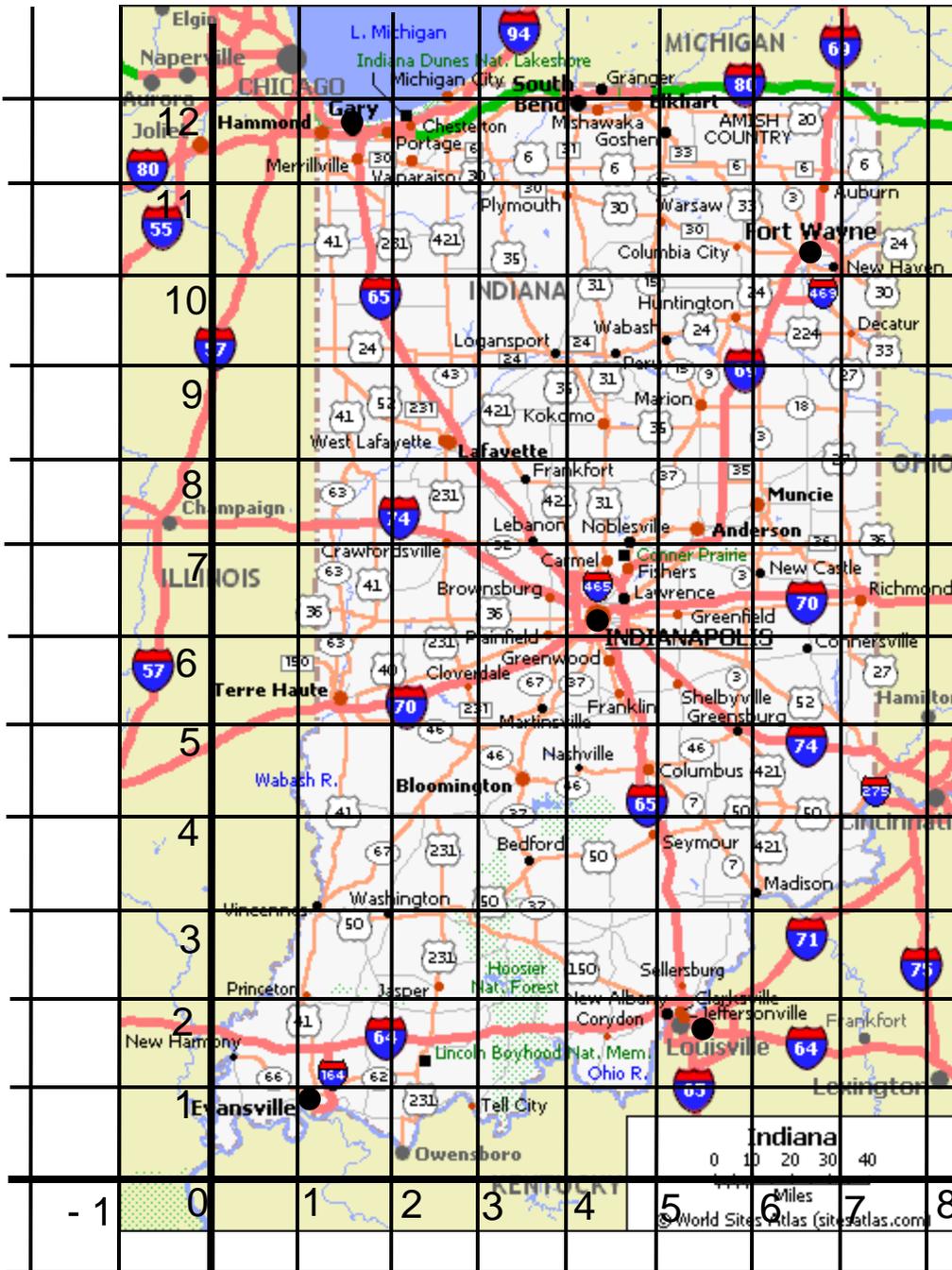
A1.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four-game average does not fall below 20 points?

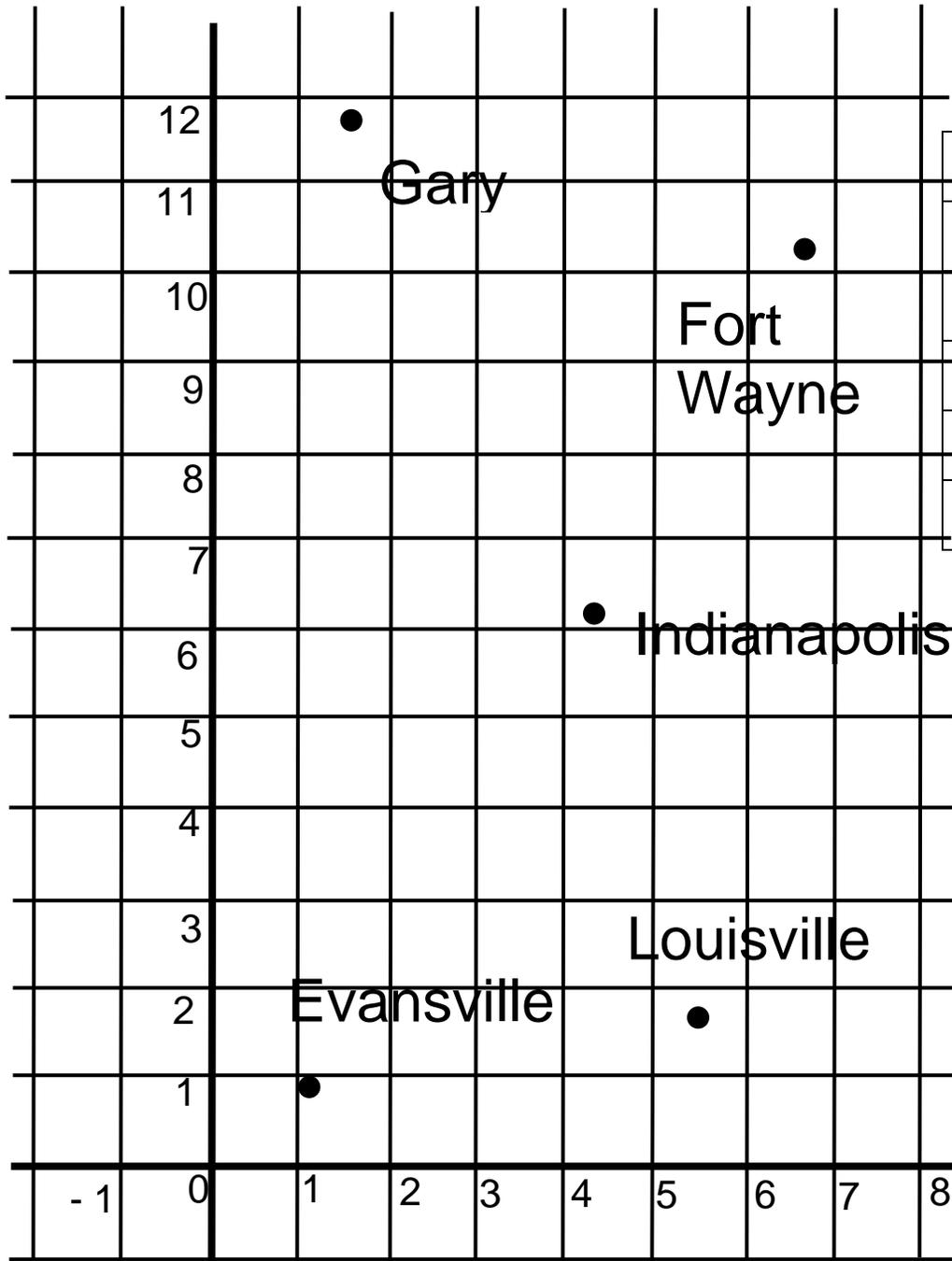
A1.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 10 points. Is his answer reasonable? Why or why not?

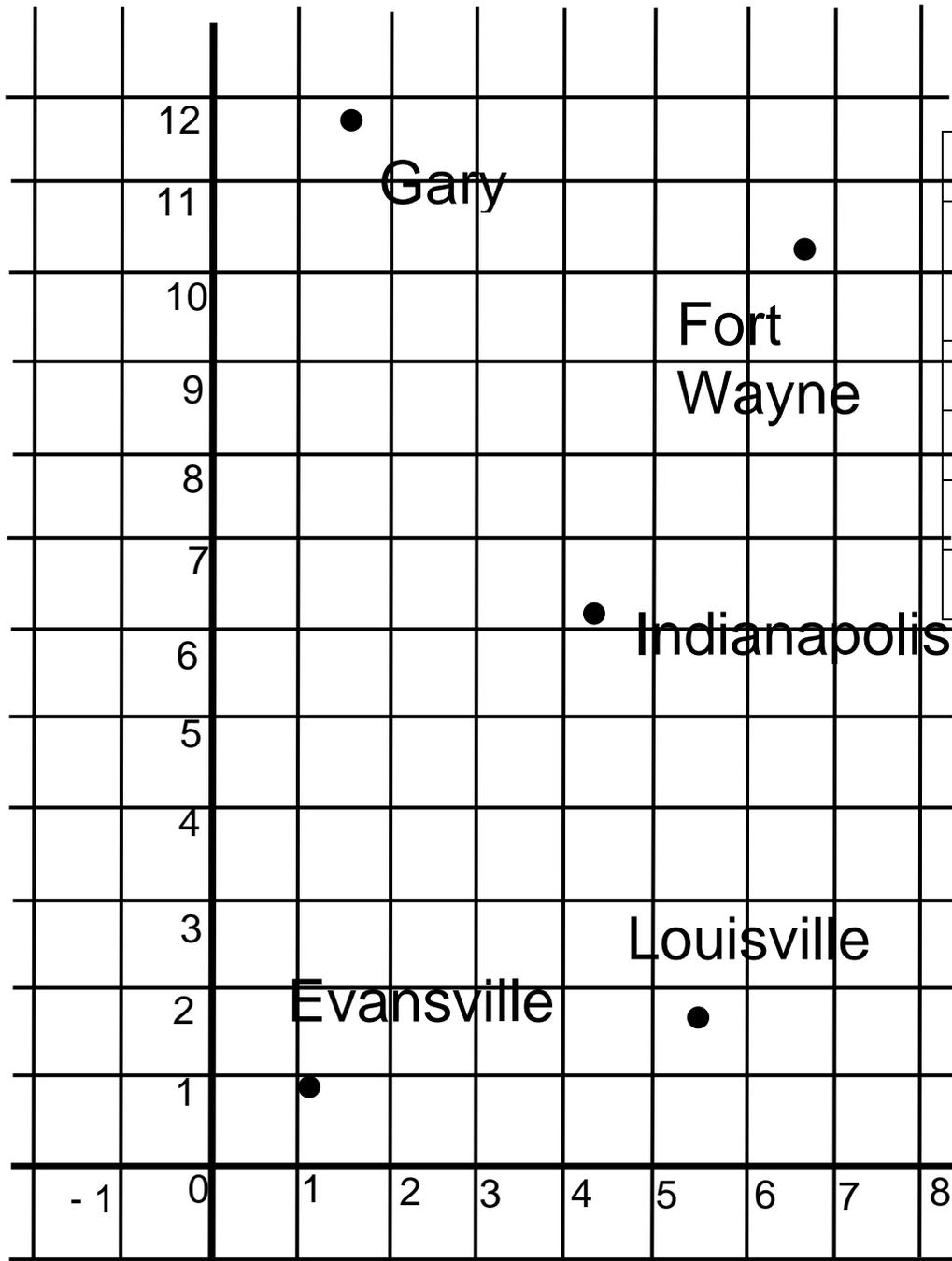
How would you decide where to locate a distribution center if you knew the number of truckloads per week going to each retail store in the state of Indiana?



City	roads	X	Y
Indianapolis	50		
Gary	40		
Fort Wayne	45		
Evansville	25		
Louisville	30		



City	oads	X	Y
Indianapolis	50		
Gary	40		
Fort Wayne	45		
Evansville	25		
Louisville	30		



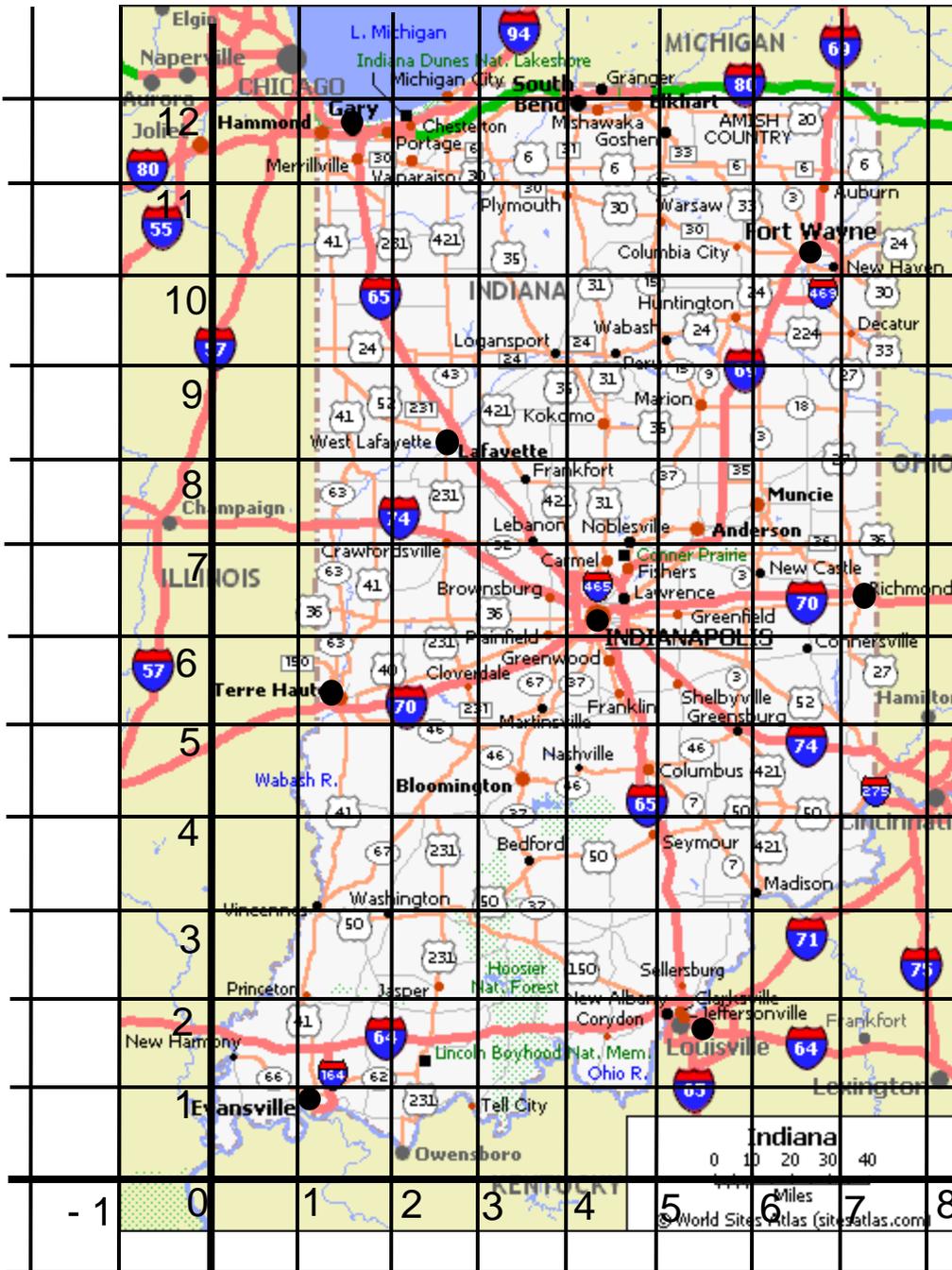
City	oads <sup>L</sup>	X	Y
Indianapolis	50	4.2	6.1
Gary	40	1.6	11.8
Fort Wayne	45	6.7	10.2
Evansville	25	1.0	1.0
Louisville	30	5.4	1.8
sum	190		

$$x = \frac{4.2(50) + 1.6(40) + 6.7(45) + 1.0(25) + 5.4(30)}{190}$$

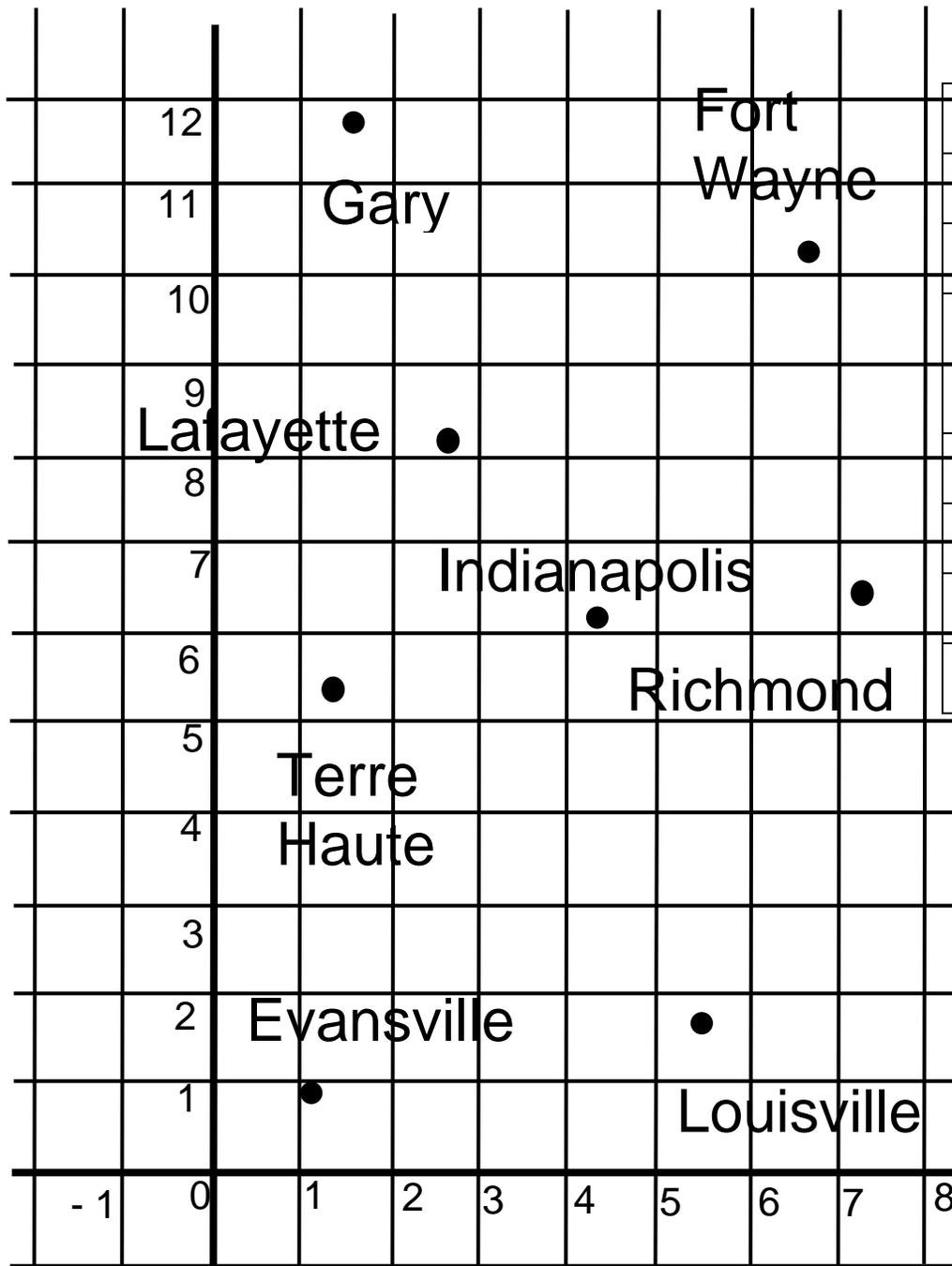
$$y = \frac{6.1(50) + 11.8(40) + 10.2(45) + 1.0(25) + 1.8(30)}{190}$$

$$x=4.0$$

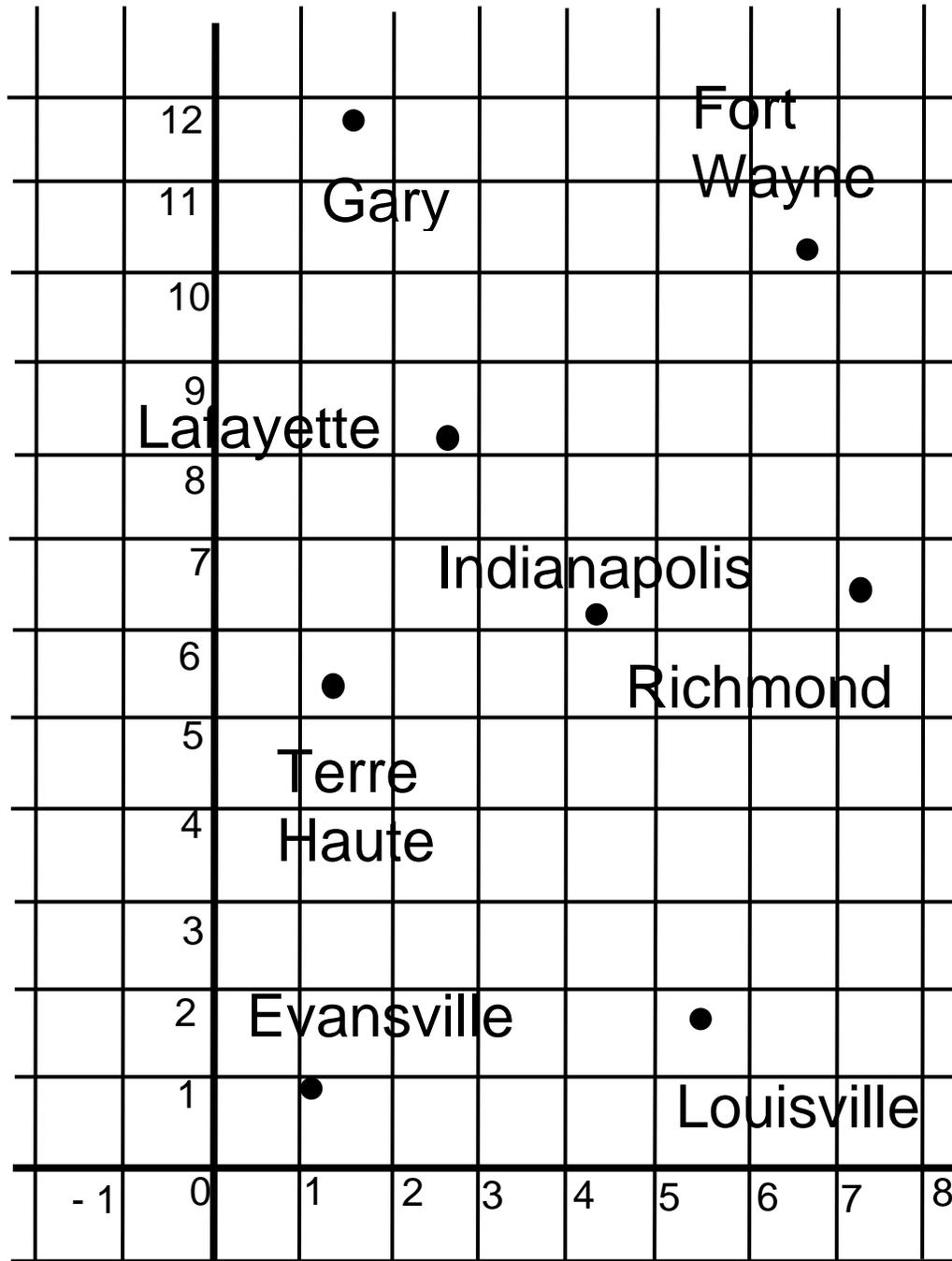
$$y=6.9$$



City	roads	X	Y
Indianapolis	10		
Gary	7		
Fort Wayne	8		
Evansville	2		
Louisville	3		
Lafayette	1		
Terre Haute	1		
Richmond	1		



City	oads	X	Y
Indianapolis	10		
Gary	7		
Fort Wayne	8		
Evansville	2		
Louisville	3		
Lafayette	1		
Terre Haute	1		
Richmond	1		



City	Loads	X	Y
Indianapolis	10	4.2	6.1
Gary	7	1.6	11.8
Fort Wayne	8	6.7	10.2
Evansville	2	1.0	1.0
Louisville	3	5.4	1.8
Lafayette	1	2.8	8.1
Terre Haute	1	1.2	5.4
Richmond	1	7.1	6.5

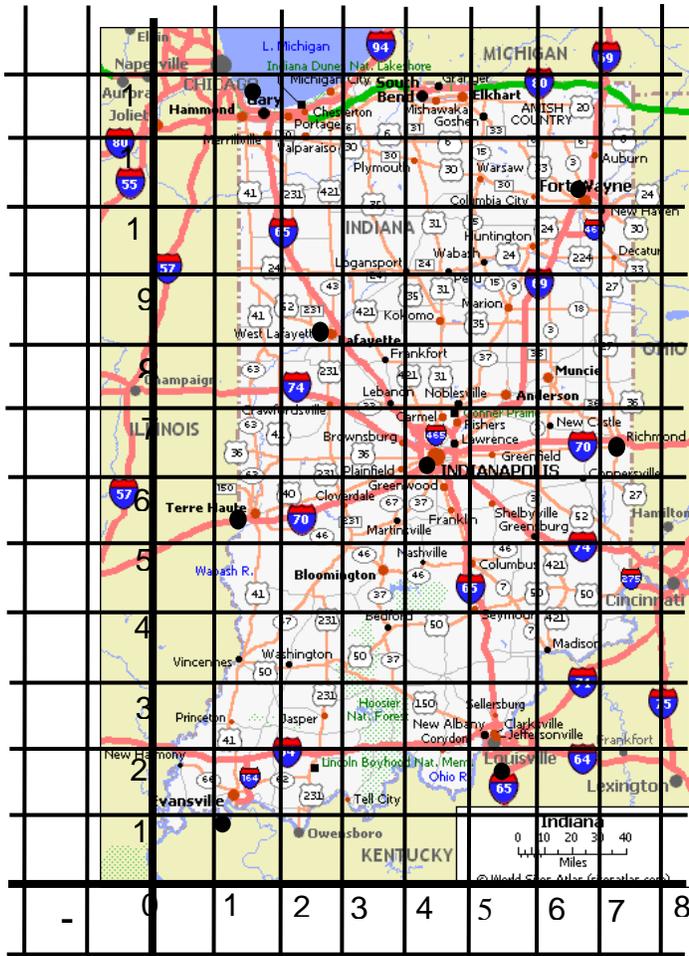
$$x = \frac{4.2(10) + 1.6(7) + 6.7(8) + 1.0(2) + 5.4(3) + 2.8(1) + 1.2(1) + 7.1(1)}{33}$$

$$y = \frac{6.1(10) + 11.8(7) + 10.2(8) + 1.0(2) + 1.8(3) + 8.1(1) + 5.4(1) + 6.5(1)}{33}$$

$$x=4.1$$

$$y=7.7$$

# Location Planning Worksheet



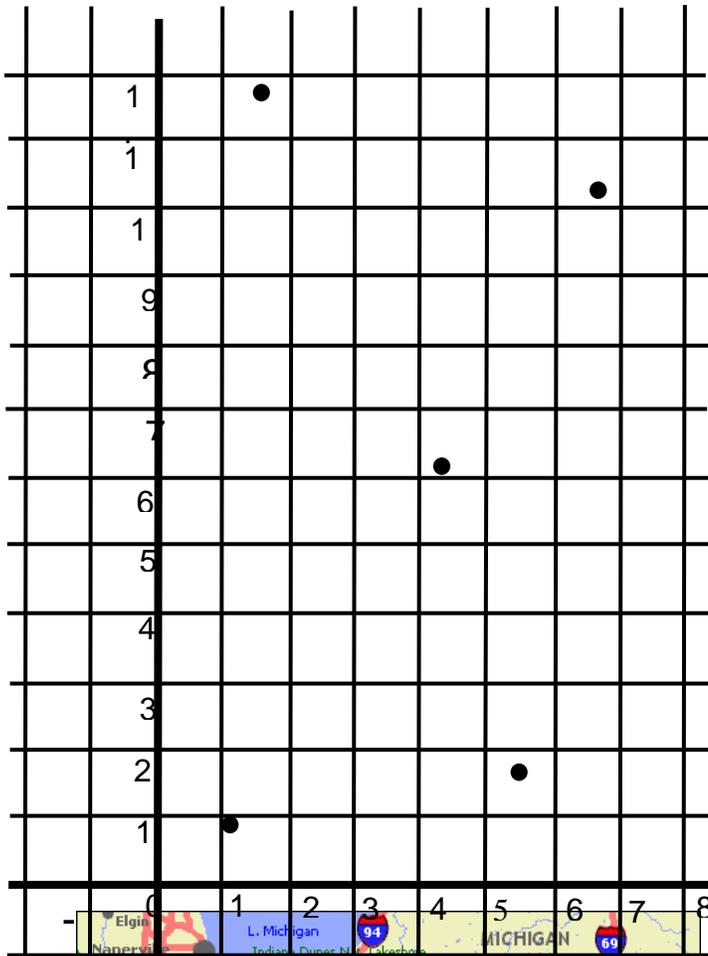
City	roads	X	Y
Indianapolis	10		
Gary	7		
Fort Wayne	8		
Evansville	2		
Louisville	3		
Lafayette	1		
Terre Haute	1		
Richmond	1		

# Location Planning Worksheet

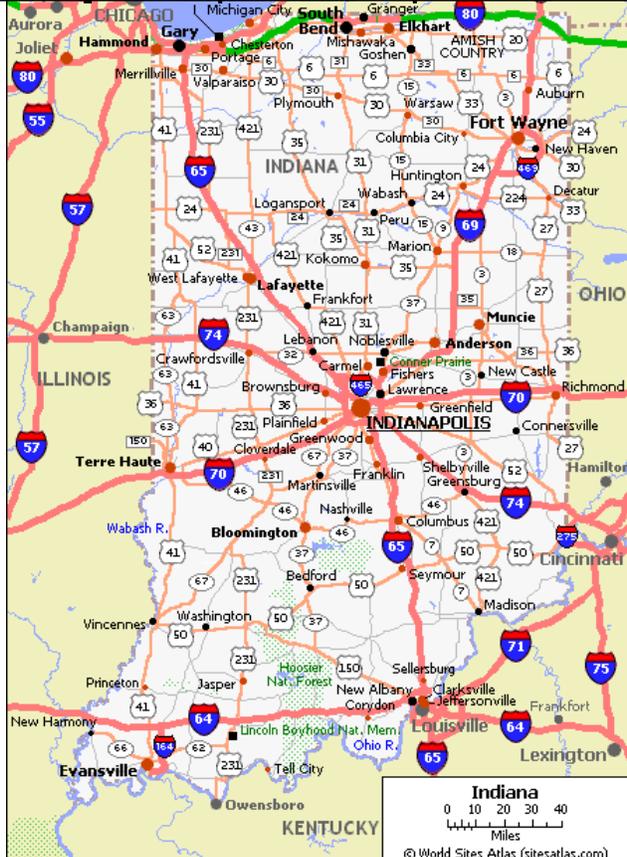


City	roads	X	Y
Indianapolis	50		
Springfield	30		
Los Angeles	85		
Denver	75		
Atlanta	50		
Las Vegas	25		
Chicago	95		
Dallas	60		
Columbus	50		

. 25. Find the best location for a Distribution center. Please give the coordinate and the nearest city



City	roads	X	Y
Indianapolis	25		
Gary	50		
Fort Wayne	35		
Evansville	4		
Louisville	15		



. 25. Find the best location for a Distribution center. Please give the coordinate and the nearest city



City	roads	X	Y
Indianapolis	50		
Springfield	30		
Los Angeles	85		
Denver	75		
Atlanta	50		
Las Vegas	25		
Chicago	95		
Dallas	60		
Columbus	50		

## Graphical Linear Programming

**Course:** Algebra 2

### **Standards:**

- A2.2.1 Graph absolute value equations and inequalities.
- A2.2.3 Use systems of linear equations and inequalities to solve word problems.
- A2.10.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, writing an equation, and working backwards.
- A2.10.2 Decide whether a solution is reasonable in the context of the original situation.
- DM.6.1 Use geometric techniques to solve optimization problems.

**Objective:** Students will learn how to deduce from a word problem constraints in order to determine the optimum values of the variables to achieve a goal.

### **Materials:**

- Calculator
- Graph Paper

### **Job Connection:**

- Dow Agroscience - [www.dowagro.com](http://www.dowagro.com)
  - **Supply Research and Development** - In this function, scientists develop products invented by Discovery into viable commercial products. Process Research Chemists determine the most effective methods for scale-up and manufacture of the active ingredient of a product. Formulation chemists work to develop stable and safe formulation and packaging systems.
  - **The Modeling Center of Expertise** - Scientists leverage modeling technology and bio-statistical techniques to facilitate field and laboratory scientists in their generation and evaluation of data.
    - [http://www.dowagro.com/careers/career/rd\\_path.htm](http://www.dowagro.com/careers/career/rd_path.htm)
- American Airlines - [www.aa.com](http://www.aa.com)
  - **Industrial Engineer** - Our Engineering professionals develop airplane performance data to ensure the safe and efficient operation of American Airlines and its affiliates. This involves developing and maintaining load planning, planning take-off performance, flight databases and related systems, payload/range mission

analysis, fuel forecasting and reporting, airframe/engine evaluations and the support of daily operational activities.

- **Airport Consulting Group** - This group focuses on assisting airport locations with technical support. Our goal is to maximize each station's productivity while providing optimal customer service. Entry level industrial engineers may have the opportunity to work on exciting assignments relating to developing and maintaining corporate work standards for passenger, ramp, and cargo services by using time study and work sampling techniques; providing technical support for evaluating new technologies, service initiatives, or future airport projects; coordinating, developing and supporting real-time labor management systems to increase labor efficiency and productivity in addition to staffing automation used for strategic planning and forecasting and/or generating productivity and methods improvement recommendations.
  - [http://www.aacareers.com/us/frame\\_index.htm?college.shtml](http://www.aacareers.com/us/frame_index.htm?college.shtml)

### **Vocabulary:**

Linear Programming: A technique that identifies the minimum or maximum value of some quantity.

Constraint: Limit on the variables

Objective Function: a model of the quantity that you want to make as large or as small as possible

Vertices: Points of intersection of the equations of the constraints.

### **Input:**

- We can use the idea of linear programming to solve a multitude of problems involving many different types of situations
- Example 1: Buying Music
  - Suppose you want to buy CDs and tapes. You can afford as many as 10 tapes or 7 CDs. You want at least 4 CDs and at least 10 hours of recorded music. Each tape holds 45 minutes and each CD holds 1 hour of recorded music.
    - Define Variables
      - $X = \# \text{ CDs}, Y = \# \text{ Tapes}$
    - Write Constraints
      - $x \geq 0, y \geq 0$
      - $x \leq 7, y \leq 10$
      - $x \geq 4$
      - $.75y + 1x \geq 10$
    - Graph Constraints
    - Write Objective Function
      - Given:  $M = 2x + 3y$

- Test Vertices
  - (7,0) (0, 10), (3, 10) (7, 4)
  - Max at (7, 10)
  - So, we should by 7 CDs and 10 Tapes
- Example 2: Manufacturing
  - A small company makes two similar products, which all follow the same three-step process, consisting of cutting, gluing, and finishing. Time requirements in minutes for each product at each operation are given below.

operation	product	
	$x$	$y$
cutting	1.5	0.8
gluing	0.6	2.2

The firm has 30 hours available in the next period for cutting, 55 hours for gluing. Product  $x$  contributes \$4.40 per unit to profit and  $y$  contributes \$4.50 per unit. Find the product mix that maximizes profit.

- Define Variables
  - $X = \text{Product 1}, Y = \text{Product 2}$
- Write Constraints
  - $x \geq 0, y \geq 0$
  - $1.5x + 0.8y \leq 1800$
  - $0.6x + 2.2y \leq 3300$
- Graph Constraints
- Write Objective Function
  - $P = 4.40x + 4.50y$
- Test Vertices
  - (0, 1500), (468, 1372), (1200, 0)
  - Max at (468, 1372)
  - So, we should make 468 product  $x$  and 1372 of product  $y$
- Example 3: Running a Bakery
  - A baking tray of corn muffins takes 4c milk and 3c flour. A tray of bran muffins takes 2c milk and 3c flour. A baker has 16c milk and 15c flour. He makes \$3 profit per tray of corn and \$2 profit per tray of bran muffins. How many trays of each type of muffin should he bake to get maximum profit?
  - Write Constraints
    - $x \geq 0, y \geq 0$
    - $4x + 2y \leq 16$
    - $3x + 3y \leq 15$
  - Graph Constraints
  - Write Objective Function
    - $P = 3x + 2y$
  - Test Vertices
    - (0, 5), (3, 2), (4,0)

- Max at (3, 2)
- So, we should make 3 batches of corn and 2 batches of bran.

**Assessment:** Students will complete the attached worksheet for homework.

**Additional Documents:**

GraphicalLPWorksheet.doc

GraphicalLPHomework.doc

GraphicalLPTest.doc

GraphicalLPSlides.pdf

GraphicalLPSlidesMarkup.pdf

GraphicalLPSlidesAdv.doc

GraphicalLPSlidesAdv.ppt

GraphicalLPWorksheetAdv.doc

# **Graphical Linear Programming**

University of Indianapolis  
Department of Mathematics

Leslie Gardner, Ph.D., Professor

## **Indiana Math Standards:**

A2.2.1 Graph absolute value equations and inequalities.

Example: Draw the graph of  $y = 2x - 5$  and use that graph to draw the graph of  $y = |2x - 5|$ .

A2.2.3 Use systems of linear equations and inequalities to solve word problems.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.10.1 Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: The swimming pool at Roanoke Park is 24 feet long and 18 feet wide. The park district has determined that they have enough money to put a walkway of uniform width, with a maximum area of 288 square feet, around the pool. How could you find the maximum width of a new walkway?

A2.10.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 20 feet. Is that reasonable?

DM.6.1 Use geometric techniques to solve optimization problems.

Example: A company produces two varieties of widgets — standard and deluxe. A standard widget takes 3 hours to assemble and 6 hours to finish. A deluxe widget takes 5 hours to assemble and 5 hours to finish. The assemblers can work no more than 45 hours per week and the finishers can work no more than 60 hours per week. The profit is \$32 on a standard widget and \$40 on a deluxe widget. Use a graph to find how many of each model should be produced each week to maximize profit.

# What are the major technological advances that came out of World War II?

- Atomic bomb
- Synthetic antibiotics
- Rocketry
- Digital computers
  - Linear Programming

# Manufacturing

A small company makes two similar products, which all follow the same three-step process, consisting of cutting, gluing, and finishing. Time requirements in minutes for each product at each operation are given below.

<b>operation</b>	<b>product</b>	
	<b>x</b>	<b>y</b>
cutting	1.5	0.8
gluing	6.0	11.0
finishing	12.0	3.0

The firm has 30 hours available in the next period for cutting, 55 hours for gluing, and 44 hours for finishing. Product x contributes \$4.40 per unit to profit and y contributes \$4.50 per unit. Find the product mix that maximizes profit.

# Variables and Objective Function

$X$  = number of product  $x$  manufactured

$Y$  = number of product  $y$  manufactured

Max profit

Max  $\$4.40x + \$4.50 y$



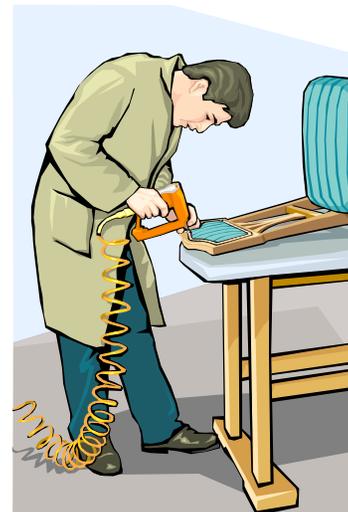
# Constraints

Cutting  $1.5x + 0.8y \leq 1800$

Gluing  $6x + 11y \leq 3300$

Finishing  $12x + 3y \leq 2640$

$x \geq 0, y \geq 0$

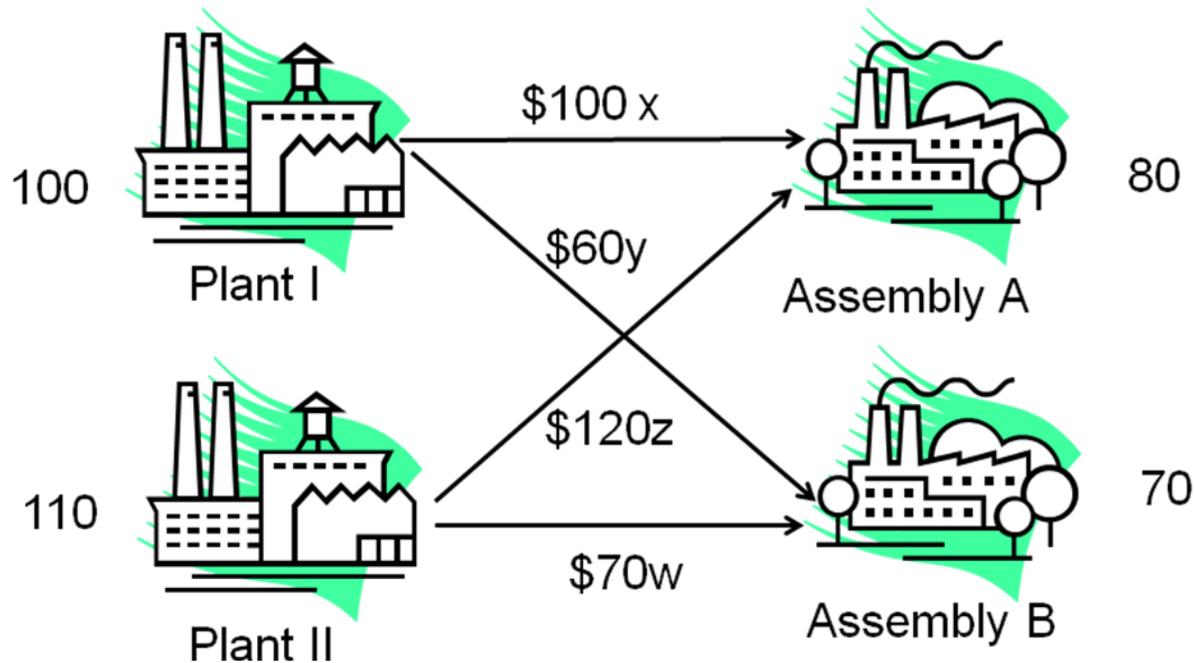


# Logistics

Allison Engine has two manufacturing plants, I and II, that produce engines for Embraer that makes commuter planes. The maximum production capabilities of these two manufacturing plants are 100 and 110 engines per month respectively. The engines are shipped to two of Embraer's assembly plants, A and B. The shipping costs in dollars per engine from plants I and II are to the assembly plants A and B are as follows:

	<i>To Assembly</i>	
<i>From</i>	A	B
Plant I	100	60
Plant II	120	70

In a certain month, assembly plant A needs 80 engines, whereas assembly plant B needs 70 engines. How many engines should be shipped from each manufacturing plant to each assembly plant to minimize shipping costs?



***x*** = number of engines shipped from plant I to plant A

***y*** = number of engines shipped from plant I to plant B

***z*** = number of engines shipped from plant II to plant A

***w*** = number of engines shipped from plant II to plant B

Bottleneck is at assembly so  $z=80 - x$  and  $w = 70 - y$

$$\begin{aligned}\text{Cost} &= 100x + 60y + 120(80 - x) + 70(70 - y) \\ &= 14,500 - 20x - 10y\end{aligned}$$

Production constraints

$$x + y \leq 100$$

$$(80 - x) + (70 - y) \leq 110$$

Which simplifies to  $x + y \geq 40$

$$x \geq 0, y \geq 0, (80 - x) \geq 0, (70 - y) \geq 0$$

$$\text{so } x \leq 80, y \leq 70$$

$$\min 14,500 - 20x - 10y$$

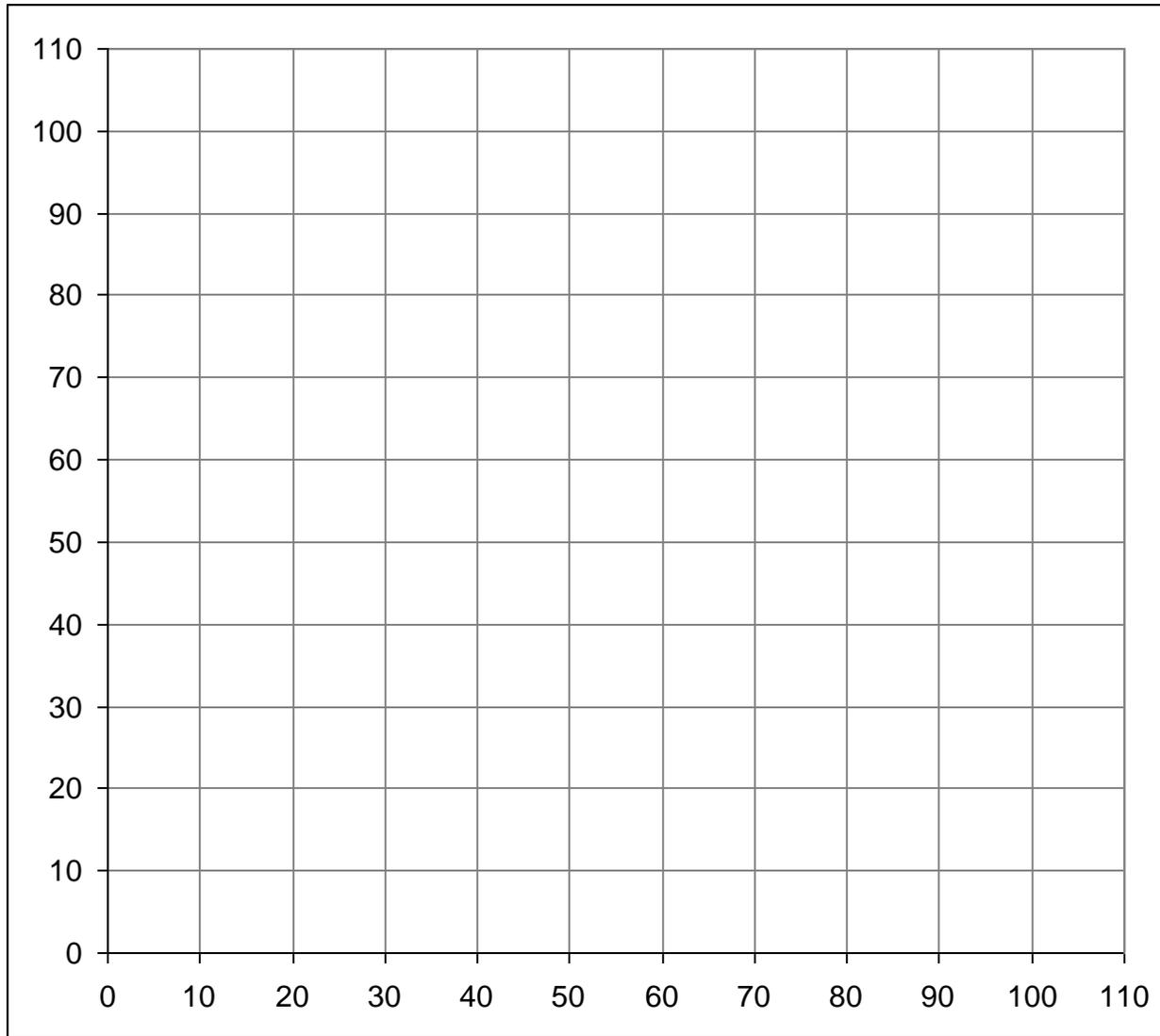
$$\text{s.t. } x + y \leq 100$$

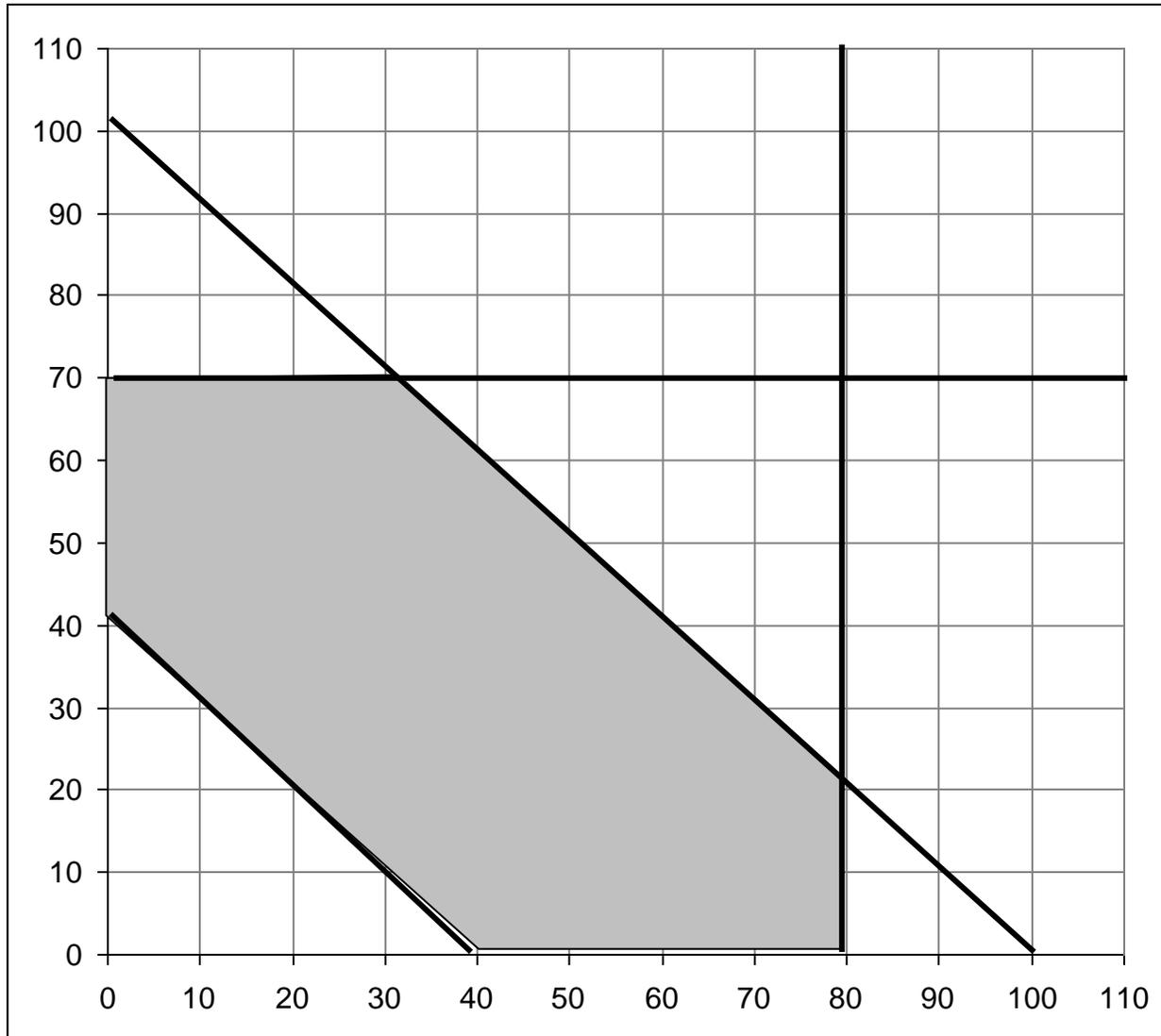
$$x + y \geq 40$$

$$x \leq 80$$

$$y \leq 70$$

$$x \geq 0, y \geq 0$$



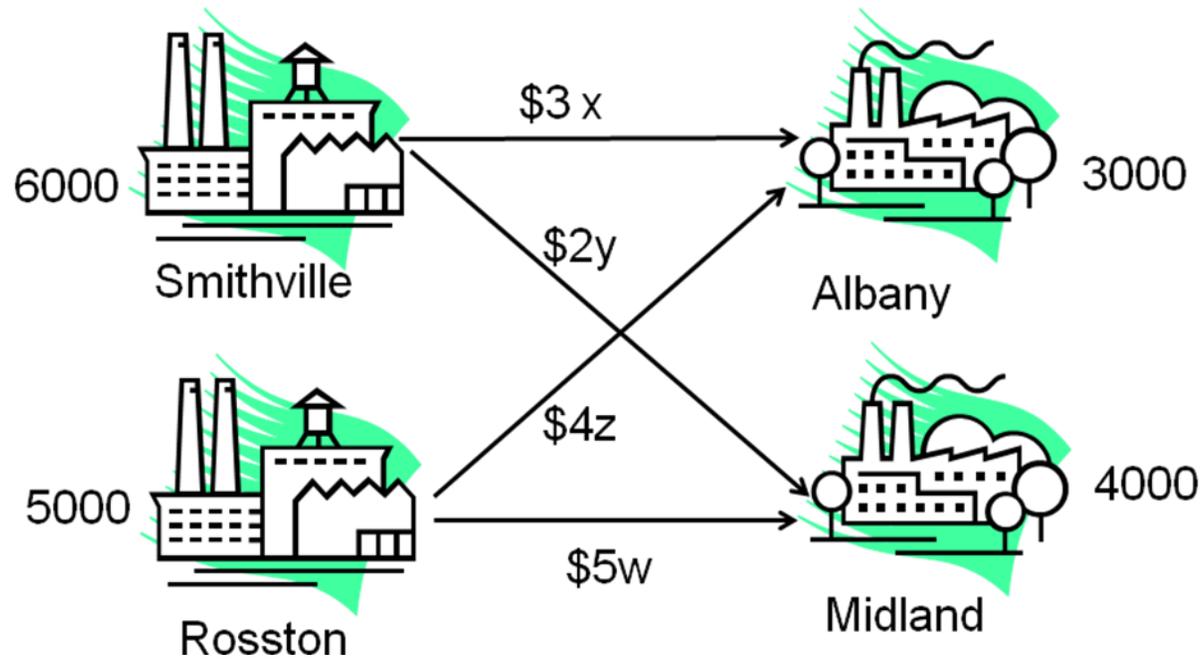


x	y	$14,500 - 20x - 10y$
40	0	13,700
80	0	12,900
80	20	12,700
30	70	13,200
0	70	13,800
0	40	14,100

The TMA Company manufactures 19 inch color LCD panels tubes at two separate locations, Smithville and Rosston. The output at Smithville is at most 6000 panels per month and the output at Rosston is at most 5000 panels. TMA is the main supplier of the Pulsar Corporation, which has priority on having all of its requirements met. In a certain month, Pulsar placed orders for 3000 and 4000 panels to be shipped to two of its factories located in Albany and Midland respectively. The shipping costs (in dollars) per LCD panel from the two TMA plants to the two Pulsar factories are:

From	<i>To Assembly</i>	
	Albany	Midland
Smithville	3	2
Rosston	4	5

How much should be shipped from each panel manufacturing plant to the television factories in order to minimize costs?



Where is the bottleneck?

How can you reduce the number of variables?

$$\min 32,000 - x - 3y$$

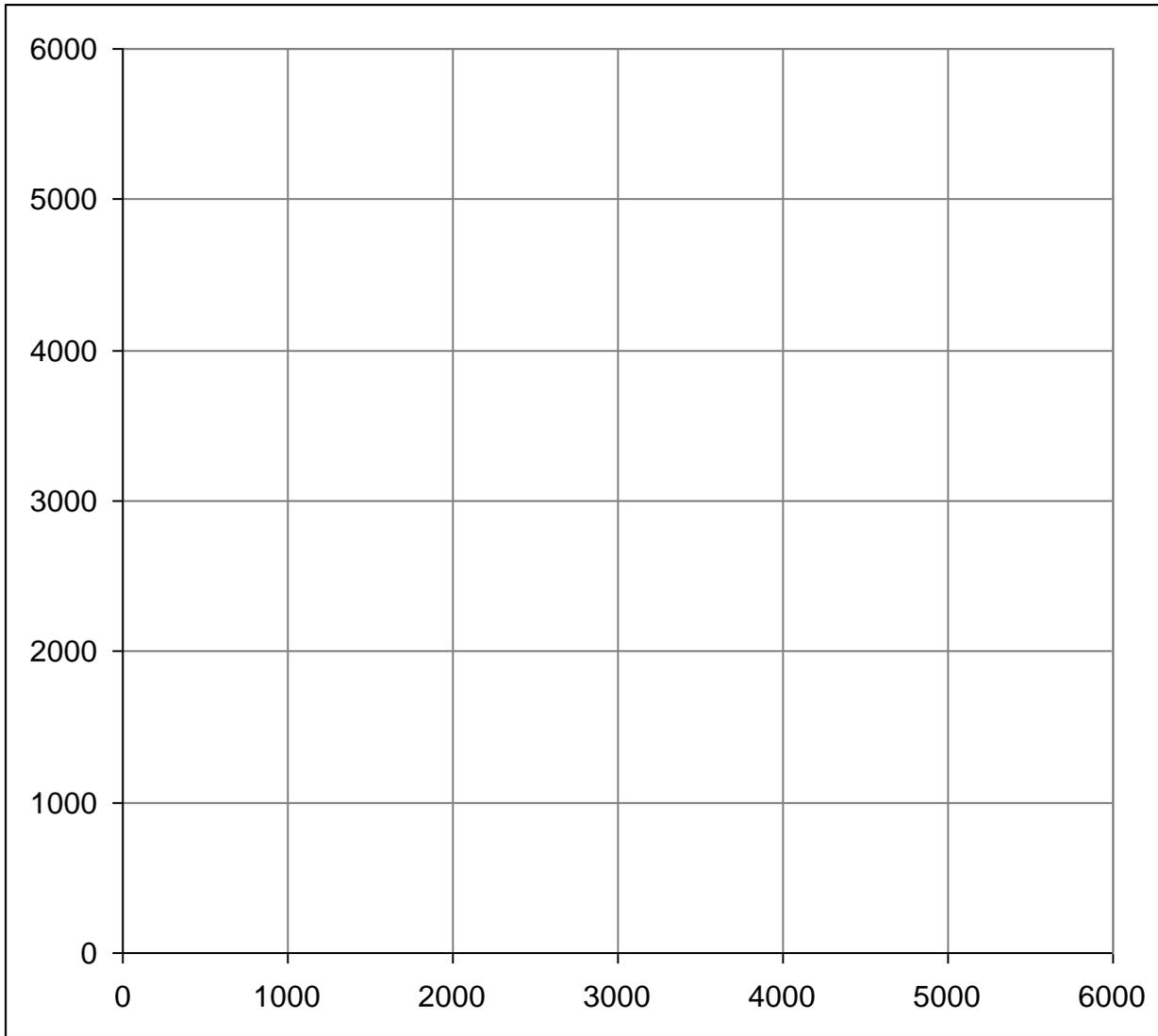
$$\text{s.t. } x + y \leq 6000$$

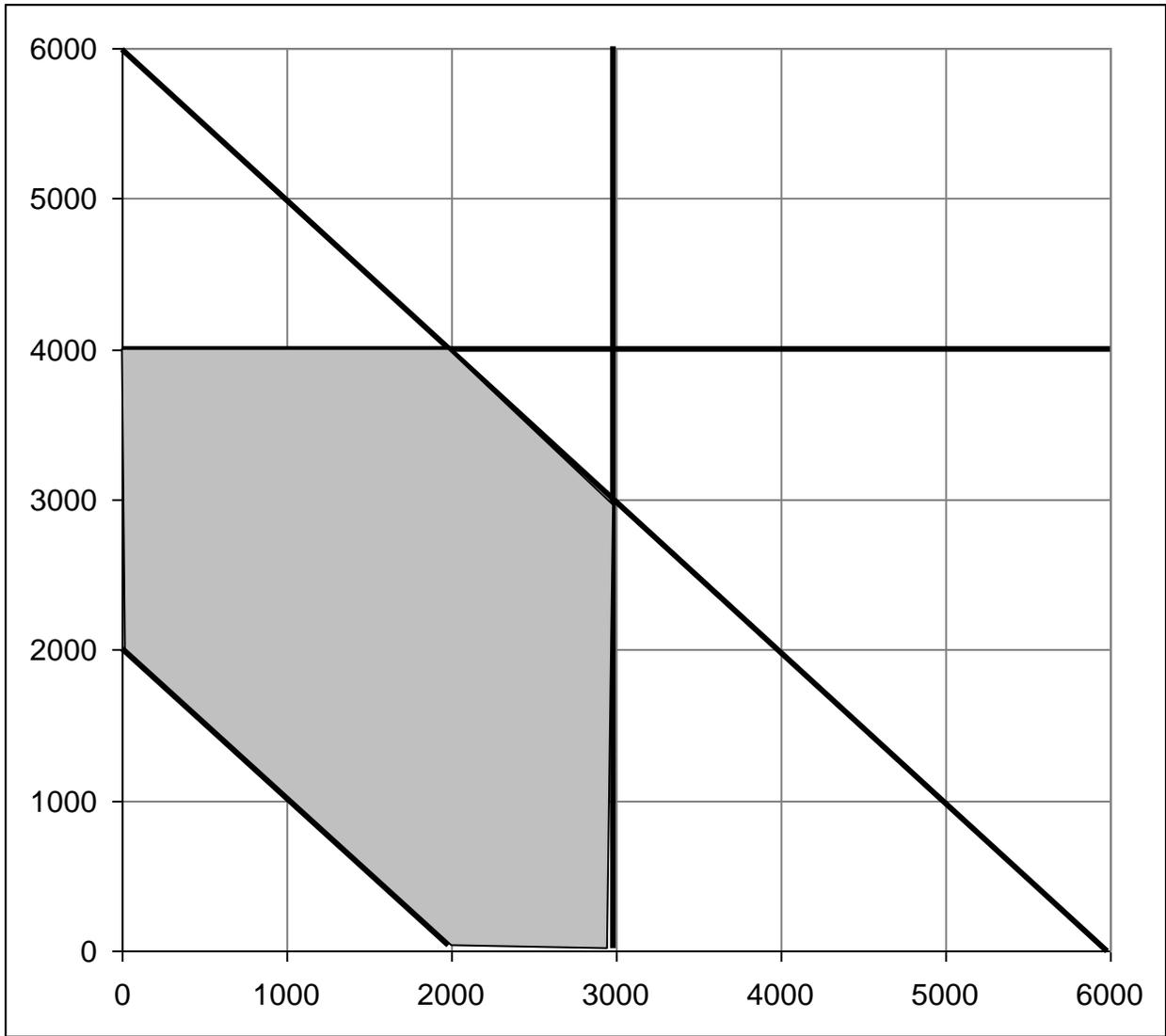
$$x + y \geq 2000$$

$$x \leq 3000$$

$$y \leq 4000$$

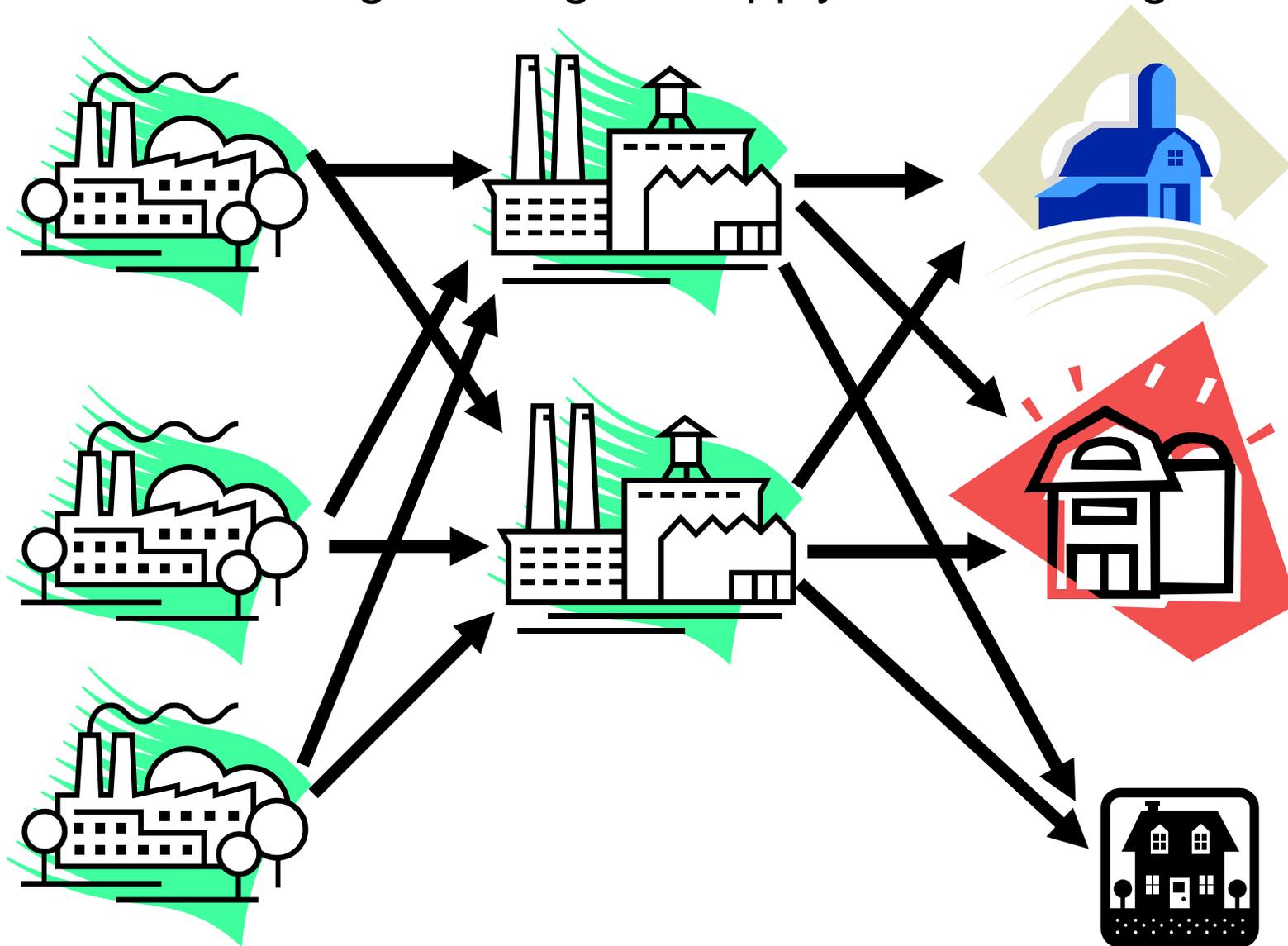
$$x \geq 0, y \geq 0$$





x	y	$32,000 - x - 3y$
2000	0	30,000
3000	0	29,000
3000	3000	20,000
2000	4000	18,000
0	4000	20,000
0	2000	26,000

# Linear Programming for Supply Chain Management



# Manufacturing

A small company makes two similar products, which all follow the same three-step process, consisting of cutting, gluing, and finishing. Time requirements in minutes for each product at each operation are given below.

operation	product	
	x	y
cutting	1.5	0.8
gluing	6	11
finishing	12	3

The firm has 30 hours available in the next period for cutting, 55 hours for gluing, and 44 hours for finishing. Product x contributes \$4.40 per unit to profit and y contributes \$4.50 per unit. Find the product mix that maximizes profit.

Can you write equations which represent this situation?

What are we looking to do here? M \_\_\_\_\_ P \_\_\_\_\_

$X \times$  \_\_\_\_\_ +  $Y \times$  \_\_\_\_\_ = \_\_\_\_\_

Number of minutes in a hour? \_\_\_\_\_

Minutes available for cutting? \_\_\_\_\_

Equation for a cutting constraint:  $X \times$  \_\_\_\_\_ +  $Y \times$  \_\_\_\_\_  $\leq$  \_\_\_\_\_

Minutes available for gluing? \_\_\_\_\_

Equation for a gluing constraint: \_\_\_\_\_ + \_\_\_\_\_  $\leq$  \_\_\_\_\_

Minutes available for finishing? \_\_\_\_\_

Equation for a finishing constraint: \_\_\_\_\_ + \_\_\_\_\_  $\leq$  \_\_\_\_\_

# Logistics

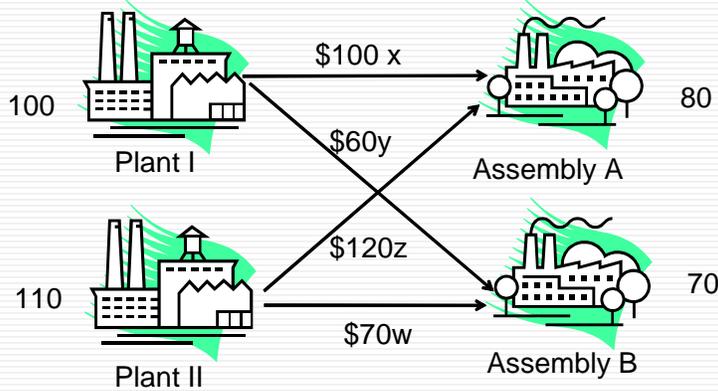
plants are 100 and 110 engines per month respectively. The engines are Allison Engine has two manufacturing plants, I and II, that produce the propjet engines for Embraer Corporation that makes commuter planes. The maximum production capabilities of these two manufacturing shipped to two of Embraer’s assembly plants, A and B. The shipping costs in dollars per engine from plants I and II are to the assembly plants A and B are as follows:

*To Assembly Plant*

From	A	B
Plant I	\$100	\$60
Plant II	\$120	\$70

In a certain month, assembly plant A needs 80 engines, whereas assembly plant B needs 70 engines. How many engines should be shipped from each manufacturing plant to each assembly plant to minimize shipping costs?

## Logistics



30

How many engines can you produce? \_\_\_\_\_

How many engines can you assemble? \_\_\_\_\_

An Equation for shipping cost would be:

\_\_\_\_\_ X + \_\_\_\_\_ Y + \_\_\_\_\_ Z + \_\_\_\_\_ W = \_\_\_\_\_

Equations to describe the production of each Assembly facility:

X + Z = \_\_\_\_\_ and \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_

Isolate a variable for substitution in these two equations:

Z = \_\_\_\_\_ - \_\_\_\_\_      \_\_\_\_\_ = \_\_\_\_\_ - \_\_\_\_\_

# Supply Chain: Graphical Linear Programming

**Linear Programming –**

**Constraint –**

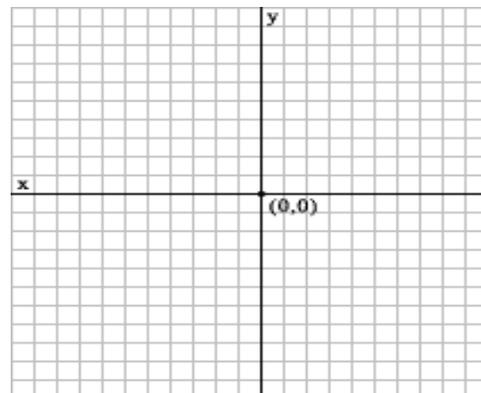
**Objective Function –**

**Vertices -**

## **Example 1: Consumer Knowledge**

Suppose you want to buy CDs and tapes. You can afford as many as 10 tapes or 7 CDs. You want at least 4 CDs and at least 10 hours of recorded music. Each tape holds 45 minutes and each CD holds 1 hour of recorded music.

1. Define Variables
2. Write Constraints
3. Graph Constraints



4. Test Vertices to find maximum and minimum values

### Example 2: Manufacturing

A small company makes two similar products, which all follow the same two-step process, consisting of cutting and gluing. Time requirements *in minutes* for each product at each operation are given below.

operation	product	
	$x$	$y$
cutting	1.5	0.8
gluing	0.6	2.2

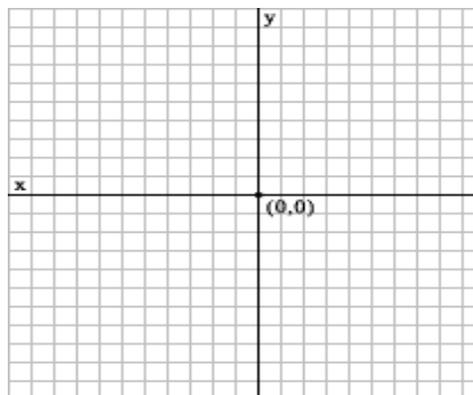
The firm has 30 hours available in the next period for cutting and 55 hours for gluing. Product  $x$  contributes \$4.40 per unit to profit and  $y$  contributes \$4.50 per unit. Find the product mix that maximizes profit.

1. Define Variables
2. Write Constraints
3. Write Objective Function
4. Graph Constraints
  - a. Use a graphing calculator (see handout)
5. Test Vertices

### Example 3: Running a Bakery

A baking tray of corn muffins takes 4c milk and 3c flour. A tray of bran muffins takes 2c milk and 3c flour. A baker has 16c milk and 15c flour. He makes \$3 profit per tray of corn and \$2 profit per tray of bran muffins. How many trays of each type of muffin should he bake to get maximum profit?

1. Define Variables
2. Write Constraints
3. Write Objective Function
4. Graph (both by hand and on calculator)



5. Check Vertices

1. Suppose you want to buy CDs and tapes. You can afford as many as 8 tapes or 12 CDs. You want at least 6 CDs and at least 15 hours of recorded music. Each tape holds 45 minutes and each CD holds 1 hour of recorded music.
  - a. Define Variables
  - b. Write Constraints
  - c. Graph Constraints
  - d. Test Vertices to find maximum and minimum values in the following Objective Equation:  $M = 3x + 4y$

2. A small company makes two similar products, which all follow the same two-step process, consisting of cutting and gluing. Time requirements *in minutes* for each product at each operation are given below.

<u>operation</u>	<u>product</u>	
	<u>x</u>	<u>y</u>
cutting	1.5	0.8
gluing	0.6	2.2

The firm has 60 hours available in the next period for cutting and 110 hours for gluing. Product  $x$  contributes \$4.60 per unit to profit and  $y$  contributes \$4.35 per unit. Find the product mix that maximizes profit.

- a. Define Variables
- b. Write Constraints
  
- c. Write Objective Function
  
- d. Graph Constraints
  - a. Use a graphing calculator (see handout)
  
- e. Test Vertices

3. A baking tray of corn muffins takes  $3c$  milk and  $4c$  flour. A tray of bran muffins takes  $4c$  milk and  $2c$  flour. A baker has  $24c$  milk and  $16c$  flour. He makes  $\$3.25$  profit per tray of corn and  $\$2.75$  profit per tray of bran muffins. How many trays of each type of muffin should he bake to get maximum profit?
- Define Variables
  - Write Constraints
  - Write Objective Function
  - Graph
  - Check Vertices

A small company makes two similar products, which all follow the same two-step process, consisting of cutting and gluing. Time requirements *in minutes* for each product at each operation are given below.

<b>operation</b>	<b>product</b>	
	<b><i>x</i></b>	<b><i>y</i></b>
cutting	1.5	0.8
gluing	0.6	2.2

The firm has 40 *hours* available in the next period for cutting and 40 *hours* for gluing. Product  $x$  contributes \$4.40 per unit to profit and  $y$  contributes \$4.50 per unit. Find the product mix that maximizes profit.

a. Define Variables

b. Write Constraints

c. Write Objective Function

d. Graph Constraints (Use Graphing Calculator)

e. Test Vertices

## Solving Systems of Equations Using Matrices: Computer Algorithms

**Course:** Algebra 2 and/or Pre-Calculus

### **Indiana Standards:**

A2.2.2 – Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables

DM.2.1 – Use matrices to organize and store data

DM.2.3 – Use row-reduction techniques to solve problems

### **Objective:**

Students will learn how to systematically solve matrices in order to better understand how computers are able to solve large systems of equations and other computer algorithms.

### **Materials:**

- Student Handout
- Calculator

### **Job Connection:**

- Bastian Material Handling - [www.bastiansolutions.com](http://www.bastiansolutions.com)
  - **Support Software Developer** - You will develop creative software programs based on received functional specifications, software design documents and end user input. Prepare test plans and quality, stress and load test software programs developed at ASAP US and ASAP India. Maintain and troubleshoot existing programs developed for existing ASAP customers that are on customer support. .Make sure that in-house test environment for customers on support is functioning correctly and that all issues being resolved and change orders being delivered are tested properly, delivered on time and will be able to satisfy performance needs of our customers.
    - Support ASAP Automation Software - <http://www.asapauto.com/>

### **Vocabulary:**

Algorithm - A set of instructions used to solve a problem or obtain a desired result.

Matrix - A rectangular (or square) array of numbers.

### **Input:**

Solving systems of equations is behind many things computers do:

- Linear Programming
- Graphics

How do computers solve systems of equations?

- They need an organized, sequential, detailed set of instructions called an algorithm.
- They use matrices.

Algorithms are the basis of almost everything a computer does. The goal of this lesson is to learn to think in an organized, sequential way that could be programmed into a computer.

To develop an algorithm, you must first find a pattern.

We can save effort by using augmented matrices so we don't have to write all of the variables:

$$\begin{array}{r} x - y = 2 \\ 2x - 3y = 2 \end{array} \quad \left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -3 & 2 \end{array} \right]$$

Multiply the first equation by  $-2$  and add it to the second equation to eliminate  $x$ :

$$\left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -3 & 2 \end{array} \right] \begin{array}{l} \longrightarrow -2x + 2y = -4 \\ \longrightarrow \underline{2x - 3y = 2} \\ \qquad \qquad \qquad -y = -2 \text{ or } y = 2 \end{array}$$

So now we have:

$$\left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 2 \end{array} \right] \begin{array}{l} x - y = 2 \\ y = 2 \end{array}$$

Adding the equations again we have:

$$\left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 2 \end{array} \right] \begin{array}{l} \longrightarrow x - y = 2 \\ \longrightarrow \underline{y = 2} \\ \qquad \qquad \qquad x = 4 \end{array}$$

This gives us the answer:

$$\left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array} \right] \begin{array}{l} x = 4 \\ \underline{y = 2} \end{array}$$

Solve using an augmented matrix:

$$2x + 2y + 2z = 12$$

$$3x + 2y - z = 4$$

$$3x + y + 2z = 11$$

Put this in matrix form:

$$\left[ \begin{array}{ccc|c} 2 & 2 & 2 & 12 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right]$$

The goal is this form:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right]$$

Rules to get there:

1. You may divide or multiply a row by a constant.
2. You may divide or multiply a row by a constant then add it to another row, replacing the row to which you added the original row.
3. You may switch rows under certain conditions.

$$\left[ \begin{array}{ccc|c} 2 & 2 & 2 & 12 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

Copy row 1 down to the next matrix and get zeroes in rows 2 and 3 of the first column.

$$\begin{array}{l} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right] \begin{array}{l} \xrightarrow{-3r1} \\ \xrightarrow{+r2} \end{array} \\ \begin{array}{cccccc} -3 & -3 & -3 & -18 \\ 3 & 2 & -1 & 4 \\ \hline 0 & -1 & -4 & -14 \end{array} \\ \begin{array}{l} \xrightarrow{-3r1} \\ \xrightarrow{+r3} \end{array} \\ \begin{array}{cccccc} -3 & -3 & -3 & -18 \\ 3 & 1 & 2 & 11 \\ \hline 0 & -2 & -1 & -7 \end{array} \end{array}$$

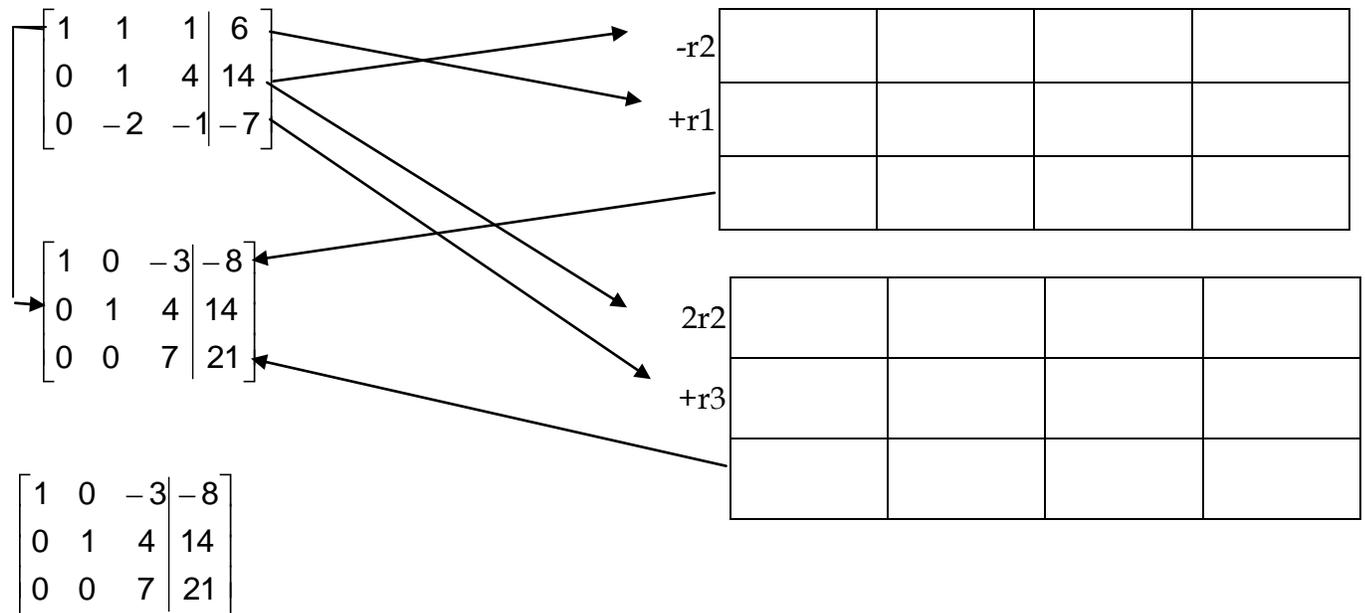
$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -2 & -1 & -7 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -2 & -1 & -7 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.



Get a 1 in the third row, third column by dividing row 3 by the number in the third row, third column.

What is that number?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array} \right]$$




What is the solution?

What is the overall pattern to what you did?

In what order did you work on columns?

How did you get 1s?

How did you get 0s?

Can you write down the pattern for someone to follow?

Solve using an augmented matrix:

$$\begin{aligned} 3x - 2y + 8z &= 9 \\ -2x + 2y + z &= 3 \\ x + 2y - 3z &= 8 \end{aligned}$$

Put this in matrix form:

$$\left[ \begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right]$$

The goal is this form:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

Copy row 1 down to the next matrix and get zeroes in rows 2 and 3 of the first column.

$$\begin{array}{c}
 \left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right] \begin{array}{l} \xrightarrow{2r1} \\ \xrightarrow{+r2} \end{array} \begin{array}{ccc|c} 2 & -4/3 & 16/3 & 6 \\ -2 & 2 & 1 & 3 \\ \hline 0 & 2/3 & 19/3 & 9 \end{array} \\
 \\
 \begin{array}{ccc|c} -1 & 2/3 & -8/3 & -3 \\ +r3 & 1 & 2 & 8 \\ \hline 0 & 8/3 & -17/3 & 5 \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 2/3 & 19/3 & 9 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right] \qquad \left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 2/3 & 19/3 & 9 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right]
 \end{array}$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\begin{array}{c}
 \left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right] \begin{array}{l} \xrightarrow{2/3r2} \\ \xrightarrow{+r1} \end{array} \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & -31 & -31 \end{array} \right] \begin{array}{l} \xrightarrow{-8/3r2} \\ \xrightarrow{+r3} \end{array} \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}
 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & -31 & -31 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by the number in the third row, third column.

What is that number?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$




What is the solution?

Try this one.

$$\begin{aligned} 2y + 3z &= 7 \\ 3x + 6y - 12z &= -3 \\ 5x - 2y + 2z &= -7 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & 3 & 7 \\ 3 & 6 & -12 & -3 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

There is no way to get a 1 in the upper left by division so switch rows.

$$\begin{array}{l} \rightarrow \\ \rightarrow \end{array} \left[ \begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Copy row 1 down to the next matrix and get zeroes in row 3 of the first column. Copy row 2 since there is already a zero in row 2 in the correct position.

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$



$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & -12 & 22 & -2 \end{array} \right]$$



$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 40 & 40 \end{array} \right]$$


Get a 1 in the third row, third column by dividing row 3 by number in the third row, third column.

What is that number?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$




What is the solution?

**Assessment:** Students will complete homework problems (from the textbook) using this method of solving matrices

**Additional Documents:**

SolvSysEqMatrixSlides.doc

SolvSysEqMatrixWorksheet.doc

SolvSysEqMatrixTest.doc

SolvSysEqMatrixSolver.xls

# **Think Like a Computer to Solve Systems of Linear Equations**

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Solving systems of equations is behind many things computers do:

- Linear Programming
- Graphics

How do computers solve systems of equations?

- They need an organized, sequential, detailed set of instructions called an **algorithm**.
- They use matrices.

Algorithms are the basis of almost everything a computer does. The goal of this lesson is to learn to think in an organized, sequential way that could be programmed into a computer.

To develop an algorithm, you must first find a pattern.

Think about how to solve a system of equations:

$$\begin{aligned}x - y &= 2 \\ 2x - 3y &= 2\end{aligned}$$

Multiply the first equation by  $-2$  and add it to the second equation to eliminate  $x$ :

$$\begin{aligned}-2x + 2y &= -4 \\ \underline{2x - 3y} &= \underline{2} \\ -y &= -2 \text{ or } y = 2\end{aligned}$$

So now we have:

$$\begin{aligned}x - y &= 2 \\ y &= 2\end{aligned}$$

Adding the equations again we have:

$$\begin{array}{r} x - y = 2 \\ \quad y = 2 \\ \hline x \quad = 4 \end{array}$$

This gives us the answer:

$$\begin{array}{r} x = 4 \\ \quad y = 2 \\ \hline \end{array}$$

We can save effort by using augmented matrices so we don't have to write all of the variables:

$$\begin{array}{r} x - y = 2 \\ 2x - 3y = 2 \end{array} \quad \left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -3 & 2 \end{array} \right]$$

Multiply the first equation by  $-2$  and add it to the second equation to eliminate  $x$ :

$$\left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -3 & 2 \end{array} \right] \begin{array}{l} \longrightarrow -2x + 2y = -4 \\ \longrightarrow \underline{2x - 3y = 2} \\ \qquad \qquad \qquad -y = -2 \text{ or } y = 2 \end{array}$$

So now we have:

$$\left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 2 \end{array} \right] \quad \begin{array}{l} x - y = 2 \\ y = 2 \end{array}$$

Adding the equations again we have:

$$\begin{array}{l} \left[ \begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 2 \end{array} \right] \longrightarrow \begin{array}{l} x - y = 2 \\ \underline{y = 2} \\ x = 4 \end{array} \end{array}$$

This gives us the answer:

$$\begin{array}{l} \left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 2 \end{array} \right] \longleftarrow \begin{array}{l} x = 4 \\ \underline{y = 2} \end{array} \end{array}$$

Try a bigger one using an augmented matrix:

$$2x + 2y + 2z = 12$$

$$3x + 2y - z = 4$$

$$3x + y + 2z = 11$$

Put this in matrix form:

$$\left[ \begin{array}{ccc|c} 2 & 2 & 2 & 12 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right]$$

The goal is this form:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right]$$

Rules to get there:

3. You may divide or multiply a row by a constant.
4. You may divide or multiply a row by a constant then add it to another row, replacing the row to which you added the original.
5. You may switch rows under certain conditions.

$$\left[ \begin{array}{ccc|c} 2 & 2 & 2 & 12 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right]$$

Copy row 1 down to the next matrix and get zeroes in rows 2 and 3 of the first column.

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 2 & -1 & 4 \\ 3 & 1 & 2 & 11 \end{array} \right] \begin{array}{l} \xrightarrow{-3r_1} \\ \xrightarrow{+r_2} \\ \xrightarrow{-3r_1} \\ \xrightarrow{+r_3} \end{array} \\
 \begin{array}{cccc} -3 & -3 & -3 & -18 \\ 3 & 2 & -1 & 4 \\ \hline 0 & -1 & -4 & -14 \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -2 & -1 & -7 \end{array} \right] \begin{array}{l} \xrightarrow{-3r_1} \\ \xrightarrow{+r_3} \end{array} \\
 \begin{array}{cccc} -3 & -3 & -3 & -18 \\ 3 & 1 & 2 & 11 \\ \hline 0 & -2 & -1 & -7 \end{array}
 \end{array}$$

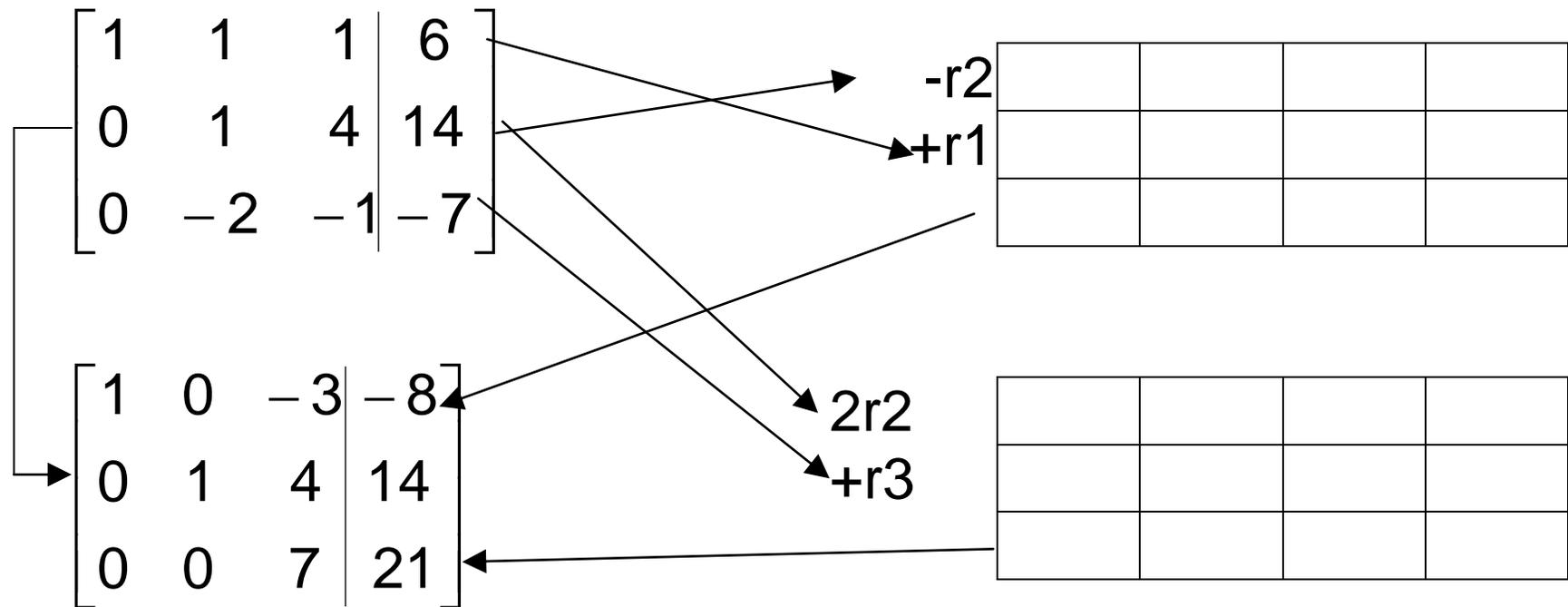
$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -2 & -1 & -7 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 14 \\ 0 & -2 & -1 & -7 \end{array} \right]$$

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.



Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 14 \\ 0 & -2 & -1 & -7 \end{array} \right] \begin{array}{l} \xrightarrow{-r2} \\ \xrightarrow{+r1} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 7 & 21 \end{array} \right] \begin{array}{l} \xrightarrow{2r2} \\ \xrightarrow{+r3} \end{array} \\
 \\
 \begin{array}{cccc} 0 & -1 & -4 & -14 \\ 1 & 1 & 1 & 6 \\ \hline 1 & 0 & -3 & -8 \\ \hline 0 & 2 & 8 & 28 \\ 0 & -2 & -1 & -7 \\ \hline 0 & 0 & 7 & 21 \end{array}
 \end{array}$$

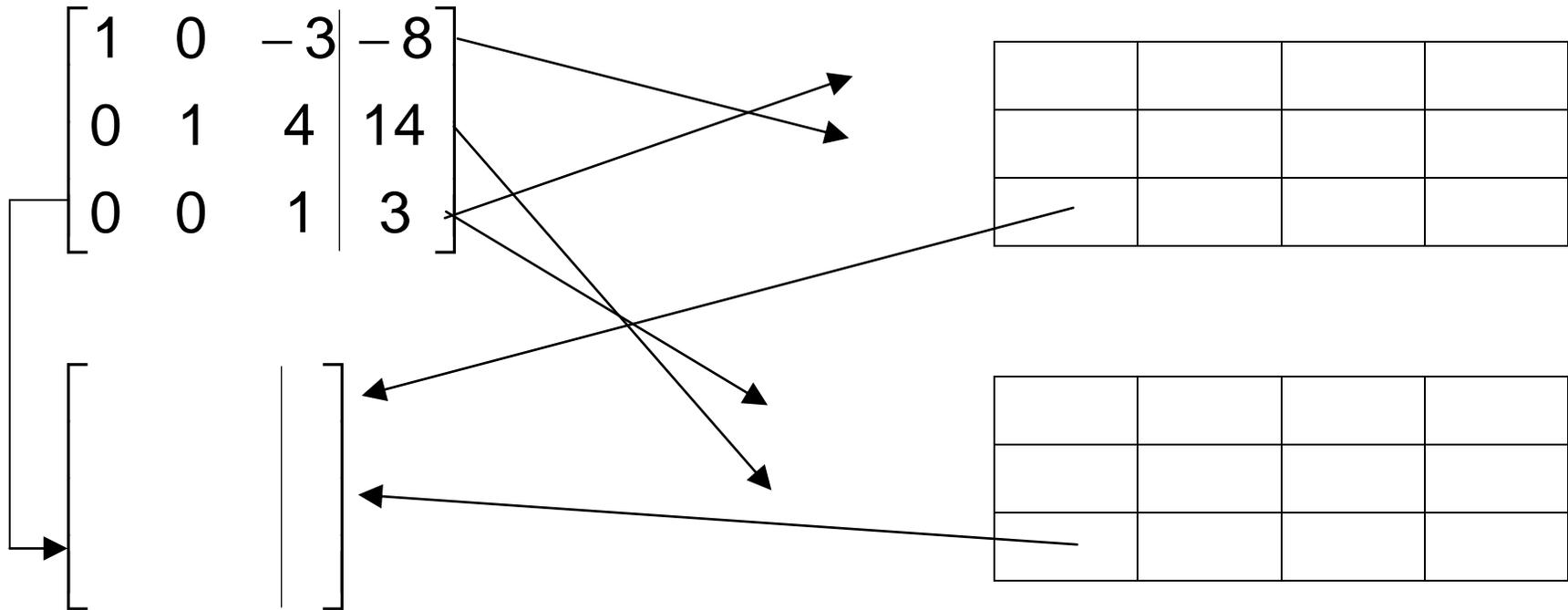
$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 7 & 21 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by the number in the third row, third column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.



What is the solution?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\begin{array}{c}
 \left[ \begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} \xrightarrow{3 r3} \\ \xrightarrow{+ r1} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} \xrightarrow{-4 r3} \\ \xrightarrow{+ r2} \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{cccc} 0 & 0 & 3 & 9 \\ 1 & 0 & -3 & -8 \\ \hline 1 & 0 & 0 & 1 \end{array} \\
 \\
 \begin{array}{cccc} 0 & 0 & -4 & -12 \\ 0 & 1 & 4 & 14 \\ \hline 0 & 1 & 0 & 2 \end{array}
 \end{array}$$

What is the solution?

$$x = 1, y = 2, z = 3$$

What is the overall pattern to what you did?

In what order did you work on columns?

How did you get 1s?

How did you get 0s?

Can you write down the pattern for someone to follow?

Solve using an augmented matrix:

$$3x - 2y + 8z = 9$$

$$-2x + 2y + z = 3$$

$$x + 2y - 3z = 8$$

$$\left[ \begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right]$$

Copy row 1 down to the next matrix and get zeroes in rows 2 and 3 of the first column.

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c}
 1 & -2/3 & 8/3 & 3 \\
 -2 & 2 & 1 & 3 \\
 1 & 2 & -3 & 8
 \end{array} \right] \begin{array}{l} \rightarrow 2r1 \\ \rightarrow +r2 \\ \rightarrow -1r1 \\ \rightarrow +r3 \end{array} \\
 \\
 \left[ \begin{array}{ccc|c}
 1 & -2/3 & 8/3 & 3 \\
 0 & 2/3 & 19/3 & 9 \\
 0 & 8/3 & -17/3 & 5
 \end{array} \right] \begin{array}{l} \\ \\ \end{array} \\
 \\
 \begin{array}{ccc|c}
 2 & -4/3 & 16/3 & 6 \\
 -2 & 2 & 1 & 3 \\
 \hline
 0 & 2/3 & 19/3 & 9 \\
 \\
 -1 & 2/3 & -8/3 & -3 \\
 1 & 2 & -3 & 8 \\
 \hline
 0 & 8/3 & -17/3 & 5
 \end{array}
 \end{array}$$

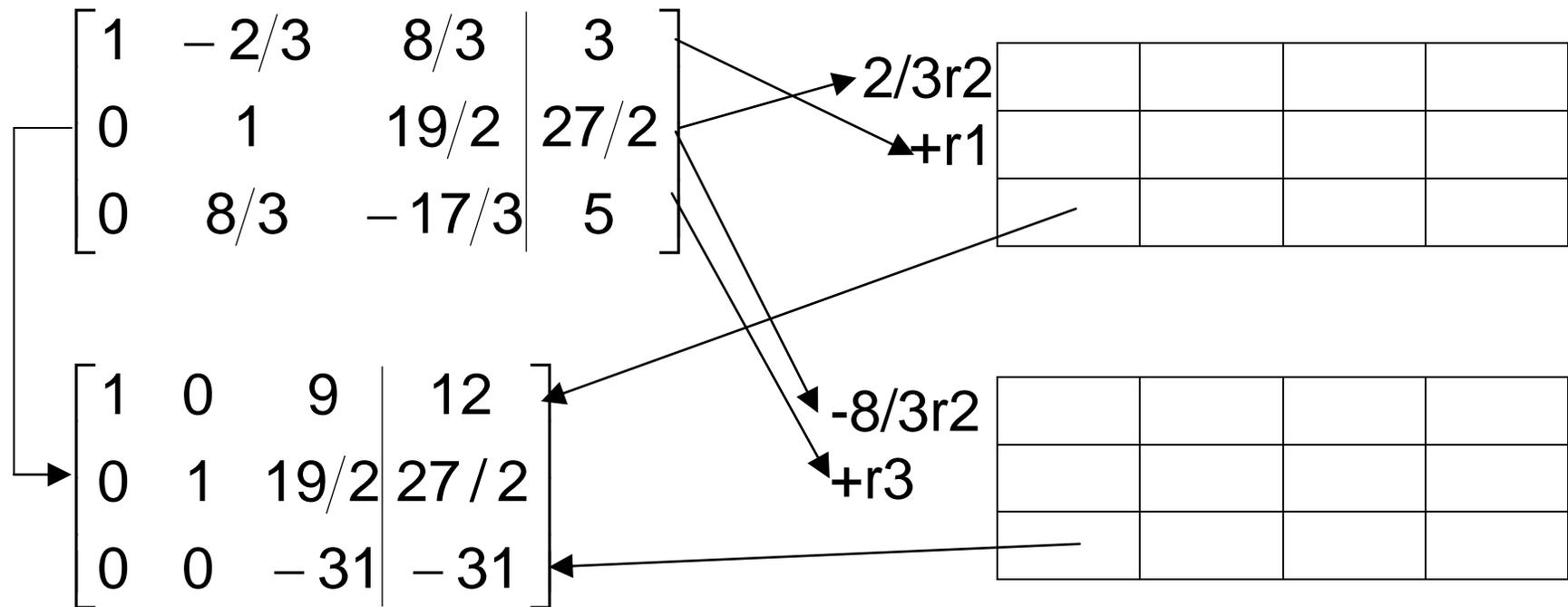
$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 2/3 & 19/3 & 9 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right]$$

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.



Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\begin{array}{c}
 \left[ \begin{array}{ccc|c}
 1 & -2/3 & 8/3 & 3 \\
 0 & 1 & 19/2 & 27/2 \\
 0 & 8/3 & -17/3 & 5
 \end{array} \right] \begin{array}{l} \xrightarrow{2/3r2} \\ \xrightarrow{+r1} \\ \xrightarrow{-8/3r2} \\ \xrightarrow{+r3} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c}
 1 & 0 & 9 & 12 \\
 0 & 1 & 19/2 & 27/2 \\
 0 & 0 & -31 & -31
 \end{array} \right] \begin{array}{l} \xrightarrow{+r1} \\ \xrightarrow{-8/3r2} \\ \xrightarrow{+r3} \end{array} \\
 \\
 \begin{array}{ccc|c}
 0 & 2/3 & 19/3 & 9 \\
 1 & -2/3 & 8/3 & 3 \\
 \hline
 1 & 0 & 9 & 12 \\
 \\
 0 & -8/3 & -76/3 & -36 \\
 0 & 8/3 & -17/3 & 5 \\
 \hline
 0 & 0 & -31 & -31
 \end{array}
 \end{array}$$

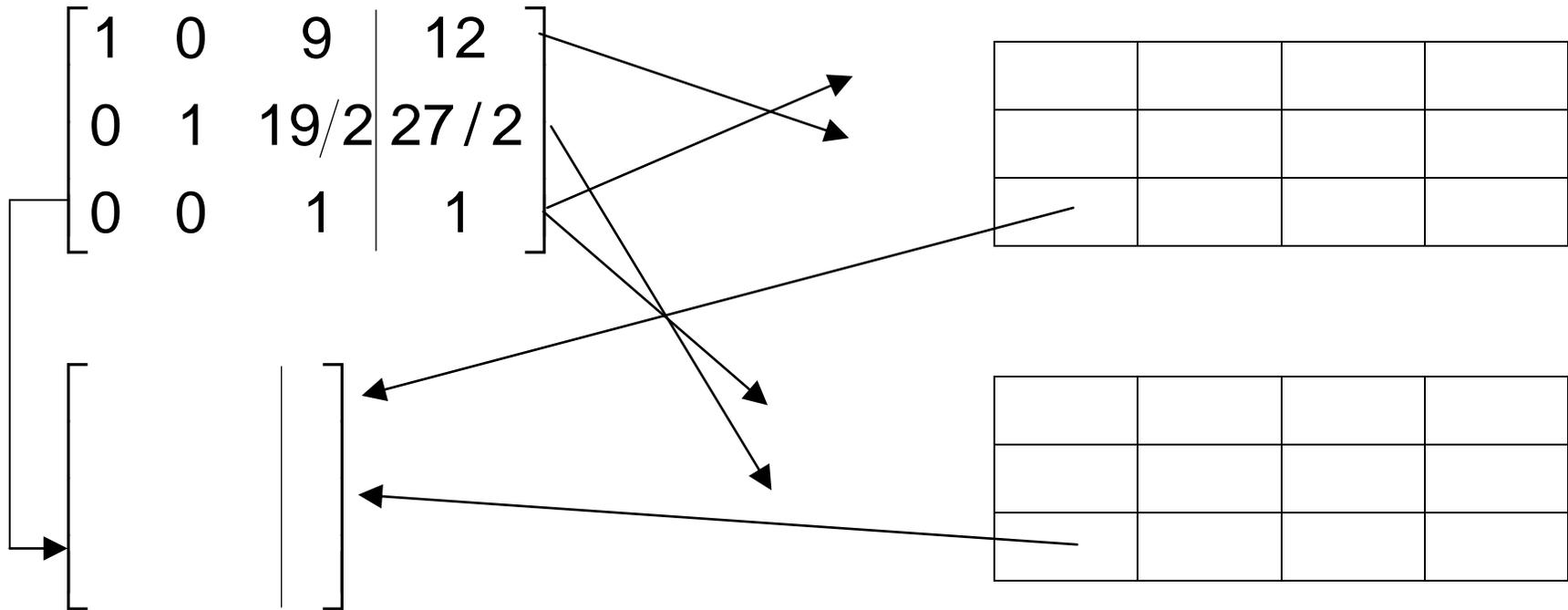
$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & -31 & -31 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by the number in the third row, third column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.



What is the solution?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\begin{array}{c}
 \left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{-9 r_3} \\ \xrightarrow{+ r_1} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{-19/2 r_3} \\ \xrightarrow{+ r_2} \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{cccc} 0 & 0 & -9 & -9 \\ 1 & 0 & 9 & 12 \\ \hline 1 & 0 & 0 & 3 \end{array} \\
 \\
 \begin{array}{cccc} 0 & 0 & -19/2 & -19/2 \\ 0 & 1 & 19/2 & 27/2 \\ \hline 0 & 1 & 0 & 4 \end{array}
 \end{array}$$

What is the solution?

$$x = 3, y = 4, z = 1$$

Try this one.

$$2y + 3z = 7$$

$$3x + 6y - 12z = -3$$

$$5x - 2y + 2z = -7$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & 3 & 7 \\ 3 & 6 & -12 & -3 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

There is no way to get a 1 in the upper left by division so switch rows.

$$\begin{array}{l} \rightarrow \\ \rightarrow \end{array} \left[ \begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

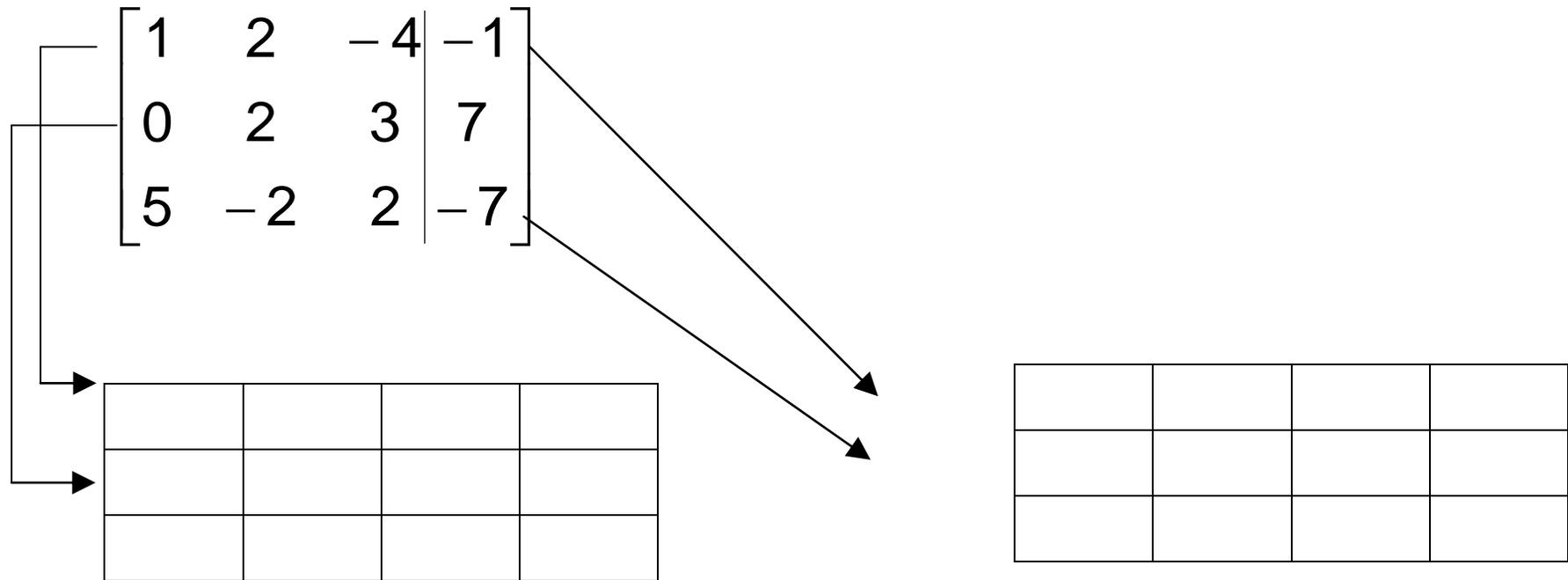
$$\left[ \begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Copy row 1 down to the next matrix and get zeroes in row 3 of the first column. Copy row 2 since there is already a zero in the correct position.



Copy row 1 down to the next matrix and get zeroes in row 3 of the first column. There is already a zero in row 2

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right] \\
 \left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array} \right]
 \end{array}$$

$-5 r1$      $-5$     $-10$     $20$     $5$   
 $+ r3$      $5$     $-2$     $2$     $-7$   


---

 $0$     $-12$     $22$     $-2$

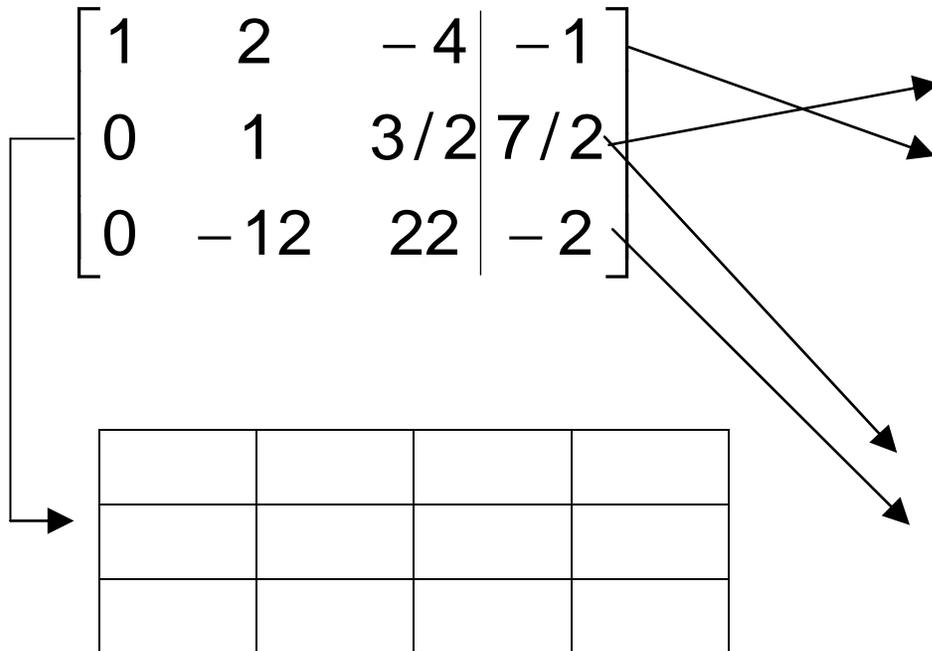
$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second row, second column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & -12 & 22 & -2 \end{array} \right]$$

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.





Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & -12 & 22 & -2 \end{array} \right] \begin{array}{l} \xrightarrow{-2 r_2} \\ \xrightarrow{+ r_1} \end{array} \begin{array}{c} 0 \quad -2 \quad -3 \quad -7 \\ 1 \quad 2 \quad -4 \quad -1 \\ \hline 1 \quad 0 \quad -7 \quad -8 \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 40 & 40 \end{array} \right] \begin{array}{l} \xrightarrow{-12 r_2} \\ \xrightarrow{+ r_3} \end{array} \begin{array}{c} 0 \quad 12 \quad 18 \quad 42 \\ 0 \quad -12 \quad 22 \quad -2 \\ \hline 0 \quad 0 \quad 40 \quad 40 \end{array}
 \end{array}$$

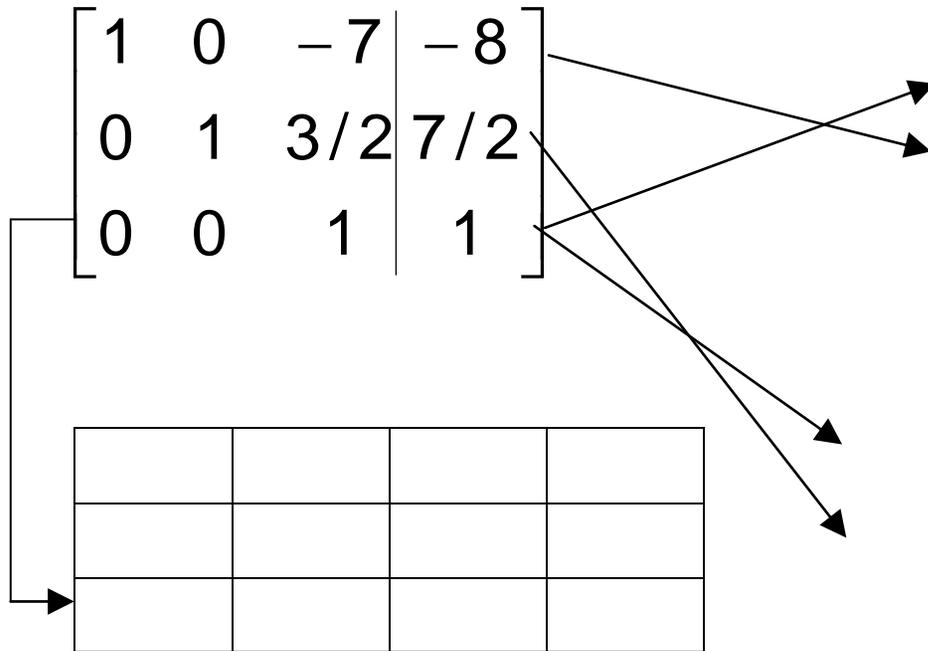
$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 40 & 40 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by the number in the third row, third column.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.





What is the solution?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\begin{array}{l}
 \left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{7 r3} \\ \xrightarrow{+ r1} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{-3/2 r3} \\ \xrightarrow{+ r2} \end{array} \\
 \\
 \left[ \begin{array}{ccc|c} 0 & 0 & 7 & 7 \\ 1 & 0 & -7 & -8 \\ \hline 1 & 0 & 0 & -1 \\ 0 & 0 & -3/2 & -3/2 \\ 0 & 1 & 3/2 & 7/2 \\ \hline 0 & 1 & 0 & 2 \end{array} \right]
 \end{array}$$

What is the solution?

$$x = -1, y = 24, z = 1$$

## Think Like a Computer to Solve Systems of Linear Equations

Solving systems of equations is behind many things computers do:

- Linear Programming
- Graphics

How do computers solve systems of equations?

- They need an organized, sequential, detailed set of instructions called an **algorithm**.
- They use matrices.

Algorithms are the basis of almost everything a computer does. The goal of this lesson is to learn to think in an organized, sequential way that could be programmed into a computer.

To develop an algorithm, you must first find a pattern.

Solve using an augmented matrix:

$$\begin{aligned} 3x - 2y + 8z &= 9 \\ -2x + 2y + z &= 3 \\ x + 2y - 3z &= 8 \end{aligned}$$

Put this in matrix form:

$$\left[ \begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right]$$

The goal is this form:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array} \right]$$

Rules to get there:

6. You may switch rows.
7. You may divide or multiply a row by a constant.
8. You may divide or multiply a row by a constant then add it to another row, replacing the row to which you added the original row.

$$\left[ \begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

Copy row 1 down to the next matrix and get zeroes in rows 2 and 3 of the first column.

$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right] \begin{array}{l} \xrightarrow{2r1} \\ \xrightarrow{+r2} \end{array} \begin{array}{cccc} 2 & -4/3 & 16/3 & 6 \\ -2 & 2 & 1 & 3 \\ \hline 0 & 2/3 & 19/3 & 9 \end{array}$$
  

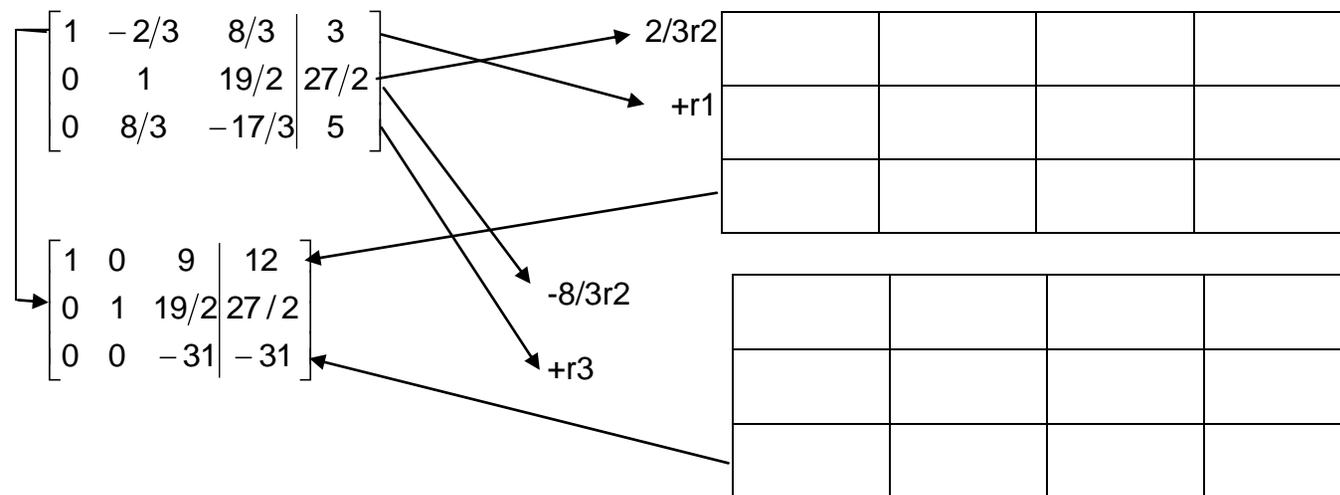
$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 2/3 & 19/3 & 9 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right] \begin{array}{l} \xrightarrow{-1r1} \\ \xrightarrow{+r3} \end{array} \begin{array}{cccc} -1 & 2/3 & -8/3 & -3 \\ 1 & 2 & -3 & 8 \\ \hline 0 & 8/3 & -17/3 & 5 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & -2/3 & 8/3 & 3 \\ 0 & 2/3 & 19/3 & 9 \\ 0 & 8/3 & -17/3 & 5 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by number in the second row, second column.

What is that number?

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.



$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & -31 & -31 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by number in the third row, third column.

What is that number?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & 19/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$




What is the solution?

What is the overall pattern to what you did?

In what order did you work on columns?

How did you get 1s?

How did you get 0s?

Can you write down the pattern for someone to follow?

Try this one.

$$\begin{aligned}2y + 3z &= 7 \\ 3x + 6y - 12z &= -3 \\ 5x - 2y + 2z &= -7\end{aligned}$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & 3 & 7 \\ 3 & 6 & -12 & -3 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

There is no way to get a 1 in the upper left by division so switch rows.

$$\left[ \begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Get a 1 in the upper left corner by dividing row 1 by the number the upper left corner.

What is that number?

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$

Copy row 1 down to the next matrix and get zeroes in row 3 of the first column. There is already a zero in row 2

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array} \right]$$



$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array} \right]$$

Get a 1 in the second row, second column by dividing row 2 by number in the second row, second column.

What is that number?

Copy row 2 down to the next matrix and get zeroes in rows 1 and 3 of the second column.

$$\left[ \begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & -12 & 22 & -2 \end{array} \right]$$




$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 40 & 40 \end{array} \right]$$

Get a 1 in the third row, third column by dividing row 3 by number in the third row, third column.

What is that number?

Copy row 3 down to the next matrix and get zeroes in rows 2 and 3 of the third column.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -7 & -8 \\ 0 & 1 & 3/2 & 7/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$




What s the solution?

Solve the following system of equations.

$$2x + 4y + 2z = 18$$

$$3x + 3y - 3z = 15$$

$$3x - y + 2z = 12$$

Rubric based on 10 points:

Students must do operations in proper order to get full points.

Do not take off points for arithmetic errors.

$$\left| \begin{array}{ccc|c} 2 & 4 & 2 & 18 \\ 3 & 3 & -3 & 15 \\ 3 & -1 & 2 & 12 \end{array} \right|$$

Get a 1 in the upper left corner by dividing row 1 by number the upper left corner.

What is that number?

2

*1 pt for dividing row 1 by 2*

$$\left| \begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 3 & 3 & -3 & 15 \\ 3 & -1 & 2 & 12 \end{array} \right|$$

*1 pt for adding -3 r1 to r2 and replacing r2*

$$\begin{array}{r} -3 r1 \\ + r2 \end{array} \quad \begin{array}{cccc|c} & -3 & -6 & -3 & -27 \\ & 3 & 3 & -3 & 15 \\ \hline & 0 & -3 & -6 & -12 \end{array}$$

Copy row 1 down to the next matrix and  
Get zeroes in rows 2 and 3 of the first  
column.

*1 pt for adding -3 r1 to r3 and replacing r3*

$$\left| \begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & -3 & -6 & -12 \\ 0 & -7 & -1 & -15 \end{array} \right|$$

$$\begin{array}{r} -3 r1 \\ + r3 \end{array} \quad \begin{array}{cccc|c} & -3 & -6 & -3 & -27 \\ & 3 & -1 & 2 & 12 \\ \hline & 0 & -7 & -1 & -15 \end{array}$$

Get a 1 in the second row, second column by dividing row 2 by the number in the second column.

What is that number? -3

*1 pt for correct division of r2*

$$\left| \begin{array}{ccc|c} 1 & 2 & 1 & 9 \\ 0 & 1 & 2 & 4 \\ 0 & -7 & -1 & -15 \end{array} \right|$$

*1 pt for correct addition of r2 and r1 with correct multiplier*

$$\begin{array}{r} -2 r2 \\ + r1 \end{array} \quad \begin{array}{ccc|c} 0 & -2 & -4 & -8 \\ 1 & 2 & 1 & 9 \\ \hline 1 & 0 & -3 & 1 \end{array}$$

Copy row 2 down to the next matrix and  
Get zeroes in rows 1 and 3 of the second column.

*1 pt for correct addition of r2 and r3 with correct multiplier*

$$\begin{array}{r} 7 r2 \\ + r3 \end{array} \quad \begin{array}{ccc|c} 0 & 7 & 14 & 28 \\ 0 & -7 & -1 & -15 \\ \hline 0 & 0 & 13 & 13 \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 13 & 13 \end{array} \right|$$

Get a 1 in the third row, third column by dividing row 3 by the number in the third column.

What is that number? 2

*1 pt for correct division of r3*

$$\left| \begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 \end{array} \right|$$

*1 pt for correct addition of r3 and r1 with correct multiplier*

$$\begin{array}{r} -3 r3 \\ + r1 \end{array} \quad \begin{array}{ccc|c} 0 & 0 & 3 & 3 \\ 1 & 0 & -3 & 1 \\ \hline 1 & 0 & 0 & 4 \end{array}$$

Copy row 3 down to the next matrix and  
Get zeroes in rows 1 and 2 of the third column.

*1 pt for correct addition of r3 and r2 with correct multiplier*

$$\begin{array}{r} -2 r3 \\ + r2 \end{array} \quad \begin{array}{ccc|c} 0 & 0 & -2 & -2 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 1 & 0 & 2 \end{array}$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right|$$

X=4      Y=2      Z=1

*1 pt for reading final answer*

## Logarithm as Applied to pH: Acids and Basis in Supply Chain

**Course:** Algebra 2 and/or Pre-Calculus

### Indiana Standards:

- A2.7.3 Understand and use the inverse relationship between exponents and logarithms.
- A2.7.4 Solve logarithmic and exponential equations and inequalities.
- PC.2.1 Solve word problems involving applications of logarithmic and exponential functions.

### Objective:

The students will understand and represent the relationship between logarithms and pH as well as pH's connection to supply chain management

### Materials:

- Scientific or Graphing calculator
- pH testing strips
- 10-15 samples of various liquids
- Pencil/pen
- Worksheet/Notes sheet
- Baking Soda
- pH paper
- graduated cylinder (?)
- Pan (for volcano)
- Q-tips

### Job Connection:

- Sentry BPS - <http://www.sentrybps.com/>
- MD Logistics - <http://www.mdlogistics.com/index.asp>
  - **Pharmaceutical Shipping Line Supervisor** - MD Logistics provides contract warehousing, inventory management, fulfillment, distribution, packaging, transportation, and global freight forwarding. MD Logistics serves a variety of industries from routine freight management to highly specialized life-saving medicines. Our success is based on our belief that successfully growing the business depends on consistently meeting the service requirements of our customers.

### Vocabulary:

Acid -  $< 7$  on the pH scale

Base -  $> 7$  on the pH scale

Logarithm - the logarithm base  $b$  of a number  $x$  is the power to which  $b$  must be raised in order to equal  $x$ .

Mole - SI unit for amount of substance, defined as the number of atoms in exactly 12 g of carbon-12.

Concentration - A measure of the amount of substance present in a unit amount of mixture. The amounts can be expressed as moles, masses, or volumes.

### **Input:**

*Model 1: pH*

**pH** - the measure of acidity of a solution; pH is a measurement of the concentration of hydrogen ions ( $H^+$ ) in a solution.

**pH Scale** - an inverse logarithmic representation of hydrogen ion ( $H^+$ ) concentration.

Note: Since it is a logarithmic scale, and not a linear scale, each individual pH unit is a factor of 10 different than the next higher or lower unit.

**Acid** -  $< 7$  on the pH scale

Examples: Vinegar, Lemon Juice, Nitric Acid (car battery acid), Cleaning products (HCl), Acid Rain

**Base** -  $> 7$  on the pH scale

Examples: Drano, Windex, Chlorox, Baking Soda

### ***Demonstrations:***

Voncano (baking soda)

pH paper (disappearing ink)

How to find the pH:

$$pH = -\log[H_3O^+]$$

*What's the pH of water?*

Neutral (7) so it's both an acid and base

### **Concentration of solutions:**

- Given in moles per liter
- Avagadro's number =  $6.02 \times 10^{23}$
- Mole of water = tablespoon
- The population of the world is 6,553,751,253.

- Suppose we split a mole of dollars among everyone in the world. If we all spend a mole of dollars at \$1,000,000 per day, it would take us more than 250,000 years to spend it all.

### How do Acids affect a solution?

Acids dissolved in water increase the concentration of hydronium ions:  $\text{HCl} + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{Cl}^-$

### How do Bases affect a solution?

Bases dissolved in water increase the concentration of hydroxide ions:  
 $\text{NaOH} \rightleftharpoons \text{Na}^+ + \text{OH}^-$

*Regardless of which ion increases, the product of the two remains the same ( $1.0 \times 10^{-14}$ )*

#### Problem 1:

Suppose we have a 0.10 M solution of HCl. This means that  $[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1}$  mol/L.

1. What is the pH?

$$\text{pH} = -\log(1.0 \times 10^{-1})$$

2. What is  $[\text{OH}^-]$ ?

$$\text{We know that } [\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

$$\text{Substituting } [\text{H}_3\text{O}^+] = 1.0 \times 10^{-1} \text{ mol/L}$$

$$(1.0 \times 10^{-1})[\text{OH}^-] = 1.0 \times 10^{-14}$$

$$[\text{OH}^-] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-1}} = 1.0 \times 10^{-13} = 0.0000000000001 \text{ mol/L}$$

#### Problem 2:

Suppose we have a 0.010 M solution of NaOH. This means that  $[\text{OH}^-] = 1.0 \times 10^{-2}$  mol/L.

1. What is  $[\text{H}_3\text{O}^+]$ ?

$$\text{We know that } [\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

$$\text{Substituting } [\text{OH}^-] = 1.0 \times 10^{-2} \text{ mol/L}$$

$$[\text{H}_3\text{O}^+] (1.0 \times 10^{-2}) = 1.0 \times 10^{-14}$$

$$[\text{H}_3\text{O}^+] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-2}} = 1.0 \times 10^{-12} = 0.0000000000001 \text{ mol/L}$$

2. What is the pH?

$$\text{pH} = -\log(0.0000000000001)$$

#### Experimentation:

Now let's test various pH's and determine what the  $[\text{H}_3\text{O}^+]$  concentration is of each.

Use the formula for pH ( $pH = -\log[H_3O^+]$ ) and what we know about solving log equations to find the missing piece.

*Model 2: Richter scale*

On the Richter scale, the magnitude of R of an earthquake of intensity I is  $R = \log(I / I_0)$ , where  $I = 1$  is the minimum intensity used for comparison. What is the Magnitude of an earthquake with intensity 80,000,000?

**Assessment:**

Students will complete the data collection sheet and determine the concentration of each of the ions.

**Additional Documents:**

LogarithmpHHomework.doc

LogarithmpHSlides.doc

LogarithmpHSlides.ppt

LogarithmpHWorksheet.doc

LogarithmpHWorksheetKey.doc

LogarithmpHDataSheet.doc

# **Acids and Bases in Supply Chains**

## **Logarithms as Applied to pH**

**Leslie Gardner, Ph.D.**

Professor of Operations Management and Mathematics  
University of Indianapolis

# Acids

Weak and diluted acids taste sour, and cause eye irritation.

Acetic acid ( $\text{HC}_2\text{H}_3\text{O}_2$ ) – vinegar

Citric acid – lemon juice

Strong acids corrode metals and burn eyes and skin – especially when concentrated.

Hydrochloric acid ( $\text{HCl}$ ) – some cleaning products, rust removers

Sulfuric acid ( $\text{H}_2\text{SO}_4$ ) – car battery acid

Nitric acid ( $\text{HNO}_3$ ) – industrial processes

Environmental hazards such as acid rain.

# Bases – Alkalis

Weak and dilute bases have a bitter taste and a slippery feel because they turn the oils in your skin into soap. Strong bases burn skin and eyes, especially when concentrated.

Sodium hydroxide ( $\text{NaOH}$ ) – Drano

Ammonia ( $\text{NH}_3$ ) – Windex

Sodium hypochlorite ( $\text{NaClO}$ ) – Chlorox

Sodium carbonate ( $\text{Na}_2\text{CO}_3$ ) – used in the manufacture of glass, pulp and paper, detergents, and to maintain pH in photographic developing agents

Sodium bicarbonate ( $\text{NaHCO}_3$ ) – baking soda

# Water as an Acid and a Base



Two water molecules  $\rightleftharpoons$  Hydronium ion + Hydroxide ion

The product of the concentration of the hydronium and hydroxide ions in water at 25° C in moles per liter is  $1 \times 10^{-14}$ .

$$K_w = [\text{H}_3\text{O}^+][\text{OH}^-] = 1 \times 10^{-14}$$

Pure (distilled) water is neutral, neither an acid nor a base. In pure water **at 25° C**, the concentration of hydronium ions is equal to the concentration of hydroxide ions.

$$K_w = [\text{H}_3\text{O}^+][\text{OH}^-] = (1 \times 10^{-7})(1 \times 10^{-7}) = 1 \times 10^{-14}$$

# Concentration of Solutions



Given in moles per liter.

A mole is Avogadro's number of things (atoms, ions, or molecules).

A mole of water is about a tablespoon.

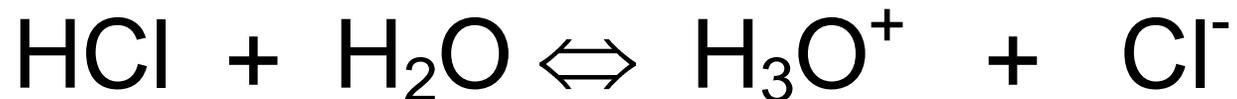
Avogadro's number =  $6.02 \times 10^{23}$

The population of the world is 6,553,751,253.

Suppose we split a mole of dollars among everyone in the world. If we all spend a mole of dollars at \$1,000,000 per day, it would take us more than 250,000 years to spend it all.

# Concentration of Acids and Bases

Acids dissolved in water increase the concentration of hydronium ions.



Bases dissolved in water increase the concentration of hydroxide ions.



Whether the concentration of hydronium ions or hydroxide ions increases, the product of the concentrations remains the same.

$$K_w = [\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

# Concentration of Acids and Bases

Suppose we have a 0.10 M solution of HCl. This means that  $[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1}$  mol/L. What is  $[\text{OH}^-]$ ?

We know that  $[\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$

Substituting  $[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1}$  mol/L

$$(1.0 \times 10^{-1})[\text{OH}^-] = 1.0 \times 10^{-14}$$

$$[\text{OH}^-] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-1}} = 1.0 \times 10^{-13} = 0.000000000000001 \text{ mol/L}$$

# Concentration of Acids and Bases

Suppose we have a 0.010 M solution of NaOH. This means that  $[\text{OH}^-] = 1.0 \times 10^{-2}$  mol/L. What is  $[\text{H}_3\text{O}^+]$ ?

We know that  $[\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$

Substituting  $[\text{OH}^-] = 1.0 \times 10^{-2}$  mol/L

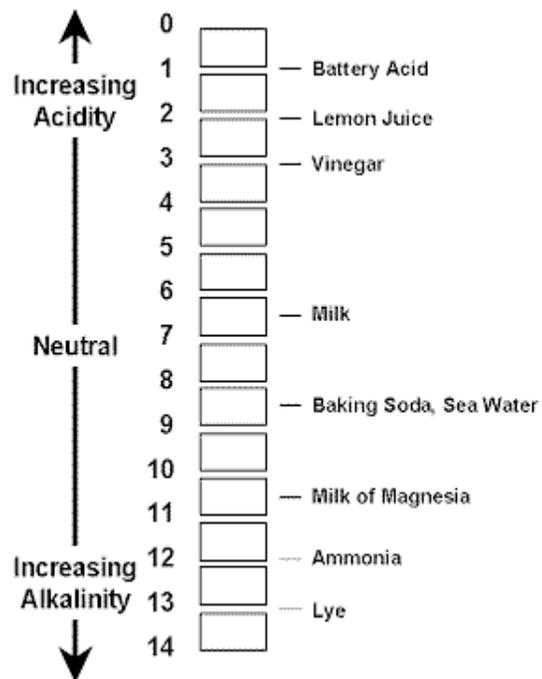
$$[\text{H}_3\text{O}^+] (1.0 \times 10^{-2}) = 1.0 \times 10^{-14}$$

$$[\text{H}_3\text{O}^+] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-2}} = 1.0 \times 10^{-12} = 0.000000000001 \text{ mol/L}$$

# The pH Scale

## Strength of Acids and Bases

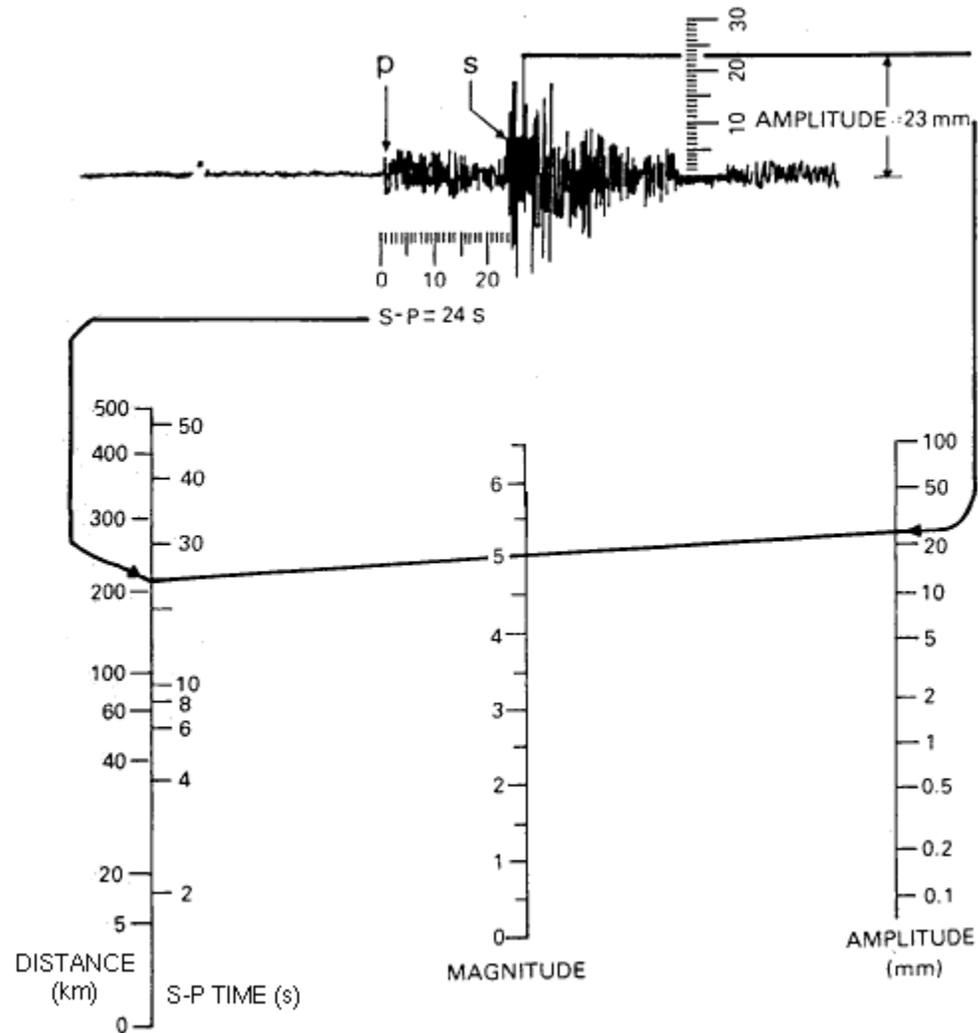
$$\text{pH} = -\log[\text{H}_3\text{O}^+]$$



## Other Logarithmic Scales – The Richter Scale

Magnitude	Earthquake Severity
Less than 3.5	Generally not felt, but recorded.
3.5-5.4	Often felt, but rarely causes damage.
Under 6.0	At most slight damage to well-designed buildings. Can cause major damage to poorly constructed buildings over small regions.
6.1-6.9	Can be destructive in areas up to about 100 kilometers across where people live.
7.0-7.9	Major earthquake. Can cause serious damage over larger areas.
8 or greater	Great earthquake. Can cause serious damage in areas several hundred kilometers across.

$$M_L = \log_{10}A(mm) + (\text{Distance correction factor})$$



# Calculating pH

What is the pH of a 0.10 M solution of HCl?

$$[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1} \text{ mol/L}$$

A logarithm is an exponent, so  $\text{pH} = -\log(1.0 \times 10^{-1}) = 1$ .

What is the pH of a 0.010 M solution of NaOH?

$$[\text{H}_3\text{O}^+] = 1.0 \times 10^{-12} \text{ mol/L}$$

A logarithm is an exponent, so  $\text{pH} = -\log(1.0 \times 10^{-12}) = 12$ .

# Why do supply chain managers care about pH?

Stability of products during production, distribution, and after sale.

Safety hazards – consumer health and safety.

- Biohazards – bacteria
- Explosions
- Burns to skin, eyes, lungs

Environmental hazards

- Acid rain
- Fish kills

## Logarithmic Models

**Model 1: pH**

**pH -**

**pH Scale -**

**Acid -**

**Base -**

How to find the pH:

$$pH = -\log[H_3O^+]$$

*What's the pH of water?*

**How do Acids affect a solution?**

Acids dissolved in water increase the concentration of  
-----:  $HCl + H_2O \Leftrightarrow H_3O^+ + Cl^-$

**How do Bases affect a solution?**

Bases dissolved in water increase the concentration of  
-----:  
 $NaOH \Leftrightarrow Na^+ + OH^-$

*Regardless of which ion increases, the product of the two remains the same -----*

**Problem 1:**

Suppose we have a 0.10 M solution of HCl. This means that  $[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1} \text{ mol/L}$ .

3. What is the pH?

4. What is  $[\text{OH}^-]$ ?

**Problem 2:**

Suppose we have a 0.010 M solution of NaOH. This means that  $[\text{OH}^-] = 1.0 \times 10^{-2} \text{ mol/L}$ .

3. What is  $[\text{H}_3\text{O}^+]?$

4. What is the pH?

**Model 2: Richter scale**

On the Richter scale, the magnitude of R of an earthquake of intensity I is  $R = \log(I / I_0)$ , where  $I = 1$  is the minimum intensity used for comparison. What is the Magnitude of an earthquake with intensity 80,000,000?

## Logarithmic Models

### **Model 1: pH**

**pH** - the measure of acidity of a solution; pH is a measurement of the concentration of hydrogen ions ( $H^+$ ) in a solution.

**pH Scale** - an inverse [logarithmic](#) representation of hydrogen ion ( $H^+$ ) concentration.

Note: Since it is a logarithmic scale, and not a linear scale, each individual pH unit is a factor of 10 different than the next higher or lower unit.

**Acid** -  $< 7$  on the pH scale

Examples: Vinegar, Lemon Juice, Nitric Acid (car battery acid), Cleaning products (HCl), Acid Rain

**Base** -  $> 7$  on the pH scale

Examples: Drano, Windex, Chlorox, Baking Soda

### **Demonstrations:**

Volcano (baking soda)

pH paper (disappearing ink)

### How to find the pH:

$$pH = -\log[H_3O^+]$$

*What's the pH of water?*

Neutral (7) so it's both an acid and base

### **Concentration of solutions:**

- Given in moles per liter
- Avagadro's number =  $6.02 \times 10^{23}$
- Mole of water = tablespoon
- The population of the world is 6,553,751,253.
  - Suppose we split a mole of dollars among everyone in the world. If we all spend a mole of dollars at \$1,000,000 per day, it would take us more than 250,000 years to spend it all.

### **How do Acids affect a solution?**

Acids dissolved in water increase the concentration of hydronium



### How do Bases affect a solution?

Bases dissolved in water increase the concentration of hydroxide ions:



*Regardless of which ion increases, the product of the two remains the same ( $1.0 \times 10^{-14}$ )*

#### Problem 1:

Suppose we have a 0.10 M solution of HCl. This means that  $[\text{H}_3\text{O}^+] = 1.0 \times 10^{-1}$  mol/L.

5. What is the pH?

$$pH = -\log(1.0 \times 10^{-1})$$

6. What is  $[\text{OH}^-]$ ?

$$\text{We know that } [\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

$$\text{Substituting } [\text{H}_3\text{O}^+] = 1.0 \times 10^{-1} \text{ mol/L}$$

$$(1.0 \times 10^{-1})[\text{OH}^-] = 1.0 \times 10^{-14}$$

$$[\text{OH}^-] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-1}} = 1.0 \times 10^{-13} = 0.0000000000001 \text{ mol/L}$$

#### Problem 2:

Suppose we have a 0.010 M solution of NaOH. This means that  $[\text{OH}^-] = 1.0 \times 10^{-2}$  mol/L.

5. What is  $[\text{H}_3\text{O}^+]$ ?

$$\text{We know that } [\text{H}_3\text{O}^+][\text{OH}^-] = 1.0 \times 10^{-14}$$

$$\text{Substituting } [\text{OH}^-] = 1.0 \times 10^{-2} \text{ mol/L}$$

$$[\text{H}_3\text{O}^+] (1.0 \times 10^{-2}) = 1.0 \times 10^{-14}$$

$$[\text{H}_3\text{O}^+] = \frac{1.0 \times 10^{-14}}{1.0 \times 10^{-2}} = 1.0 \times 10^{-12} = 0.000000000001 \text{ mol/L}$$

6. What is the pH?

$$pH = -\log(0.000000000001)$$

#### Experimentation:

Now let's test various pH's and determine what the  $[\text{H}_3\text{O}^+]$  concentration is of each.

*Use the formula for pH ( $pH = -\log[\text{H}_3\text{O}^+]$ ) and what we know about solving log equations to find the missing piece.*

#### Model 2: Richter scale

On the Richter scale, the magnitude of R of an earthquake of intensity I is  $R = \log(I / I_0)$ , where  $I = 1$  is the minimum intensity used for comparison. What is the Magnitude of an earthquake with intensity 80,000,000?

<b>Fluid</b>	<b>pH</b>	<b>Acid/Base/Neutral</b>	<b>Log Equation</b>	<b>OH+</b>
<b>AST</b>				
Engine Coolant				
Brake Fluid				
Heavy Duty Hand Soap				
<b>Cosmetology</b>				
Perm Solution				
Neutralizer				
Shampoo				
Sink Sanitizer				
<b>Foods</b>				
Coca-Cola				
Diet Coke				
Fryer Oil				
Vinegar				
Worcestershire Sauce				
Orange Juice				
Counter Sanitizer				
<b>A+/Net+</b>				
CPU Adhesive Prep				
<b>Child Care</b>				
Sanitizer (Diaper Station)				
<b>Medical Careers</b>				
Hand Sanitizer				
<b>McKenzie</b>				
Hand Soap				
Coffee				

## Intersection of Circles: Global Positioning System

**Course:** Algebra 2 and/or Pre-Calculus

### **Indiana Standards:**

A1.8.6 – Solve quadratic equations using the quadratic formula

A1.8.7 – Use quadratic equations to solve word problems

A2.3.3 – solve quadratic equations in the complex number system

A2.3.5 – Solve word problems using quadratic equations

A2.4.1 – Write the equation of conic sections

A2.4.2 – Graph conic sections

### **Objective:**

Students will be able to connect the intersection of circles to the real-world application of Global Positioning Systems

### **Materials:**

- Graph Paper
- Graphing Software (optional)
- Graphing Calculator

### **Job Connection:**

- U.S. Xpress Enterprises, Inc. – [www.usxpress.com](http://www.usxpress.com)
  - **Fleet Manager** - Manage a fleet of designated drivers and tractors. Maintain relationships with the Dispatch and Customer Service personnel to ensure company goals are met. In addition, manages fleet uptime and make capacity availability, manage driver performance and promote safe operations.

**Vocabulary:**

Global Positioning System (GPS) - Global navigation system based on a constellation of 24 satellites orbiting the earth

**Input:**

Global Positioning System (GPS)

- *Global navigation system accessible to everyone*
- Created by United States Department of Defense (\$12 billion)
- Based on a constellation of 24 satellites orbiting the earth at very high altitude
- Impervious to jamming and interference

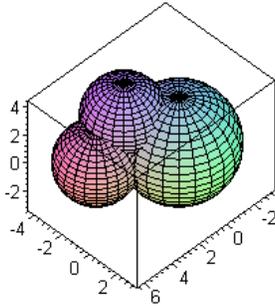
How does GPS work?

- Satellite Ranging - Measure distance from group of satellites (3 or 4 satellites)

How is distance to a satellite measured?

- Velocity times travel time  
(rate  $\times$  time = distance)
- Time how long it takes for a radio signal to reach a receiver on earth and calculate distance from time.
- Radio signals travel at the speed of light (186,000 miles per second)

So once I know the distance to 3 satellites, how do I know my location?



All points at a given distance from a point (satellite) are on a sphere.

The intersection of the three spheres is your location.  
How do you find the intersection of three spheres?

Let's start with a simpler 2-dimensional problem:

A circle is a 2-dimensional sphere.

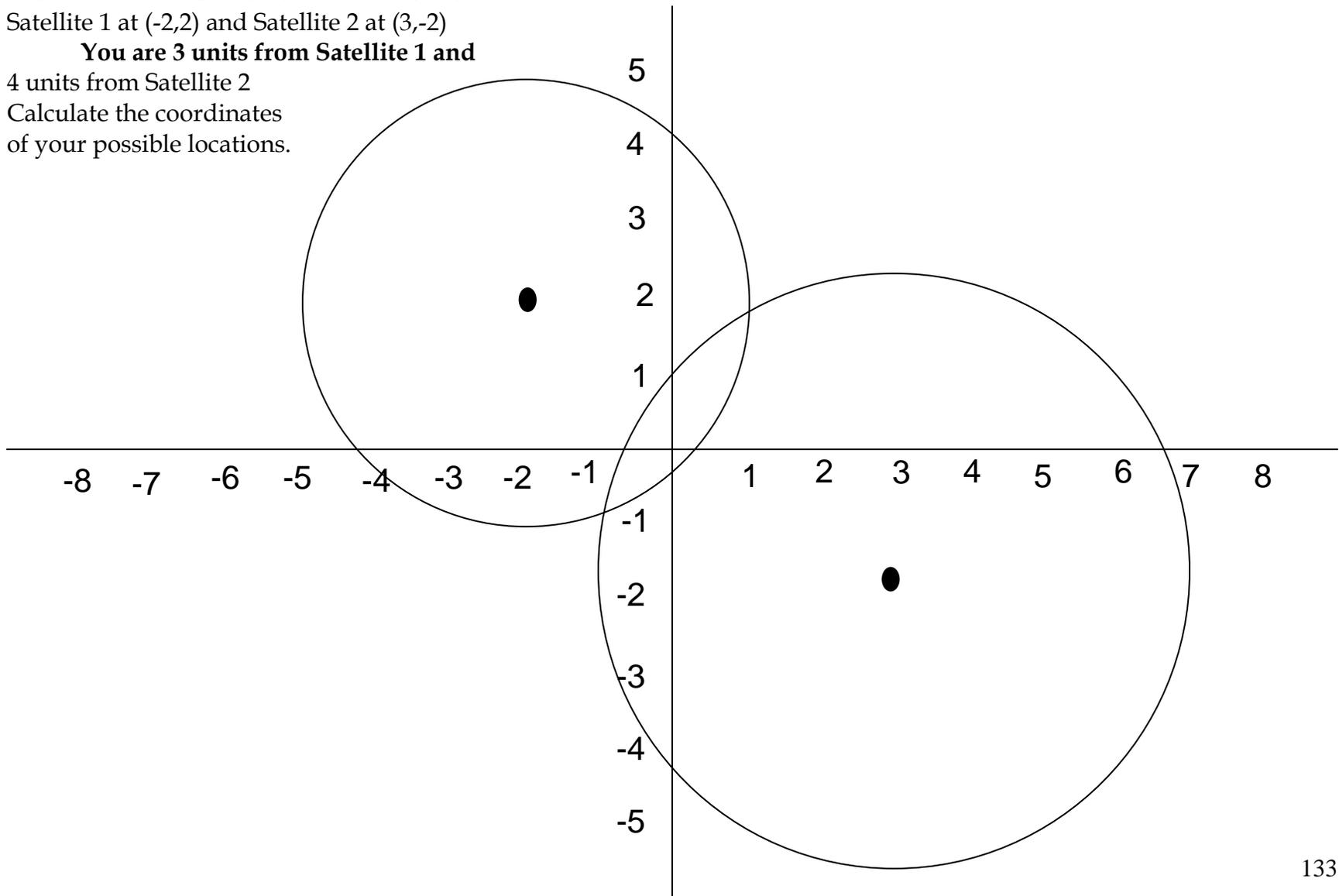
Let's find the intersection of two circles first.

Satellite 1 at  $(-2, 2)$  and Satellite 2 at  $(3, -2)$

**You are 3 units from Satellite 1 and**

4 units from Satellite 2

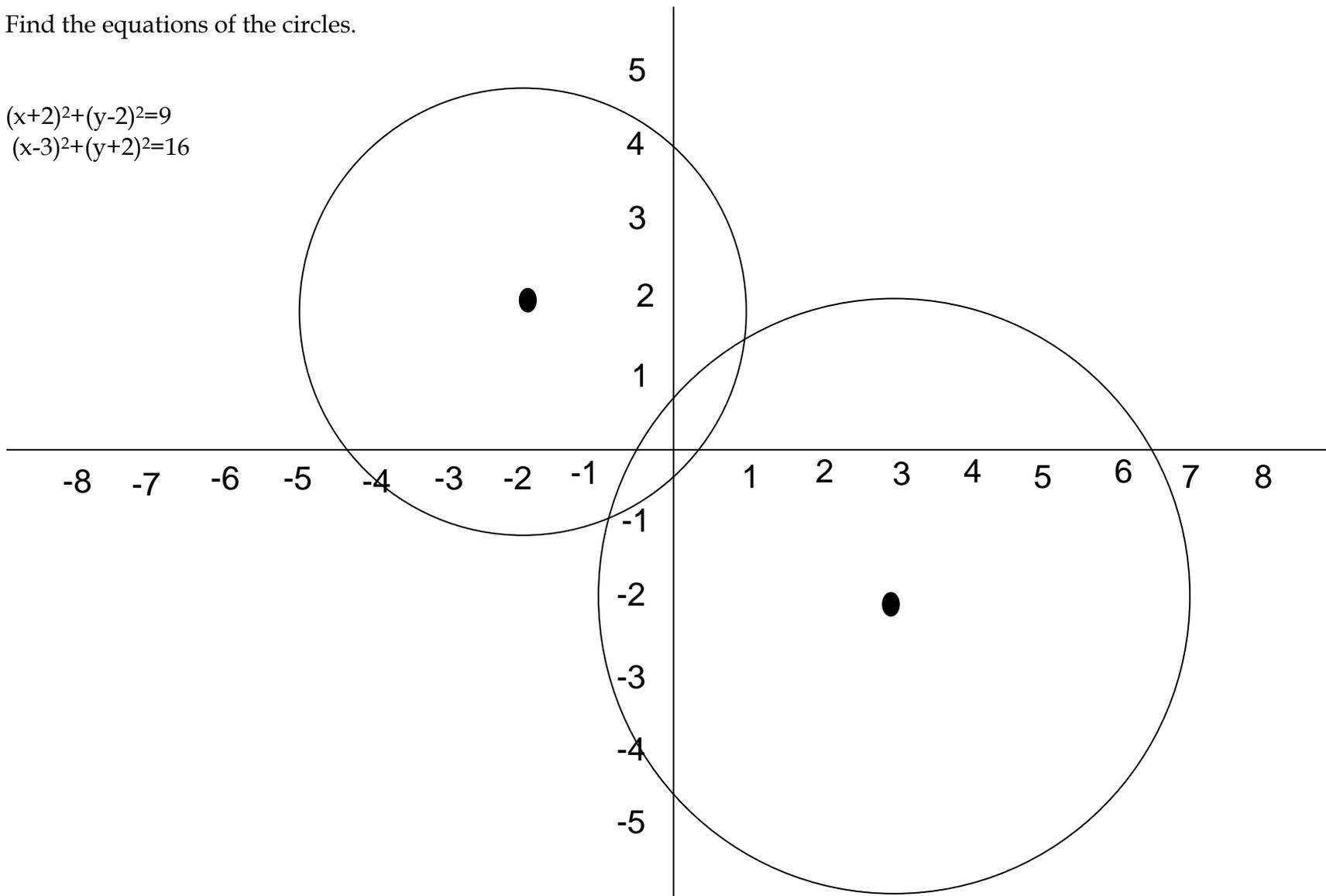
Calculate the coordinates  
of your possible locations.



Find the equations of the circles.

$$(x+2)^2+(y-2)^2=9$$

$$(x-3)^2+(y+2)^2=16$$



Square the quantities in parentheses:

Equation 1:  $(x+2)^2+(y-2)^2=9$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 9$$

$$x^2 + 4x + y^2 - 4y - 1 = 0$$

Equation 2:  $(x-3)^2+(y+2)^2=16$

$$x^2 - 6x + 9 + y^2 + 4y + 4 = 16$$

$$x^2 - 6x + y^2 + 4y - 3 = 0$$

Subtract equation 1 from equation 2:

$$x^2 - 6x + y^2 + 4y - 3 = 0$$

$$\underline{- (x^2 + 4x + y^2 - 4y - 1 = 0)}$$

$$-10x + 8y - 2 = 0$$

$$4y = 5x + 1$$

$$y = 5x/4 + 1/4$$

Substitute  $y = 5x/4 + 1/4$  into equation 1:

$$x^2 + 4x + y^2 - 4y - 1 = 0$$

$$x^2 + 4x + (5x/4 + 1/4)^2 - 4(5x/4 + 1/4) - 1 = 0$$

$$x^2 + 4x + (25x^2/16 + 10x/16 + 1/16) - (5x + 1) - 1 = 0$$

$$41x^2/16 - 3x/8 - 2 = 0$$

Solve using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

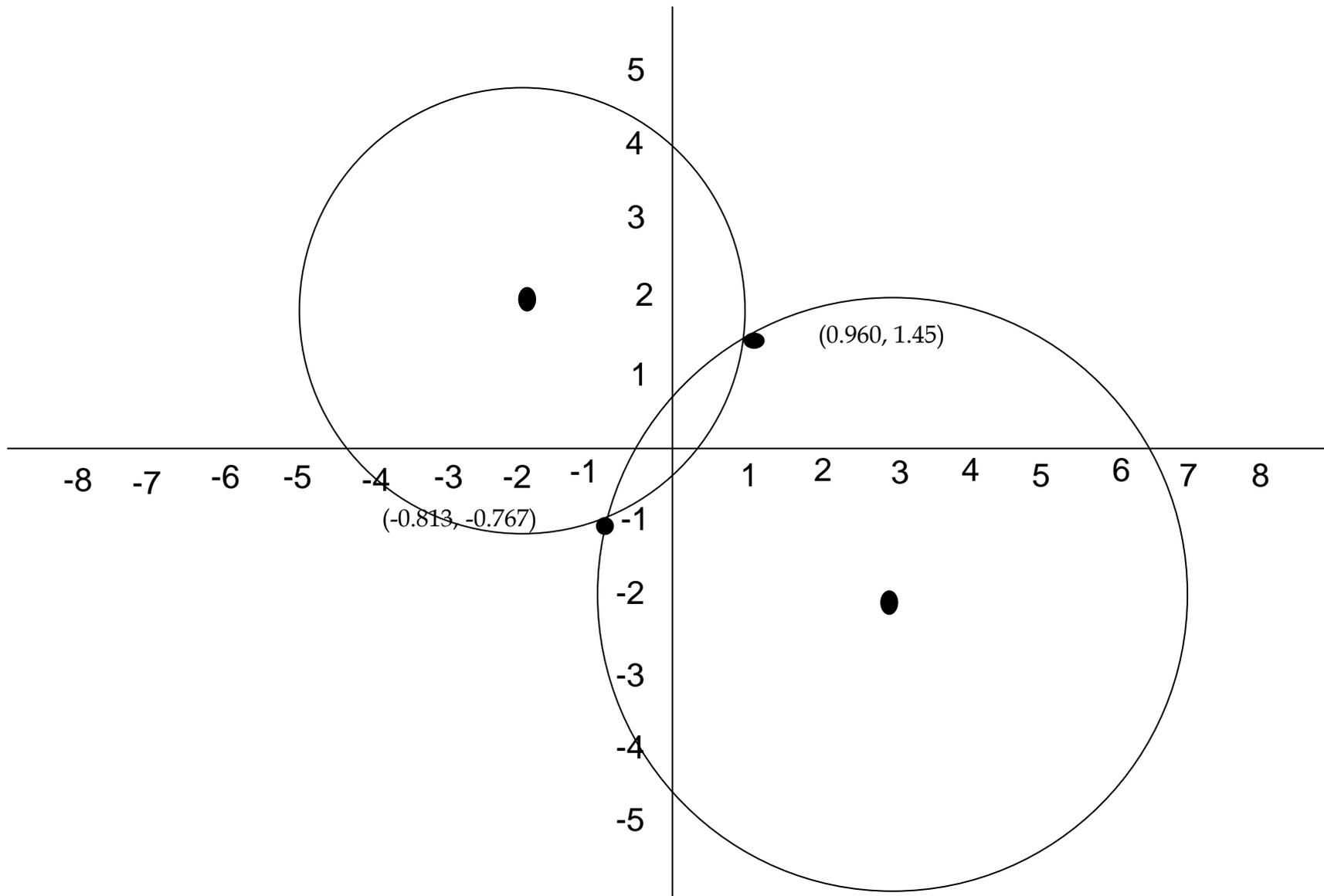
$$\begin{aligned} x &= \frac{-\frac{3}{8} \pm \sqrt{\frac{3}{8}^2 - 4(\frac{41}{16})(-2)}}{2(\frac{41}{16})} \\ &= 0.960 \text{ and } -0.813 \end{aligned}$$

Substituting  $x = 0.960$  and  $-0.813$  into  $y = 5x/4 + 1/4$ :

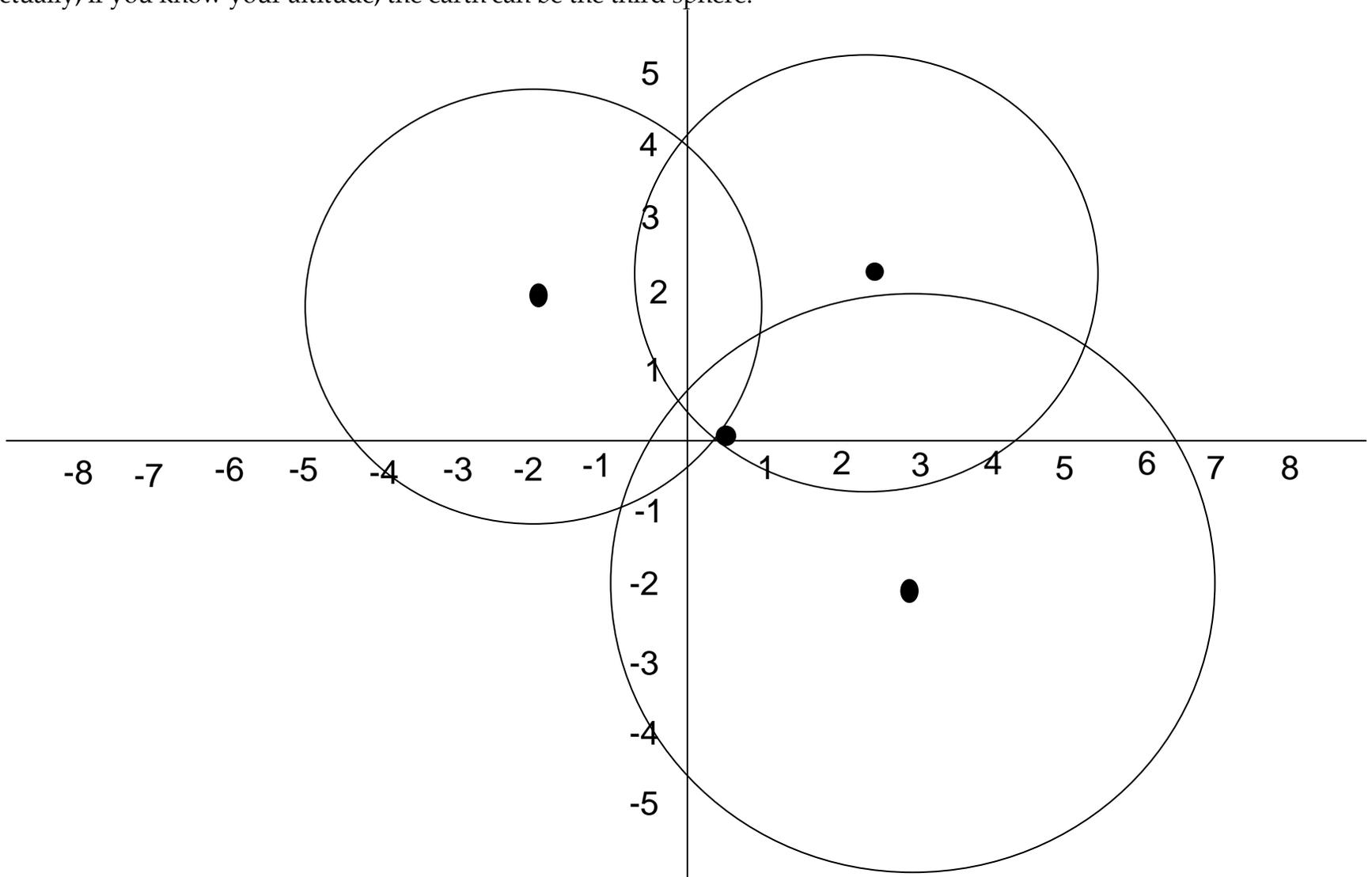
$$y = 5(0.960)/4 + 1/4 = 1.45$$

$$y = 5(-0.813)/4 + 1/4 = -0.767$$

The two solutions are the points:  
(0.960, 1.45) and (-0.813, -0.767)



So this narrows us down to two locations. How do we get down to one location?  
This is where we need the third circle 2 dimensions or sphere in 3 dimensions.  
Actually, if you know your altitude, the earth can be the third sphere.



**Assessment:**

Students will complete a handout with a similar problem as the one introduced in the lesson

**Additional Documents:**

CirclesGPSHomework.doc

CirclesGPSSlides.doc

CirclesGPSTest.doc

# **Global Positioning System Intersection of Circles**

Supply Chain Management Group  
University of Indianapolis  
School of Business

Leslie Gardner, Ph.D.,  
Professor of Operations Management and Mathematics

# Global Positioning System (GPS)

- *Global navigation system accessible to everyone*
- Created by United States Department of Defense (\$12 billion)
- Based on a constellation of 24 satellites orbiting the earth at very high altitude
- Impervious to jamming and interference

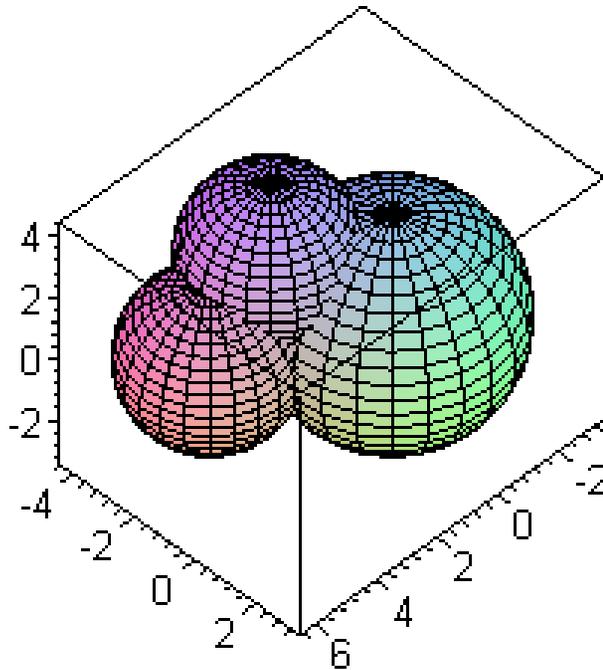
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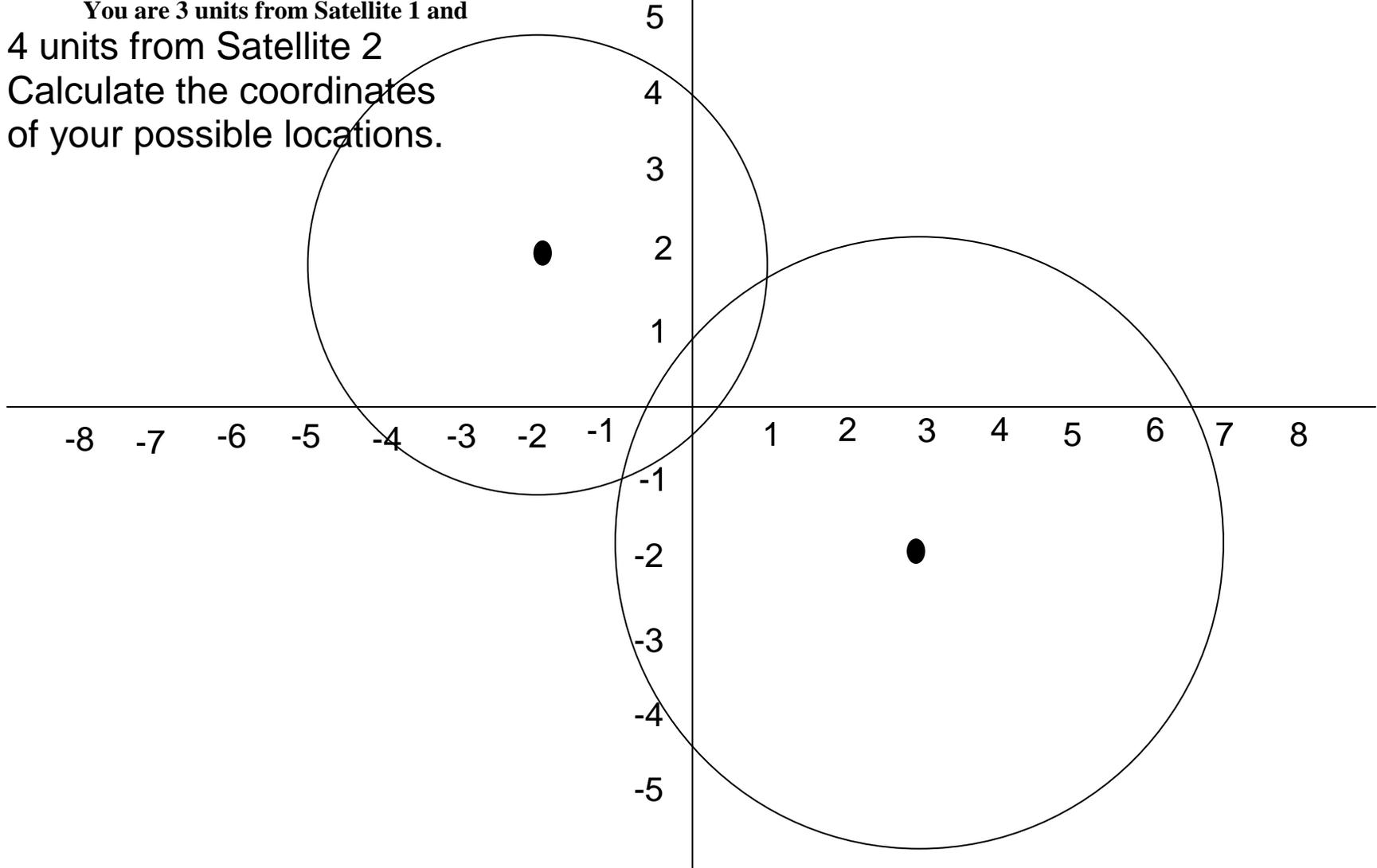
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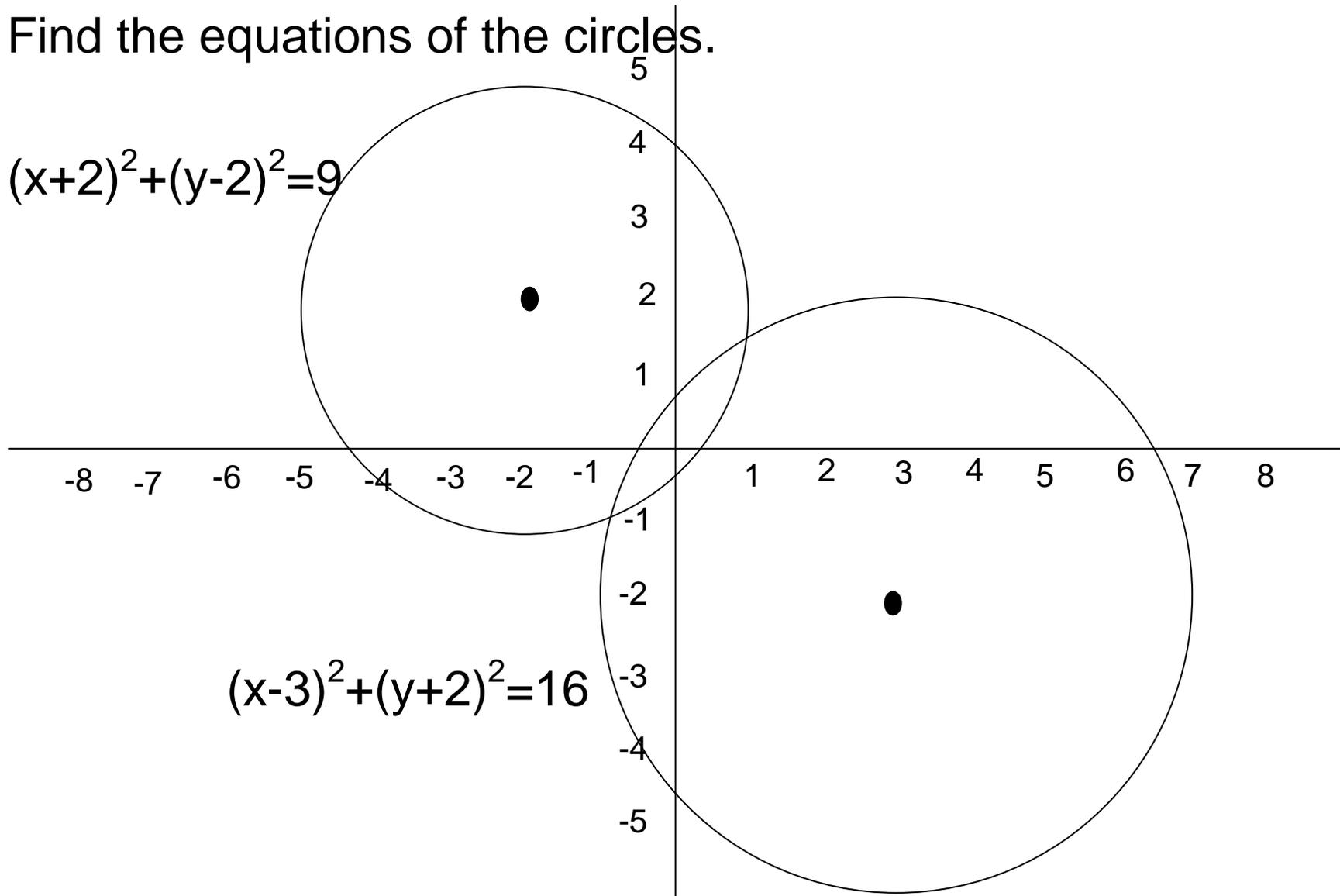
Satellite 1 at  $(-2, 2)$  and Satellite 2 at  $(3, -2)$

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Subtract equation 1 from equation 2:

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$$- (x^2 + 4x + y^2 - 4y - 1 = 0)$$

---

$$- 10x + 8y - 2 = 0$$

$$4y = 5x + 1$$

$$y = 5x/4 + 1/4$$

Substitute  $y = 5x/4 + 1/4$  into equation 1:

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$$x^2 + 4x + (5x/4 + 1/4)^2 - 4(5x/4 + 1/4) - 1 = 0$$

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$$41x^2/16 - 3x/8 - 2 = 0$$

Solve using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

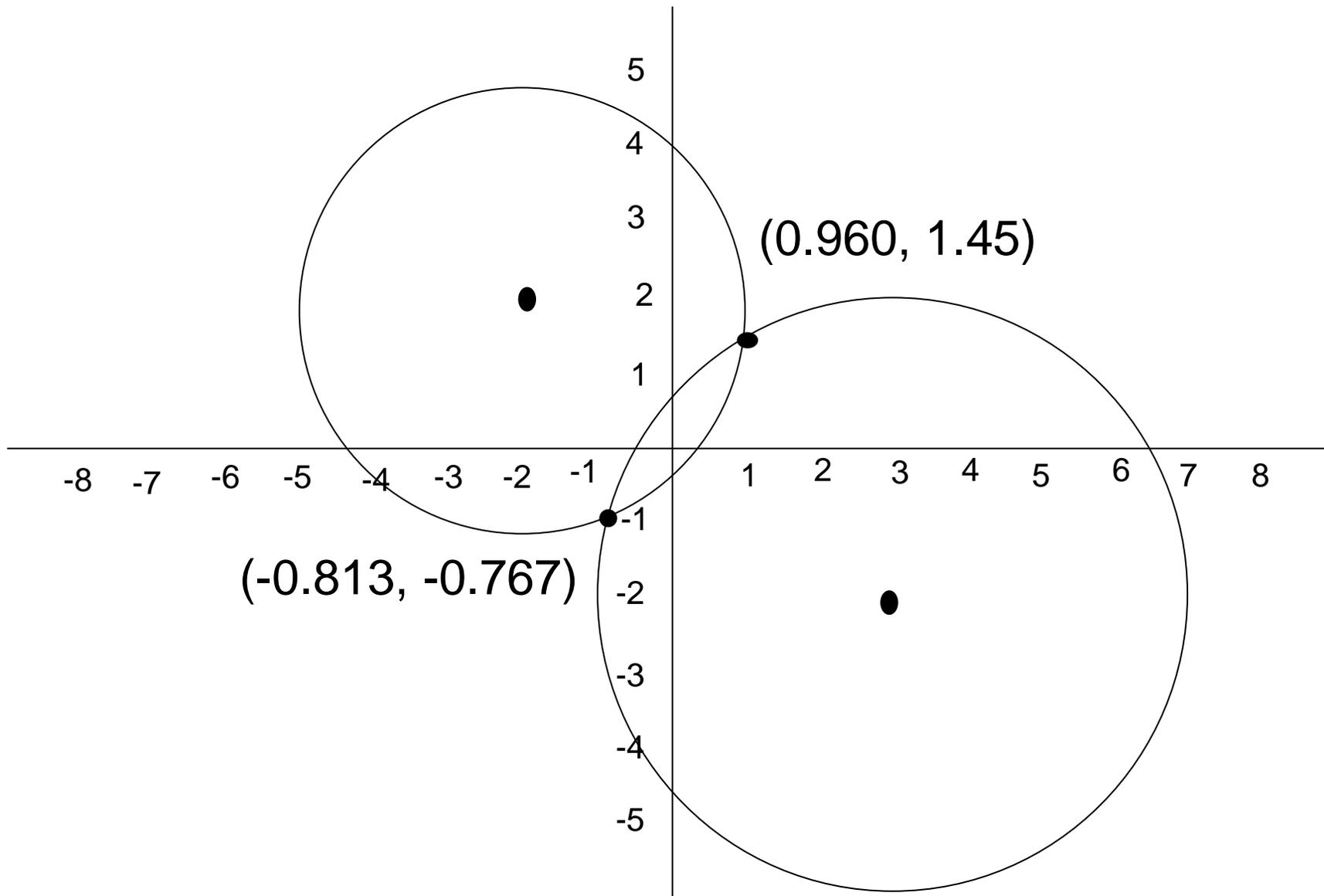
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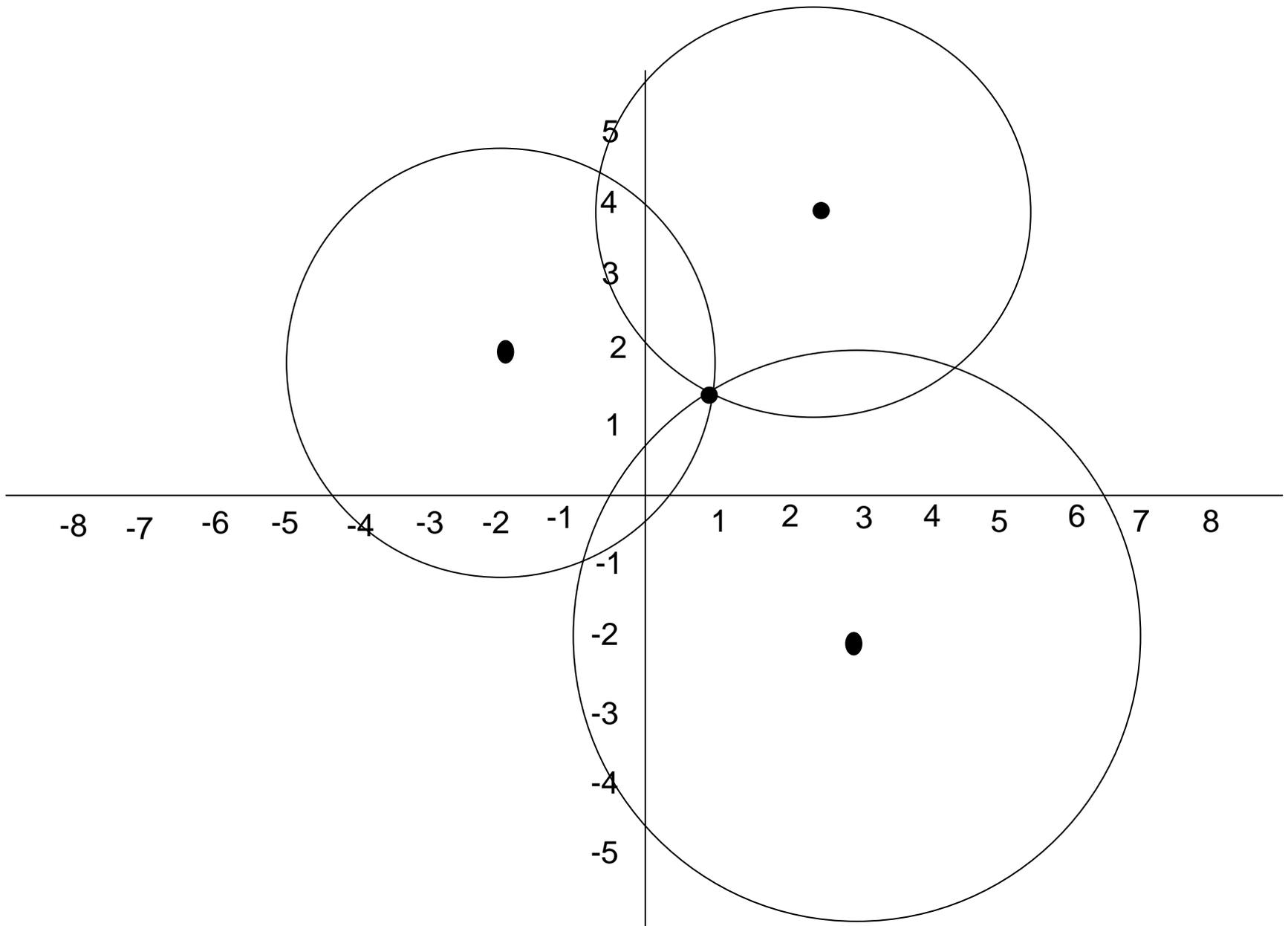
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So this narrows us down to two locations. How do we get down to one location?

This is where we need the third circle 2 dimensions or sphere in 3 dimensions.

Actually, if you know your altitude, the earth can be the third sphere.



## GPS Homework

1. Satellite 1 at  $(-2,3)$  and Satellite 2 at  $(4,-2)$ . You are 2 units from Satellite 1 and 3 units from Satellite 2. Calculate the coordinates of your possible locations. Include your graphs.
2. Satellite 1 at  $(3,-3)$  and Satellite 2 at  $(-2,-2)$ . You are 2 units from Satellite 1 and 2 units from Satellite 2. Calculate the coordinates of your possible locations. Include your graphs.
3. Satellite 1 at  $(-1,4)$  and Satellite 2 at  $(2,-4)$ . You are 3 units from Satellite 1 and 3 units from Satellite 2. Calculate the coordinates of your possible locations. Include your graphs.

## GPS Test Problems

1. Satellite 1 at  $(-2,3)$  and Satellite 2 at  $(-4,-2)$ . You are 3 units from Satellite 1 and 3 units from Satellite 2. Calculate the coordinates of your possible locations. Include your graphs.

## Step Functions: Quantity Discounts

**Course:** Algebra 2 and/or Pre-Calculus

**Indiana Standards:**

A2.1.7 – Graph functions defined piece-wise.

**Objective:**

Students will analyze the use of piece-wise step functions in order to purchase different items.

**Materials:**

- Calculator
- Pricing Information Sheet

**Job Connection:**

- Zimmer – [www.zimmer.com](http://www.zimmer.com)
  - Through innovation, we have become a worldwide leader in orthopedic surgical products. The Zimmer team is more than 7,000 employees strong, and we are dedicated to producing top quality products and services that make a difference in our communities. We are proud that we help people every day to live fuller and healthier lives. Additionally, our employees enjoy the many benefits of working for a growing, successful company, including exciting career opportunities and a competitive total rewards program.
  - **Pricing Analyst** – Prepare reports and analyze Zimmer sales, average selling price, and price impact analysis as necessary. Responsible for company level contract compliance review and reporting. Develop and maintain appropriate reports and analysis tools to be used by field organization in applying good contract management principles. Prepare, report, and analyze monthly sales rebates and administrative fees. Assist in administering and analyzing contracts with national group purchasing organizations, regional buying groups, integrated health systems, and individual facilities as necessary. Provide technical expertise for enterprise systems and sales reporting software.

**Vocabulary:**

Piecewise Function: A function whose definition is given differently on disjoint subsets of its domain

Holding Costs: Money spent to keep and maintain a stock of goods in storage (e.g. insurance, cost of capital)

Board Foot: The volume of a one foot length of board one foot wide and one inch thick

Ordering Costs: A function of the number of orders per year

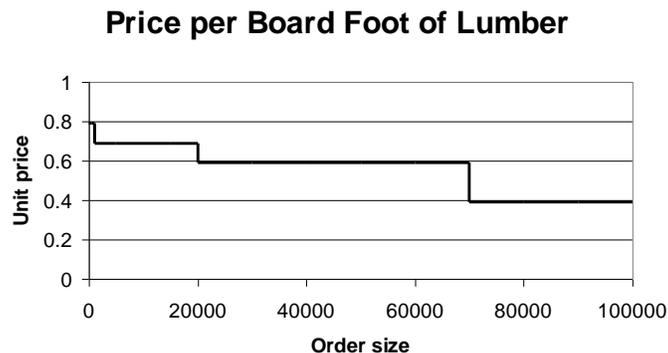
**Input:**

- When ordering mass quantities of items, a different type of function is used to determine prices. Usually a discount is given for buying more of a certain product at one time.
  - What kind of benefits come from ordering in bulk?
  - What are the downfalls?
- Remember, piecewise functions are defined between certain x-values
  - A special type of piecewise function is called the step function (because it looks like steps)
  - Look at a sample graph of a piecewise - step function.
- Today we're going to work with step and piecewise functions in order to buy lumber for a company
- Example 1: A contractor uses 40,000 board feet of a certain type of lumber each year. The lumber prices depend on the order size.

Price per board foot	Order size in board feet
\$0.79	0-999
0.69	1,000-19,999
0.59	20,000-69,999
0.39	More than 70,000

The holding costs (insurance, cost of capital) are 5% of the value of the lumber in inventory. Average inventory is half the order size. It costs about \$100 to place an order. In what quantities should the contractor buy lumber?

- Here's the graph of the functions and the corresponding equation

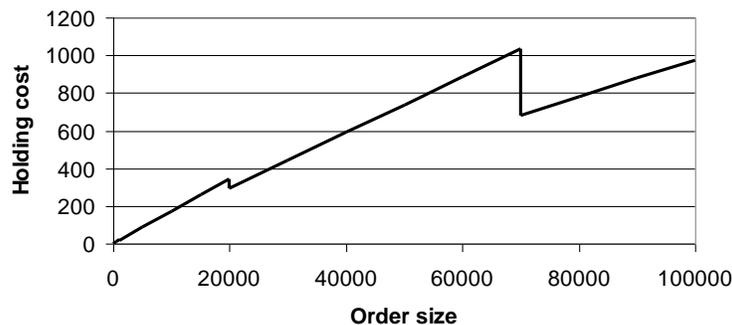


Price is a step function of order size  $x$ .

$$f(x) = \begin{cases} \$0.79 & \text{if } x < 1,000 \\ \$0.69 & \text{if } 1000 \leq x < 20,000 \\ \$0.59 & \text{if } 20,000 \leq x < 70,000 \\ \$0.39 & \text{if } 70,000 \leq x \end{cases}$$

- In addition to the actual price of lumber, there is an additional holding cost that is modeled in the graph below.

### Holding Cost



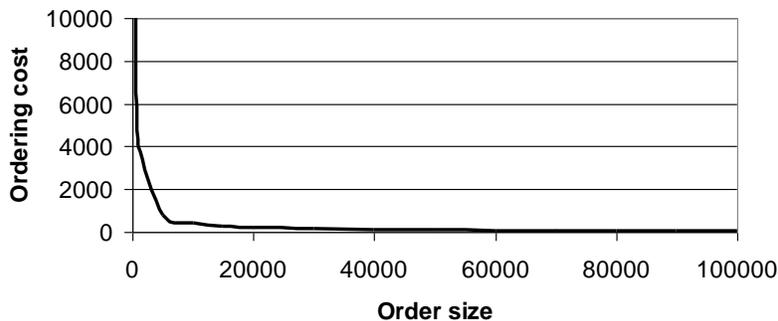
Holding costs are the costs involved in keeping inventory on hand. They include insurance, rent on storage space and the interest paid on the money to buy the inventory. In this case they are 5% of the average value of the lumber in inventory.

$$\text{Holding costs} = (0.05)(\text{unit price})(\text{order size})/2$$

Holding costs are a step function because unit price is a step function.

- Finally, we have to look at the actual cost to place an order.

### Ordering Cost

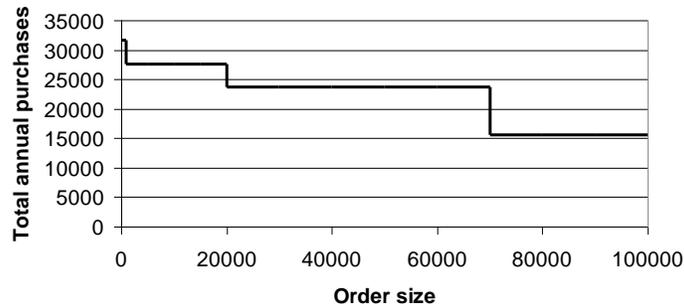


Every time an order is placed and received, it requires time that people could be spending on something else. It may require postage or internet charges. Ordering cost is a function of the number of orders per year.

The number of orders per year is  $\frac{40,000}{x}$  where  $x$  is the order size.

- Let's also take a look at the Total Annual Purchase of Lumber graph

**Total Annual Purchases of Lumber**

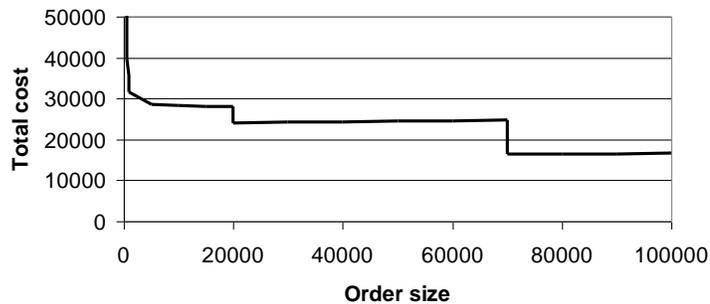


$$\begin{aligned} \text{The total annual purchases} &= (\text{demand})(\text{unit price}) \\ &= 40,000(\text{unit price}) \end{aligned}$$

This is also a step function because unit price is a step function.

- So, now we have to put all of our data together in one huge piece-wise graph...

**Total Cost**



You should make your decision on order size based on the least total annual cost.

Annual Total Cost = Holding cost + Order cost + Total annual purchases

How much should you order? Lowest point:  $x = 70,000$

How often will you order? 1.75 years, 1 yr 9 months

How many orders per year? .57

Are you concerned about placing orders so far apart? What are potential problems with this?

**Assessment:** Students will complete a homework assignment in which they will be given a total cost function and they must determine the best decision for their company.

**Additional Documents:**

StepFuncQuantDiscSlides.doc

StepFuncQuantDiscSolver.xls

StepFuncQuantDiscWorksheet.doc

StepFuncQuantDiscHomework.doc

StepFuncQuantDiscHomeworkKey.doc

# **Quantity Discounts - Step Functions**

Supply Chain Management Group  
University of Indianapolis  
School of Business

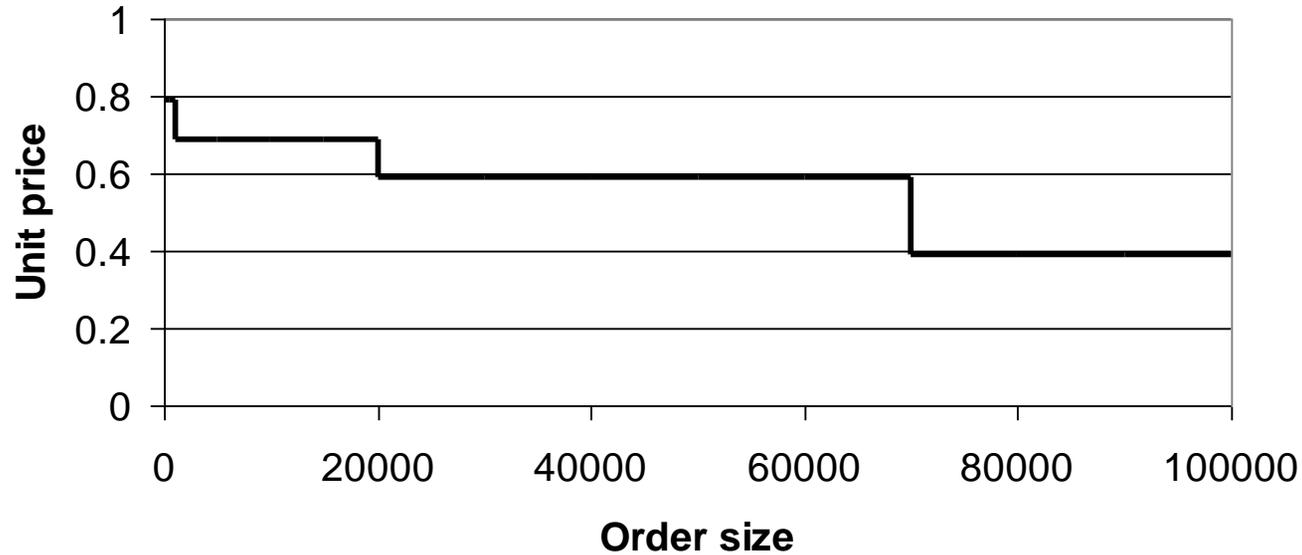
Leslie Gardner, Ph.D., Professor

A contractor uses 40,000 board feet of a certain type of lumber each year. The lumber prices depend on the order size:

Price per board foot	Order size in board feet
\$0.79	0-999
0.69	1,000-19,999
0.59	20,000-69,999
0.39	More than 70,000

The holding costs (insurance, cost of capital) are 5% of the value of the lumber in inventory. Average inventory is half the order size. It costs about \$100 to place an order. In what quantities should the contractor buy lumber?

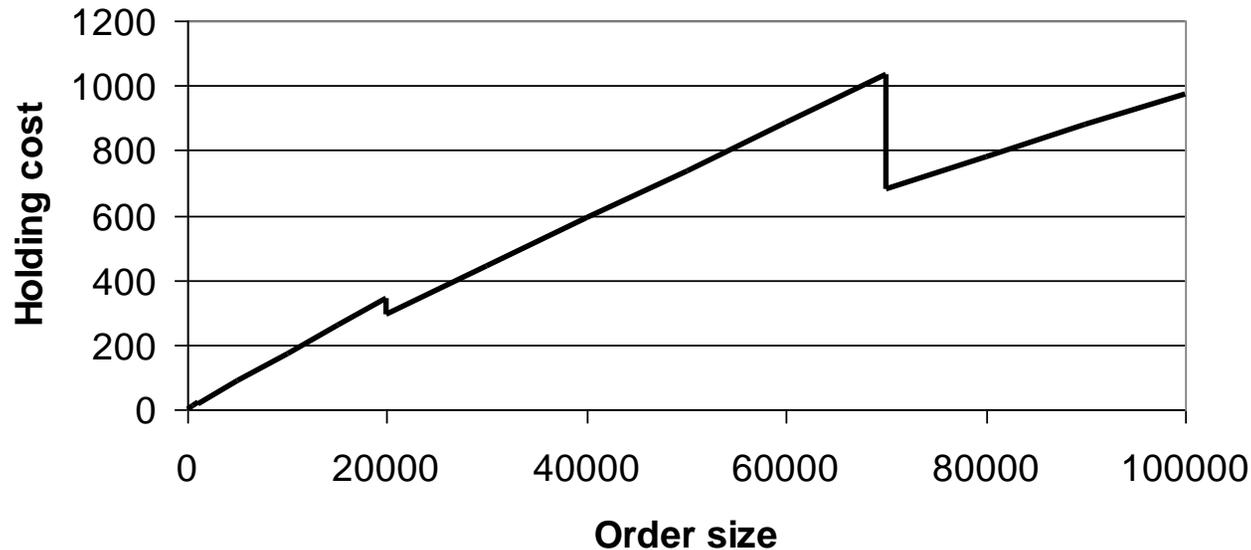
## Price per Board Foot of Lumber



Price is a step function  
of order size  $x$ .

$$f(x) = \begin{cases} \$0.79 & \text{if } x < 1,000 \\ \$0.69 & \text{if } 1,000 \leq x < 20,000 \\ \$0.59 & \text{if } 20,000 \leq x < 70,000 \\ \$0.39 & \text{if } 70,000 \leq x \end{cases}$$

## Holding Cost

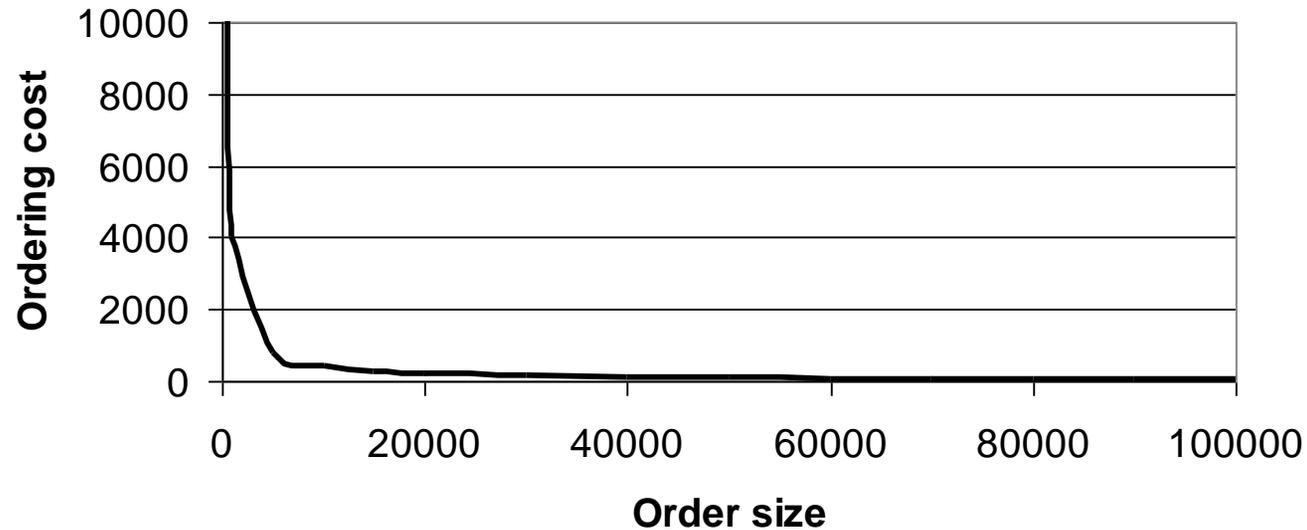


Holding costs are the costs involved in keeping inventory on hand. They include insurance, rent on storage space and the interest paid on the money to buy the inventory. In this case they are 5% of the average value of the lumber in inventory.

$$\text{Holding costs} = (0.05)(\text{unit price})(\text{order size})/2$$

Holding costs are a step function because unit price is a step function.

## Ordering Cost

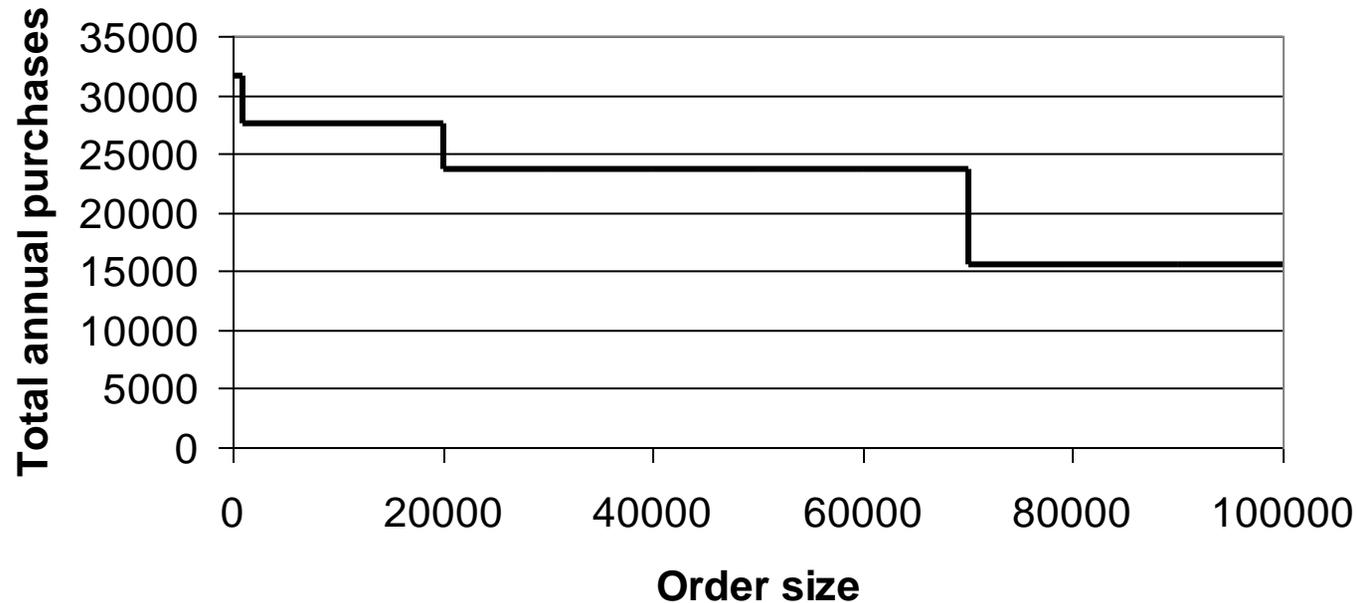


Every time an order is placed and received, it requires time that people could be spending on something else. It may require postage or internet charges. Ordering cost is a function of the number of orders per year.

The number of orders per year is  $\frac{40,000}{x}$  where  $x$  is the order size.

Notice the shape of the graph.

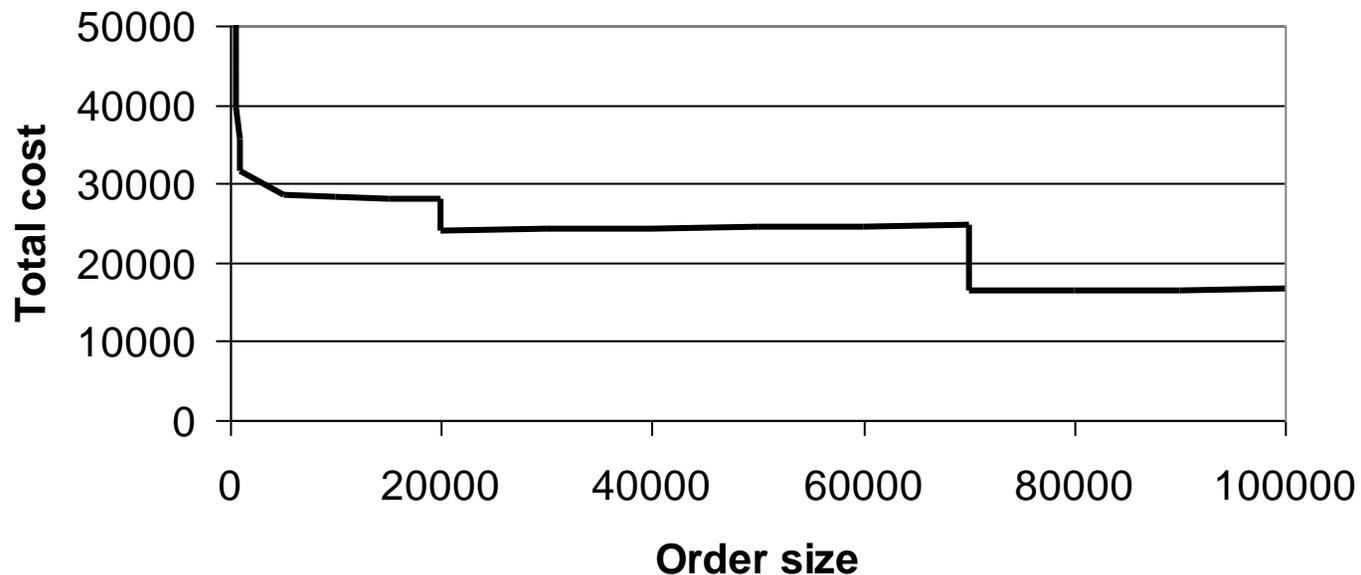
## Total Annual Purchases of Lumber



The total annual purchases = (demand)(unit price)  
= 40,000(unit price)

This is also a step function because unit price is a step function.

## Total Cost



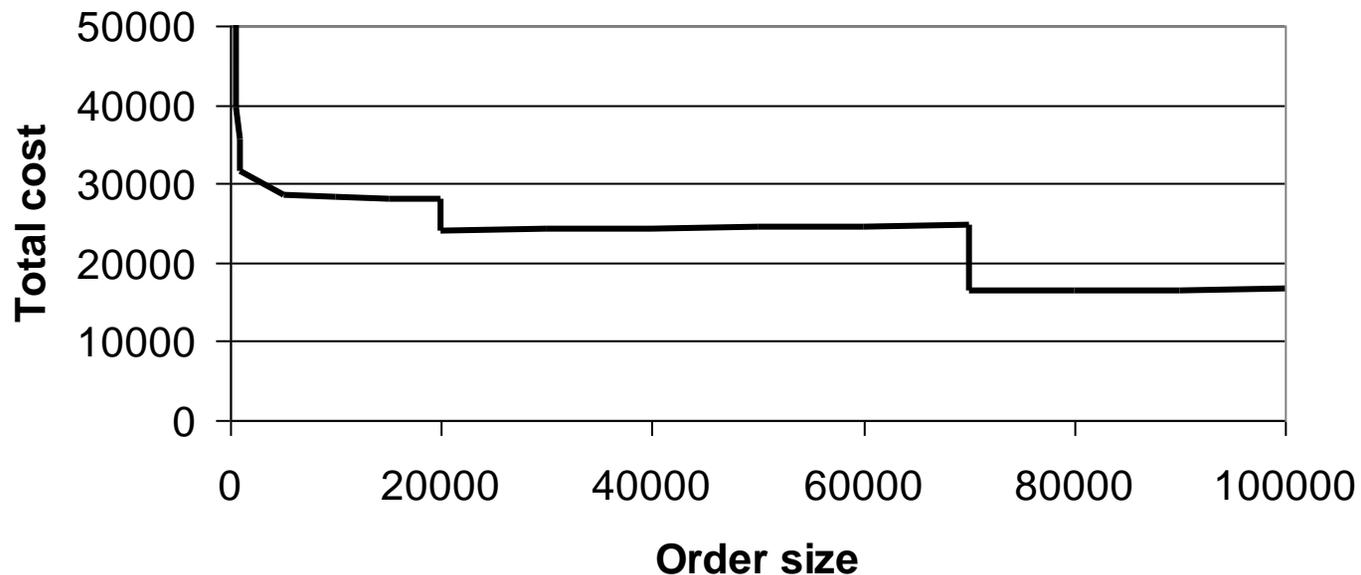
You should make your decision on order size based on the least total annual cost.

Annual Total Cost = Holding cost + Order cost + Total annual purchases

How much should you order?

How often will you order?

## Total Cost



How much should you order? Lowest point:  $x=70,000$

How often will you order?  $70,000/40,000 = 1.75$  years = 1 yr. 9 mo.

How many orders per year? 0.57 orders per year

Are you concerned about placing orders so far apart? What are potential problems with this?

## Step Functions: Quantity Discounts

Discussion:

What kind of benefits come from ordering in bulk?

What are the downfalls?

Piecewise Function:

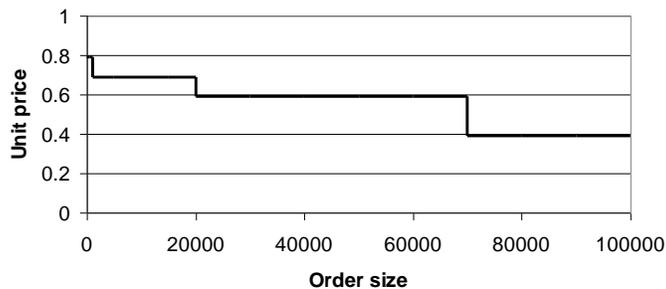
A special type of piecewise function is called the \_\_\_\_\_ (because it looks like \_\_\_\_\_)

**Example 1:** A contractor uses 40,000 board feet of a certain type of lumber each year. The lumber prices depend on the order size.

Price per board foot	Order size in board feet
\$0.79	0-999
0.69	1,000-19,999
0.59	20,000-69,999
0.39	More than 70,000

The holding costs (insurance, cost of capital) are 5% of the value of the lumber in inventory. Average inventory is half the order size. It costs about \$100 to place an order. In what quantities should the contractor buy lumber?

**Price per Board Foot of Lumber**

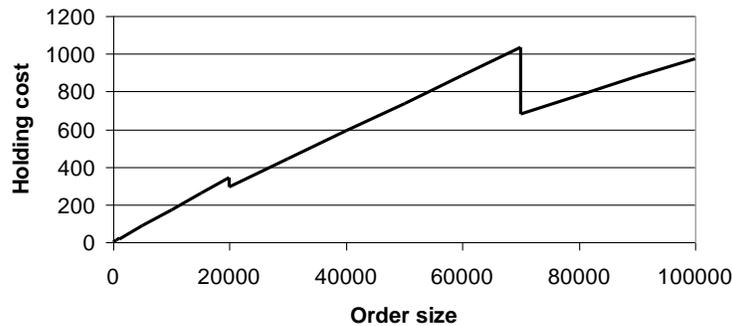


Price is a step function of order size  $x$ .

$$f(x) = \begin{cases} \$0.79 & \text{if } x < 1,000 \\ \$0.69 & \text{if } 1000 \leq x < 20,000 \\ \$0.59 & \text{if } 20,000 \leq x < 70,000 \\ \$0.39 & \text{if } 70,000 \leq x \end{cases}$$

In addition to the actual price of lumber, there is an additional \_\_\_\_\_ that is modeled in the graph below.

### Holding Cost

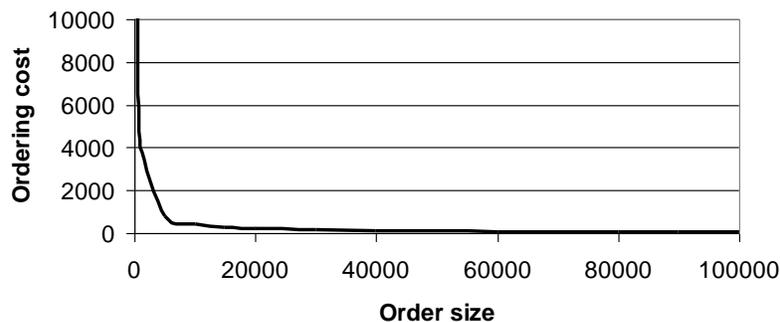


Holding costs are the costs involved in keeping \_\_\_\_\_ on hand. They include \_\_\_\_\_, rent on storage space and the \_\_\_\_\_ paid on the money to buy the inventory. In this case they are \_\_\_\_\_ of the average value of the lumber in inventory.

Holding costs =

Finally, we have to look at the actual cost to place an order.

### Ordering Cost

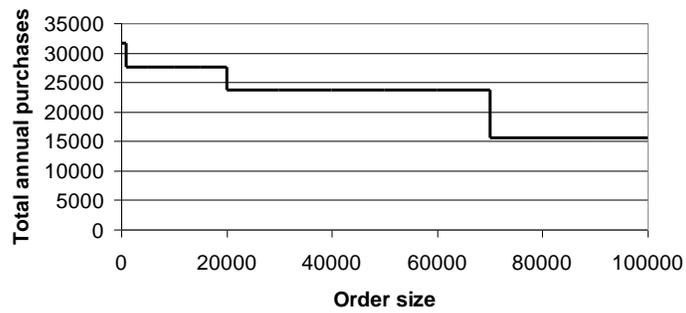


Every time an order is placed and received, it requires \_\_\_\_\_ that people could be spending on something else. It may require \_\_\_\_\_ or internet charges. Ordering cost is a function of the number of orders \_\_\_\_\_.

The number of orders per year is  $\frac{40,000}{x}$  where  $x$  is the order size.

Let's also take a look at the Total Annual Purchase of Lumber graph

**Total Annual Purchases of Lumber**

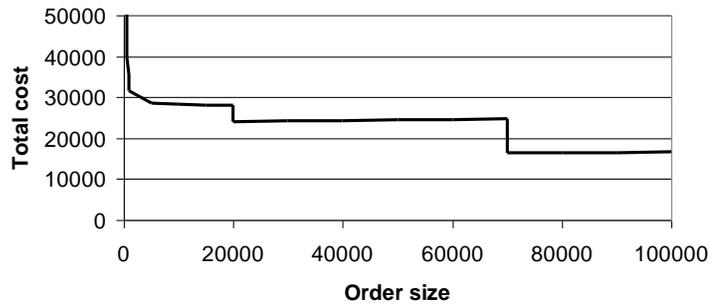


The total annual purchases =

This is also a step function because unit price is a step function.

So, now we have to put all of our data together in one huge piece-wise graph...

**Total Cost**



You should make your decision on order size based on the least total annual cost.

Annual Total Cost =

How much should you order?

How often will you order?

How many orders per year?

Are you concerned about placing orders so far apart? What are potential problems with this?

## Bulk Quantity Discounts and Wholesale Pricing on Party Supplies

We want to help you party, Cheap! PartyCheap.com offers increasing discounts when you buy in bulk quantities. You save so much it is like buying party supplies at wholesale prices.

It's easy to get wholesale pricing on party supplies, here is all you have to do.

1. Find the specific party supply or decoration you are looking to purchase.
2. Click the "View Quantity Discounts" button and you will get a table to pop open that looks something like this.

### Bulk Discount EXAMPLE

Buy...	Save...	New Price...
12 - 23	15%	<b>\$1.63 ea.</b>
24 - 71	20%	<b>\$1.54 ea.</b>
72 or more	30%	<b>\$1.34 ea.</b>

Note: Quantity discounts shown above will be automatically applied to your order.

3. Here you will see the minimum purchases you will need to purchase to receive the bulk discount. The more you purchase the more you save on your party supplies.
4. Purchase at the highest tier level to receive wholesale party supply pricing.

Almost all of our party supplies are eligible for bulk discounts including all of our birthday party supplies. And remember, no matter how much you buy, you can still get free shipping on ground deliveries to the lower 48 states. This is as close as you can get to buying wholesale party supplies.

If you have any questions on our quantity discount or wholesale pricing on party supplies please give us a call at (800) 224-3143. PartyCheap.com is your party bulk warehouse.

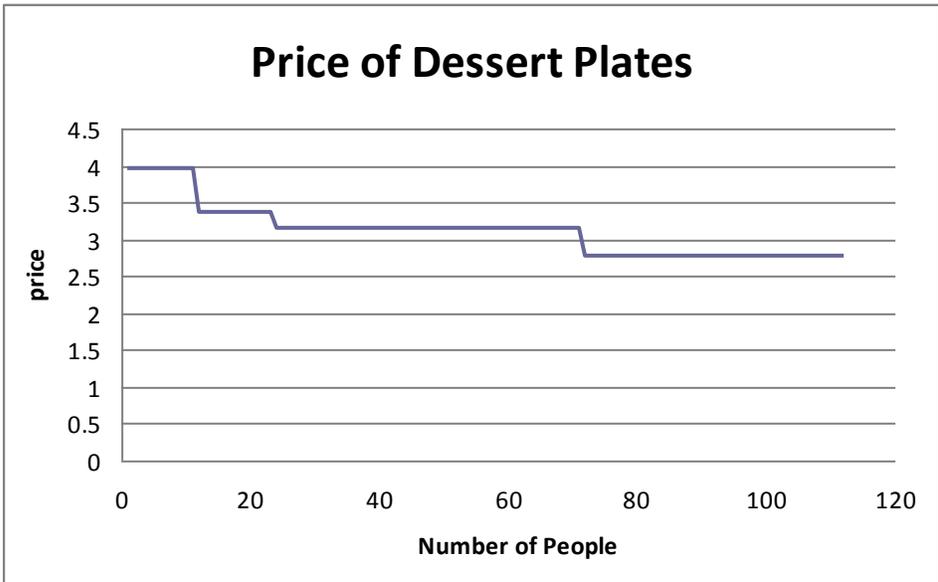
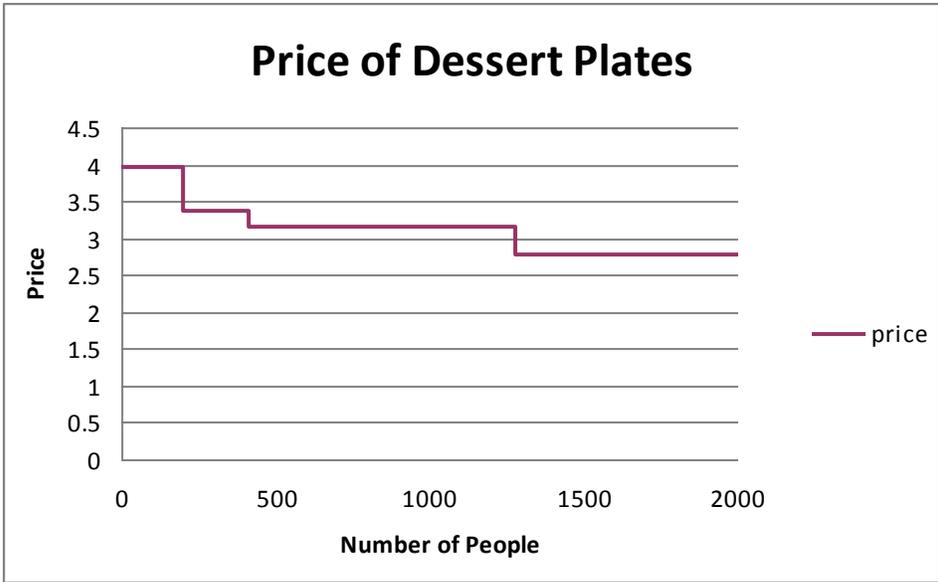
[http://www.partycheap.com/Quantity\\_Discounts\\_on\\_Party\\_Supplies\\_s/514.htm](http://www.partycheap.com/Quantity_Discounts_on_Party_Supplies_s/514.htm)

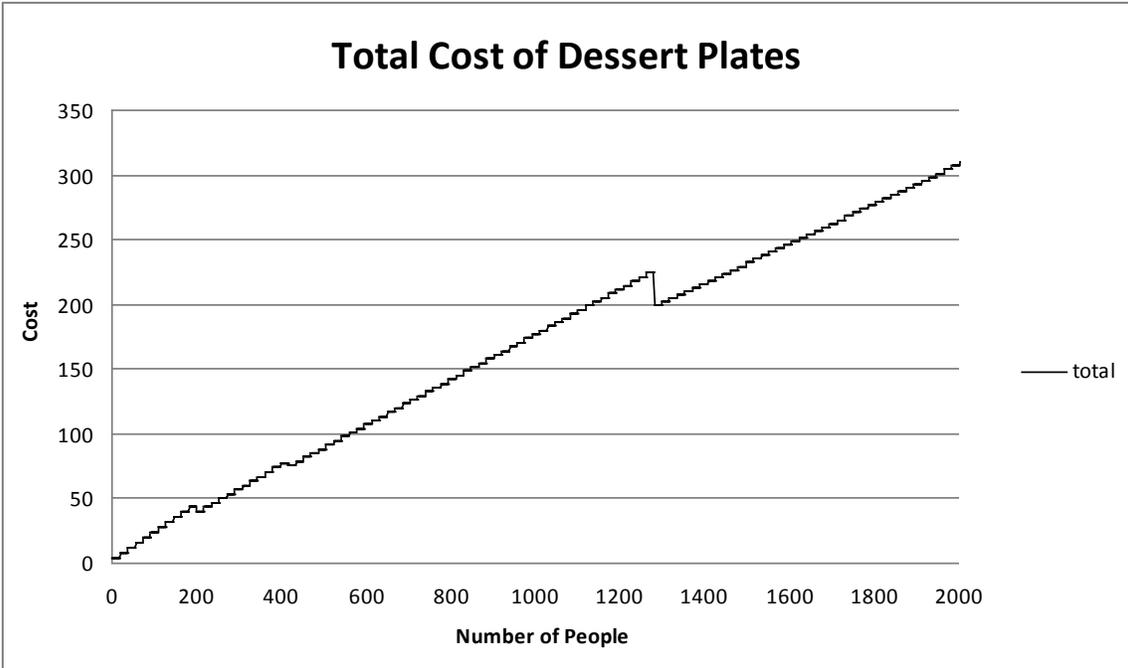
### **Congrats Grad Dessert Plates (18/pkg) Our Price: \$3.95**

Suppose you are having a graduation party and expect 30 people to come. How many packages of dessert plates do you need? How much would you pay per package? What would be the total cost?

How many people are in your graduating class? If the school gave a graduation reception for the graduates and their parents, how many dessert plates would the school have to buy? How many packages would that be? How much would the school pay per package? What would the total cost be?

Graph the cost function





## Statistics

**Course:** Discrete Math and/or Precalculus

**Standards:**

DM.2.1 Use matrices to organize and store data.

PC.8.1 Find linear models using the median fit and least squares regression methods.  
Decide which model gives a better fit.

**Objective:** Students will collect data organized in matrices and display it graphically.

**Materials:**

- Excel

**Job Connection:**

- Beckman Coulter – [www.beckmancoulter.com](http://www.beckmancoulter.com)
  - A leading manufacturer of biomedical testing instruments systems, tests and supplies that simplify and automate laboratory processes. Spanning the biomedical testing continuum - pioneering medical research and clinical trails to laboratory diagnostics and point-of-care testing - Beckman Coulter's 200,000 installed systems provide essential biomedical information to enhance health care around the world. The company, based in Fullerton, Calif. with operations in Indiana, reported annual sales of \$2.44 billion in 2005; 64 percent of these dollars were derived from the sale of after-market reagents, test kits, consumables and service.
- **Manager of Strategic Marketing** - Manages the worldwide strategic vision and direction for the (ultracentrifugation -- or -- high performance centrifugation) business. This includes effective market/industry analysis, strategic plan development, and accountability for successful implementation of the plan in all markets and geographic regions. Responsible for defining products, services, support required to serve the market and achieve strategic plan financial goals; accountable for business leadership to the development/technical team in development of products/services. The incumbent is accountable for achievement of the (ultracentrifugation – or – high performance) global operating plan; including revenue, profit, inventory and overall operating income goals; global leadership to commercial operations for achievement of operating plan. Provides business/market leadership to key constituents: supply chain management, quality, and technical support.

**Vocabulary:**

Bar Chart - A diagram consisting of a sequence of vertical or horizontal bars

Pie Graph - A circular diagram divided into sectors of which the areas are proportional to the magnitudes of the quantities represented

Time Plot - An x,y graph where x is time and y is the quantity being studied

**Input:**

***Data Collection and Analysis - Pick Your Product***

Your first task is to pick a product type that your group would like to sell to other students during lunch such as soda, candy bars, chips, sandwiches, or pizza.

What different choices could you offer for this product? For example, if your product is soda then your choices could be Coke, Diet Coke, Mountain Dew, or Sprite.

***Collect Your Data***

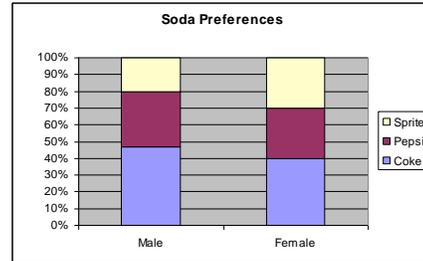
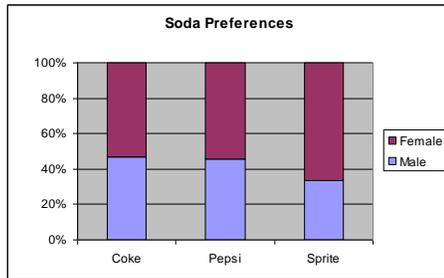
Next you will need to collect data from the class about their preferences to help you make marketing decisions.

Choices	Males	Females	Total

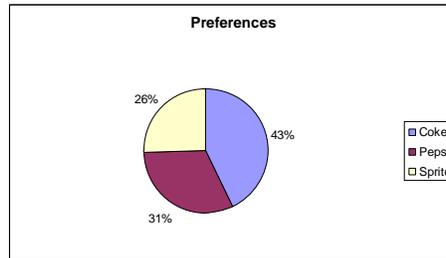
Frequency	Tally
None	
1-2	
3-4	
5 or more	

## Analyze Your Data

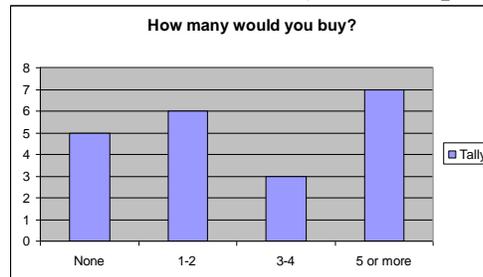
1. Enter your data into an excel spreadsheet.
2. Select the range of cells that contain your labels and data for males and females. Then click on the chart wizard and create a 100% stacked column chart that shows preferences for men and women. Make another 100% stacked column chart, but this time change the data series to rows to make a slightly different chart. (See examples below!)



3. Select the range of cells that contain your labels and totals. Then click on the chart wizard and create a pie chart that shows preferences overall. (See example below!)



4. Select the range of cells that contain the frequency data and click on the chart wizard to create a column chart. (See example below!)



5. Go to file - page setup - and under scaling choose fit to one page. Then print your charts out to turn in.

### What Do the Results Tell You?

1. If you were advertising in a men's magazine such as Sports Illustrated, which product would you feature in the ad? Why?
2. If you were advertising in a women's magazine such as Glamour or Vogue, which product would you feature? Why?
3. Which products should you order? Should you order the same amount of each item? Why or why not?

### Crude Oil Prices

1. Go to [http://tonto.eia.doe.gov/dnav/pet/pet\\_pri\\_wco\\_k\\_w.htm](http://tonto.eia.doe.gov/dnav/pet/pet_pri_wco_k_w.htm) and click on Download Series History.
2. Save this file as oilprices.xls wherever your teacher tells you to save it.
3. Go to the Tab Data 1.
4. Select the first column and reformat the dates as your instructor tells you.
5. Select the first two columns starting on line 3 and going to the end of the data.
6. Insert a scatterplot of the data. We may have to unfreeze the panes to format the chart.
7. Delete the legend since we only have one data series.
8. Reformat the x-axis to make the data fill the chart as your instructor tells you.
9. Now look at <http://www.eia.doe.gov/emeu/cabs/AOMC/Overview.html>

Does your chart look like the one on this webpage? Can you identify any of the historic events on your chart? What has happened since the end of 2006 that you can see on your chart?

If we have time, we may predict the future using this data.

**Assessment:** Students will print out their work from Excel for evaluation. They will also complete the worksheet for homework.

### Additional Documents:

StatisticsWorksheet.doc  
Data-petroleum prices.xls  
Data-oilprices.xls  
StatisticsHomework.doc

## Statistics in EXCEL

### Data Collection and Analysis

#### ***Pick Your Product***

Your first task is to pick a product type that your group would like to sell to other students during lunch such as soda, candy bars, chips, sandwiches, or pizza.

Record your product type here:

What different choices could you offer for this product? For example, if your product is soda then your choices could be Coke, Diet Coke, Mountain Dew, or Sprite.

Record your choices here:

#### ***Collect Your Data***

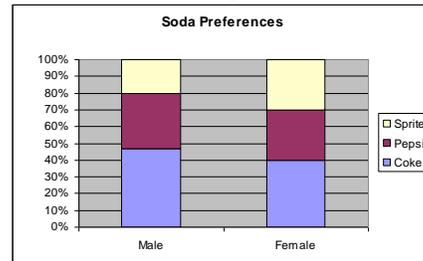
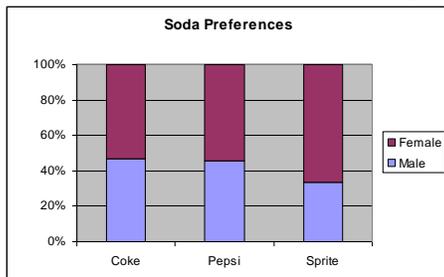
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Choices	Males	Females	Total

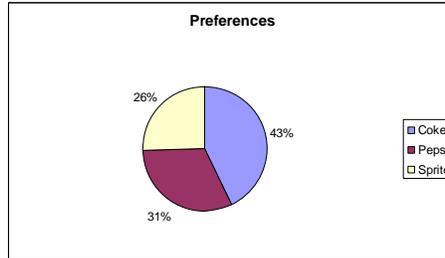
Frequency	Tally
None	
1-2	
3-4	
5 or more	

#### ***Analyze Your Data***

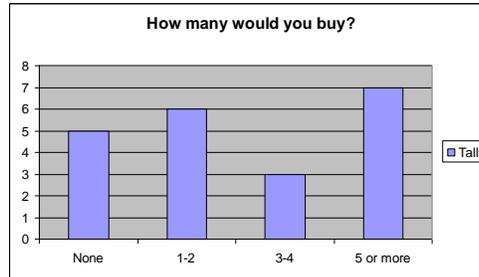
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### What Do the Results Tell You?

4. If you were advertising in a men's magazine such as Sports Illustrated, which product would you feature in the ad? Why?
  
5. If you were advertising in a women's magazine such as Glamour or Vogue, which product would you feature? Why?
  
6. Which products should you order? Should you order the same amount of each item? Why or why not?

## Crude Oil Prices

Go to [http://tonto.eia.doe.gov/dnav/pet/pet\\_pri\\_wco\\_k\\_w.htm](http://tonto.eia.doe.gov/dnav/pet/pet_pri_wco_k_w.htm) and click on Download Series History.

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Insert a scatterplot of the data. We may have to unfreeze the panes to format the chart.

Delete the legend since we only have one data series.

Reformat the x-axis to make the data fill the chart as your instructor tells you.

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Does your chart look like the one on this webpage? Can you identify any of the historic events on your chart? What has happened since the end of 2006 that you can see on your chart?

If we have time, we may predict the future using this data.

## Statistics Problems

1. The data below represents the contents of a load of trash from a city. Find the percentage of the total for each category of trash. Use Excel to make bar charts and pie charts of the data.

Material	Weight
FoodScraps	25.9
Glass	12.8
Metals	18
PaperPaperboard	86.7
Plastics	24.7
RubberLeatherTextiles	15.8
Wood	12.7
YardTrimmings	27.7
Other	7.5

2. The data below shows the average annual temperature in two cities in California over a period of 50 years. Use Excel to make a timeplot (x,y-scatterplot) of the data. Describe the main features of the data. Are the data linear? Use Excel to find a least squares regression line for the data set that is more nearly linear.

year	pasadena	reading
1951	62.27	62.02
1952	61.59	62.27
1953	62.64	62.06
1954	62.88	61.65
1955	61.75	62.48
1956	62.93	63.17
1957	63.72	62.42
1958	65.02	64.42
1959	65.69	65.04
1960	64.48	63.07
1961	64.12	63.5
1962	62.82	63.97
1963	63.71	62.42
1964	62.76	63.29
1965	63.03	63.32
1966	64.25	64.51
1967	64.36	64.21
1968	64.15	63.4
1969	63.51	63.77
1970	64.08	64.3
1971	63.59	62.23
1972	64.53	63.06
1973	63.46	63.75
1974	63.93	63.8
1975	62.36	62.66
1976	64.23	63.51
1977	64.47	63.89
1978	64.21	64.05
1979	63.76	60.38
1980	65.02	60.04
1981	65.8	61.95
1982	63.5	59.14
1983	64.19	60.66
1984	66.06	61.72
1985	64.44	60.51
1986	65.31	61.76
1987	64.58	62.94
1988	65.22	63.7
1989	64.53	61.5
1990	64.96	62.22
1991	65.6	62.73
1992	66.07	63.59
1993	65.16	61.55
1994	64.63	61.63
1995	65.43	62.62
1996	65.76	62.93
1997	66.72	62.48
1998	64.12	60.23
1999	64.85	61.88
2000	66.25	61.58

3. Marine biologists warn that the growing number of powerboats registered in Florida threatens the existence of manatees. The data set for this problem gives the number of powerboat registrations and manatee fatalities due to powerboats during the years 1977-2004.

- a) In this context, what do you think is the explanatory variable?
- b) Make a scatterplot of the data and describe the association you see.
- c) Find the equation of the regression line of the association between manatee deaths and powerboat registrations. What is the slope of the line telling us?
- d) Plot the line on the scatterplot.

Year	Powerboat registrations (1000)	Manatee fatalities
1977	447	13
1978	460	21
1979	481	24
1980	498	16
1981	513	24
1982	512	20
1983	526	15
1984	559	34
1985	585	33
1986	614	33
1987	645	39
1988	675	43
1989	711	50
1990	719	47
1991	716	53
1992	716	38
1993	716	35
1994	735	49
1995	741	42
1996	749	60
1997	796	54
1998	806	66
1999	805	82
2000	841	78
2001	903	81
2002	923	95
2003	940	73
2004	946	69

## Probability and Uncertainty: Just-In-Time

**Course:** Algebra 2 and/or Discrete Math

### Standards:

A2.9.2 Use the basic counting principle, combinations, and permutations to compute probabilities.

Example: You are on a chess team made up of 15 players. What is the probability that you will be chosen if a 3-person team is selected at random?

DM.3.1 Use recursive thinking to solve problems.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 15.

**Objective:** Students will learn the affect of cumulative variation in causing backups and delays in supply chains, both external and internal to companies.

### Materials:

- Poker Chips (at least the number of players  $\times$  8)
- Dice (one per player)
- Incoming dock/outgoing dock placemats
- Scoresheet

### Job Connection:

- Toyota – [http://www.toyota.co.jp/en/vision/production\\_system/](http://www.toyota.co.jp/en/vision/production_system/)
  - <http://www.toyotawhynot.com/#/home>
  - <http://www.toyota.com/about/careers/tmmi/>
- Subaru – [www.subaru-sia.com](http://www.subaru-sia.com)
  - **Group Leader – Paint Division** - Group Leader responsible for 2 shifts of Associates and Team Leaders as well as 4 Process Engineers, Budget controller and shop ISO14001 coordinator, Leader of dirt reduction and pre-treat, ED and paint system quality improvement, MIS contract controller and primary contact for paint vendors, Responsible for BIWEC cost reduction activity, Control of shop facility to operation standard B requirements, Process audits of system to ensure compliance with standards, On call to response to paint system spills. Required: Bachelor or Science in Chemistry

**Vocabulary:**

Just-In-Time (JIT) - A manufacturing strategy to reduce costs, improve return on investment, and improve quality by totally eliminating waste that originated at Toyota. The first target of reducing waste is to reduce inventory and carrying costs by producing only what is needed by the next process in a continuous flow with the final process being sales to the customer. It has evolved into a variety of quality and environmental initiatives.

Kanban - A signal to tell the production line to make the next piece for a product. Kanbans were originally cards or tickets but have evolved into a variety of simple visual signals, such as the absence or presence of a piece needed in the manufacturing process in a square painted on the floor or an empty bin.

Variation - The difference between an ideal and an actual situation.

Variety - A diversity of possibilities.

Probability - A measure of how likely it is that some event will occur; a number expressing the ratio of favorable cases to the whole number of cases possible.

Average or Expected Value - The sum of all possible values divided by the number of values.

Range - A measure of variation that is the difference of the largest value less the smallest.

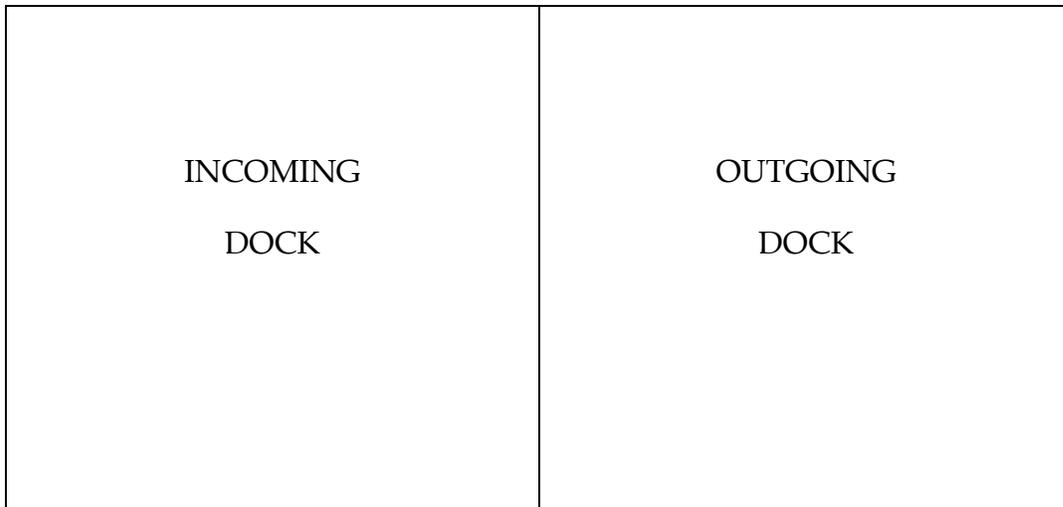
Simulation - A mathematical model that represents the internal processes of the system being simulated to predict results. It may use random numbers and statistical sampling to explore the spectrum of possible outcomes.

**Introduction:**

Do the Just-In-Time simulation described in the attached instructions. Prior to the first month simulation, have the students calculate the average number of chips processed per day, the range for the number of chips processed each day, and the expected output for 20 days. Prior to each month, have them recalculate the average and the range and verify that the expected output is the same.

**Input:****JUST-IN-TIME MANUFACTURING SIMULATION**

This is a hands-on learning activity that simulates the operation of a manufacturing system over a 4 month time period using a different operating strategy each month. Students are seated at a long table, each having a place mat as illustrated below, four poker chips on their incoming dock, and a die.



Each student represents a worker in an assembly line, the poker chips are the product he/she is working on, and the die is used to introduce randomness into the operation of the system. The first student in line is receiving and the last student is shipping. The students roll their dice 20 times to represent twenty working days in a month according to the following operating strategies.

### MONTH 1

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. Process number of chips shown on die (Move from incoming to outgoing dock)
3. Transfer chips on your outgoing dock to next person in multiples of 4 (Lot size / transfer batch = 4). Do not transfer what your upstream neighbor just put on your incoming dock.

On average, how many chips can you process each shift?

### MONTH 2

#### Lot size reduction

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. Process number of chips shown on die (Move from incoming to outgoing dock)
3. Transfer all chips on your outgoing dock to next person (Lot size / transfer batch = 1). Do not transfer what your upstream neighbor just put on your incoming dock.

### **MONTH 3**

#### **Variance reduction**

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. If you roll 1, 2, or 3 process **3** chips. If you roll 4, 5, or 6 process **4** chips. (Move from incoming to outgoing dock)
3. Transfer all chips on your outgoing dock to next person (Lot size / transfer batch = 1). Do not transfer what your upstream neighbor just put on your incoming dock.

### **MONTH 4**

#### **Pull System**

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Shipping (last person) rolls die
2. If he/she rolls 1, 2, or 3, he/she processes **3** chips. If he/she rolls 4, 5, or 6, he/she processes **4** chips.
3. Shipping takes however many chips he/she processes from person immediately upstream. Person immediately upstream replenishes his/her chips from next person upstream, and this continues to ripple upstream until receiving is reached. Receiving replenishes number taken.

After each roll of the dice, the number of chips shipped is recorded in a table like the one on the following page. The first column designates the day of the month. The second column is the cumulative number of chips that should have been shipped by the end of that day. Note that on average, 3.5 chips should be processed per day because the average of the numbers that appear on a die is 3.5.

If at least 10 students participate in this exercise, the differences in performance of the four operating strategies is quite dramatic.

A good follow-up to this exercise is to have students write minute papers on what they learned or to have a discussion as to why what they observed happened.

DAY	PLAN	MONTH 1	MONTH 2	MONTH 3	MONTH 4
1	3.5				
2	7.0				
3	10.5				
4	14.0				
5	17.5				
6	21.0				
7	24.5				
8	28.0				
9	31.5				
10	35.0				
11	38.5				
12	42.0				
13	45.5				
14	49.0				
15	52.5				
16	56.0				
17	59.5				
18	63.0				
19	66.5				
20	70.0				

**Assessment:**

Students should answer the following questions either as part of a discussion or homework.

- What is Just-In-Time manufacturing? At what company did it originate?
- What did you learn from the Just-In-Time demonstration?
- What did we do to improve the amount of poker chips moving through the line?
- Did changing the transfer batch size or reducing variation have more effect?

**Follow-up:**

Have students explore the websites listed on the job connection to find out more about JIT and environmental issues in manufacturing.  
 Have students experiment with the spreadsheet simulation in this packet.

**Additional Documents:**

ProbJITInstructions.doc

ProbJITScoresheetSlide.DOC

ProbJITSlides.DOC

JIT-spreadsheetsim.xls

ProbJITWorksheet.doc

## MONTH 1

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. Process number of chips shown on die  
(Move from incoming to outgoing dock)
3. Transfer chips on your outgoing dock to next person  
in multiples of 4 (Lot size / transfer batch = 4). Do not  
transfer what your upstream neighbor just put on your  
incoming dock.

On average, how many chips can you process each shift?

## MONTH 2

### ***Lot size reduction***

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. Process number of chips shown on die  
(Move from incoming to outgoing dock)
3. Transfer all chips on your outgoing dock to next person (Lot size / transfer batch = 1). Do not transfer what your upstream neighbor just put on your incoming dock.

## MONTH 3

### *Variance reduction*

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Roll die
2. If you roll 1, 2, or 3 process **3** chips  
If you roll 4, 5, or 6 process **4** chips  
(Move from incoming to outgoing dock)
3. Transfer all chips on your outgoing dock to next person (Lot size / transfer batch = 1). Do not transfer what your upstream neighbor just put on your incoming dock.

## MONTH 4

### ***Pull System***

Start: 4 units of inventory (4 colored poker chips)

Each shift:

1. Shipping (last person) rolls die
2. If he/she rolls 1, 2, or 3, he/she processes **3** chips  
If he/she rolls 4, 5, or 6, he/she processes **4** chips
3. Shipping takes however many chips he/she processes from person immediately upstream. Person immediately upstream replenishes his/her chips from next person upstream, and this continues to ripple upstream until receiving is reached. Receiving replenishes number taken.

DAY	PLAN	MONTH 1	MONTH 2	MONTH 3
1	3.5			
2	7.0			
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9	31.5			
10	35.0			
11	38.5			
12	42.0			
13	45.5			
14	49.0			
15	52.5			
16	56.0			
17	59.5			
18	63.0			
19	66.5			
20	70.0			

What is a supply chain?

What is supply chain management?

Name some careers in supply chain management.

What do you need to do to have a career in supply chain management?

What is Just-In-Time manufacturing? At what company did it originate?

What did you learn from the Just-In-Time demonstration?

## Networks, Projects, and Routings

**Subject:** Discrete Math

**Standards:**

DM.4.1 Use graphs consisting of vertices and edges to model a problem situation.

DM.4.2 Use critical path analysis to solve scheduling problems.

**Objective:** Students will become familiar with real applications of graphs in project management and traffic and material flows.

**Materials:**

- Optional no-bake Key Lime Pie ingredients and utensils

**Job Connection:**

- Pepsi Americas - <http://www.pepsiamericas.com/>
  - PepsiAmericas is an anchor bottler and distributor of Pepsi and related beverages. We have operations in 18 states, Eastern Europe and the Caribbean with over \$3 billion in annual revenues and 17,0 employees globally. The Warehouse Certifier ensures and verifies that all orders are accurately and timely filled according to warehouse policy.
  - **Warehouse Certifier** - The Warehouse Certifier owns check in/out procedures for all vehicles entering & exiting the gate. The position will be responsible for planning and coordinating activities of employees engaged in the building of pallets, certifies the accuracy of the product on the pallets and ensuring the loads are shipped to the correct customer. Participate in period close inventory count. Coordinate activities of employees engaged in the building of pallets, and traffic flow with the warehouse.

**Vocabulary:**

Graph - A set of points and line segments that connect the points representing connections and relationships among the objects represented by the points. For example, points may represent cities and lines may represent roads.

Network - A directed graph

Project - An ordered set of tasks

**Input:**

How do you make an apple pie?

**Crust**

2 cups flour

1 teaspoon salt

½ cup cooking oil

¼ cup + 1 tablespoon water

**Filling**

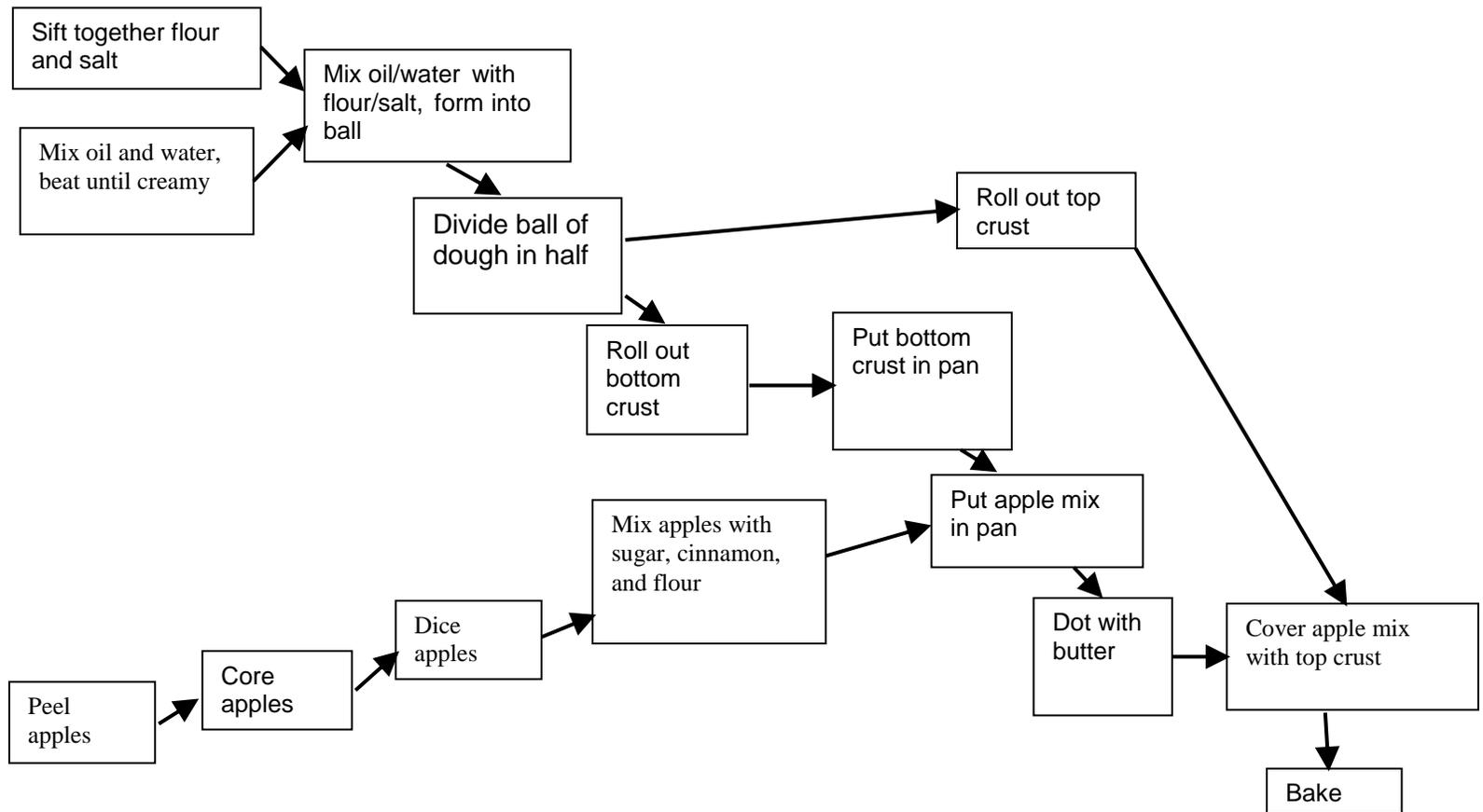
3 cups diced apples

2/3 cup sugar

1 tablespoon flour

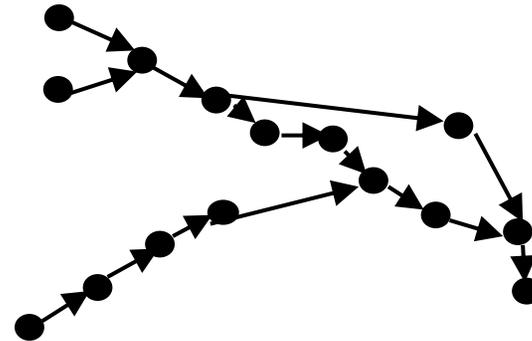
½ teaspoon cinnamon

2 tablespoons butter



### *Project Planning*

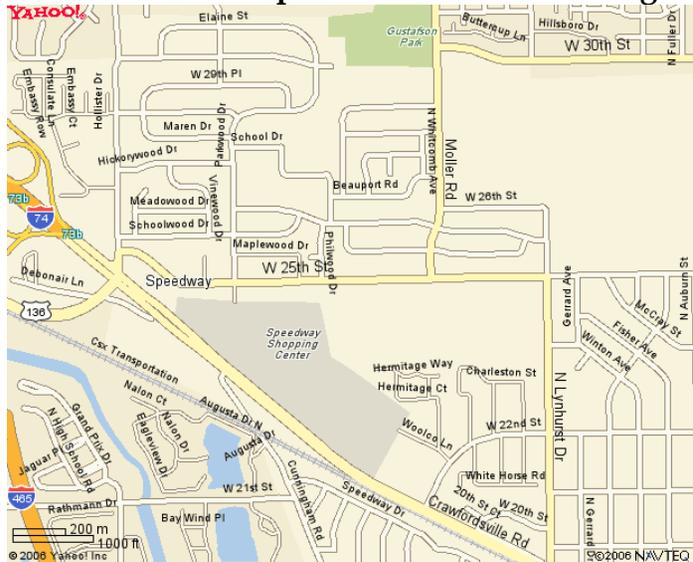
- Arrows represent precedence
- Dots are tasks
- Add times to dots - figure out
  - how long project will take
  - critical path
  - how to divide up tasks between people
  - how many people needed



### *Design of Assembly Lines And Sorting Systems (FedEx)*

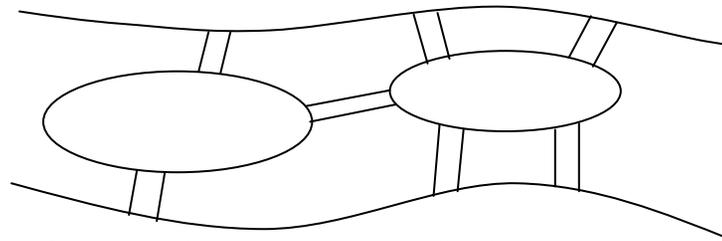
- Assign people to tasks so that everyone has the same amount of work.
- Rebalance as workflow changes

## Trash Pickup and Snow Plow Routing

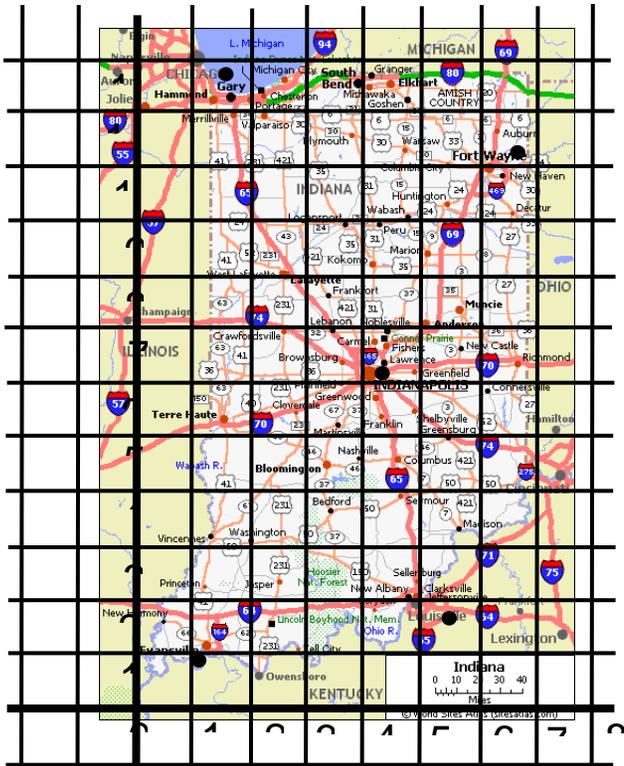


Is it possible to plow every street without repeating?

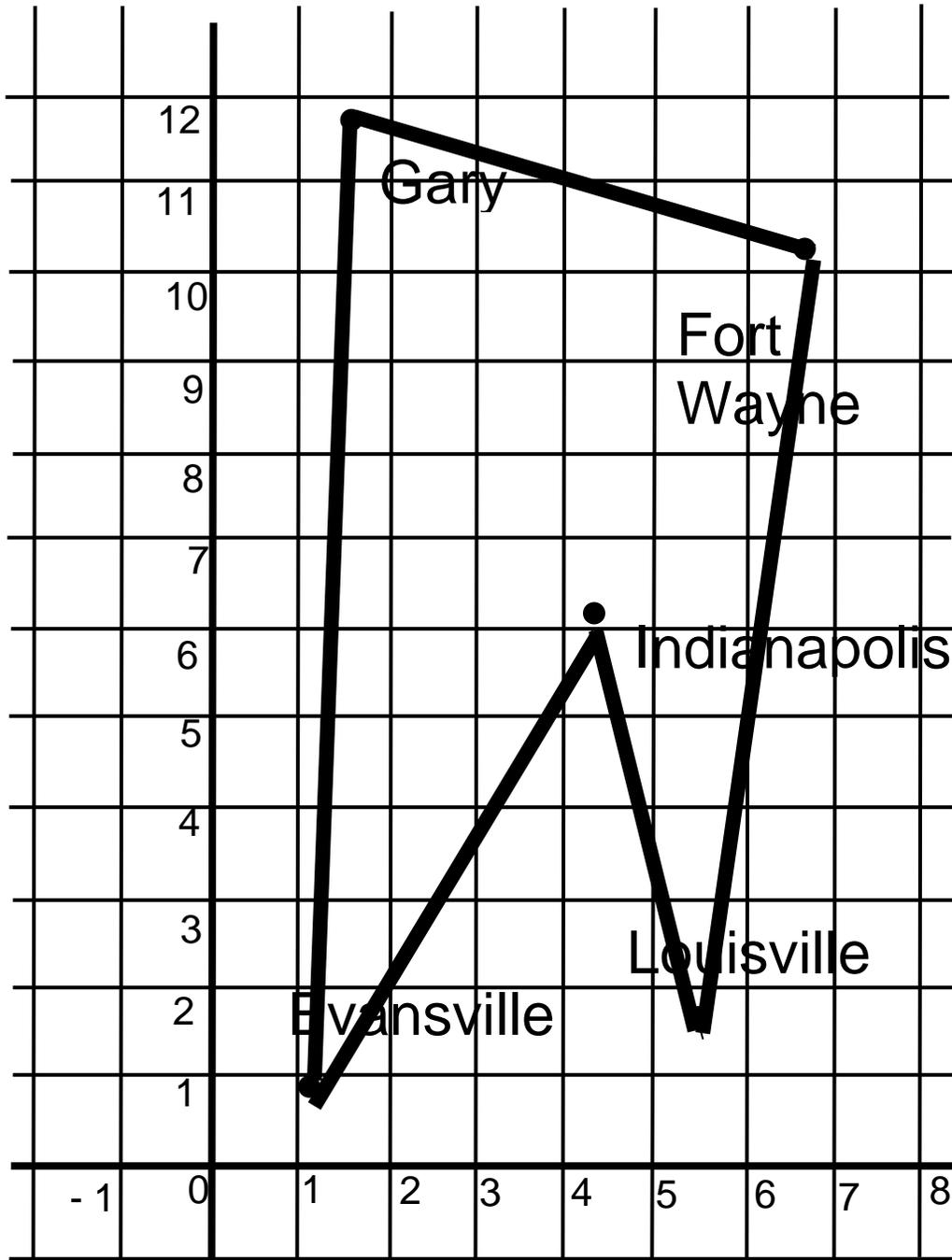
## Koenigsberg Bridge Problem



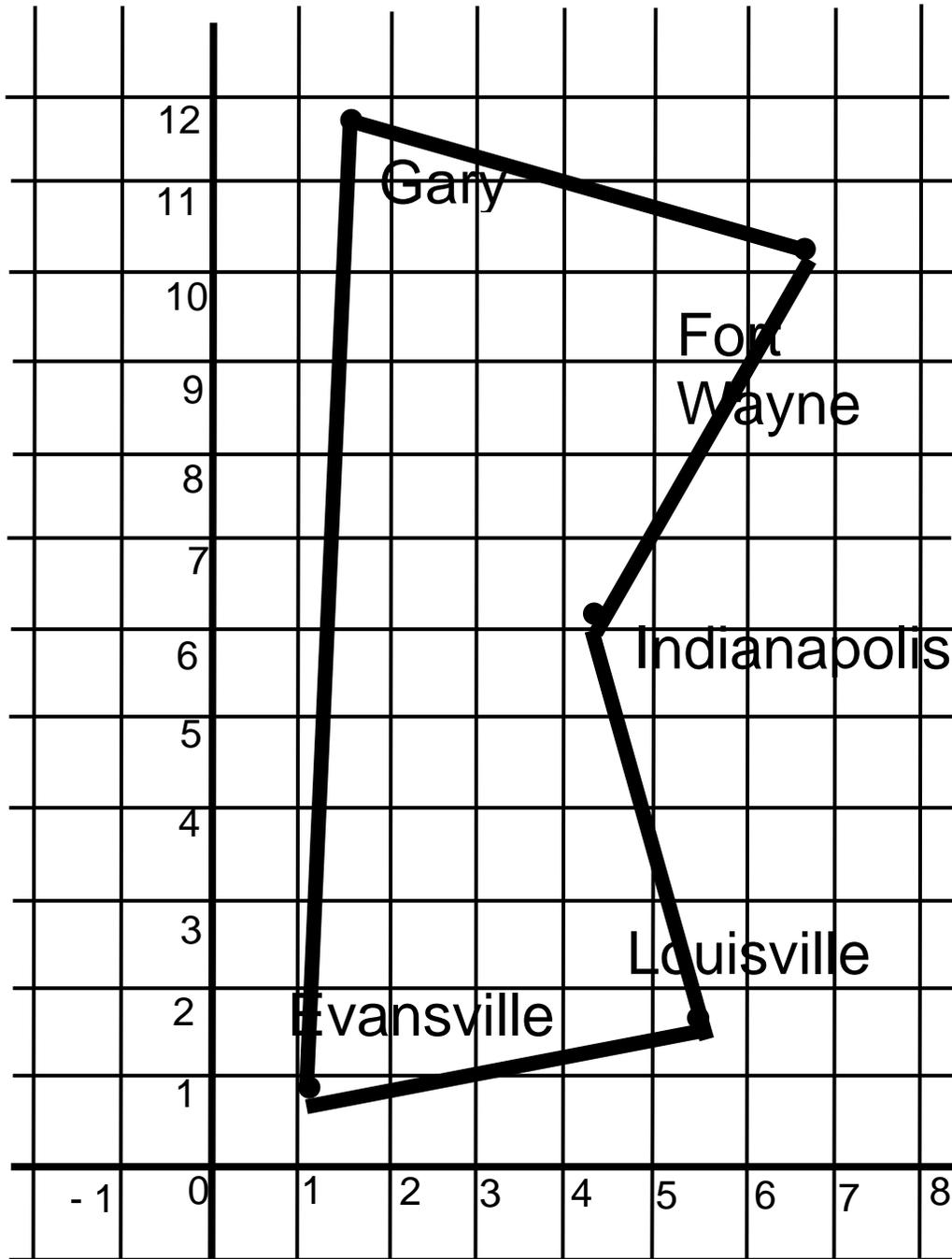
Can you take a walking tour of the town that crosses every bridge once and only once returning to your starting point?  
(called an Euler tour or Euler circuit)



What is the shortest route passing through Indianapolis, Fort Wayne, Gary, Evansville, and Louisville, returning to the starting point and not passing through any city twice?



Here is a possible route.



Here is a shorter one.

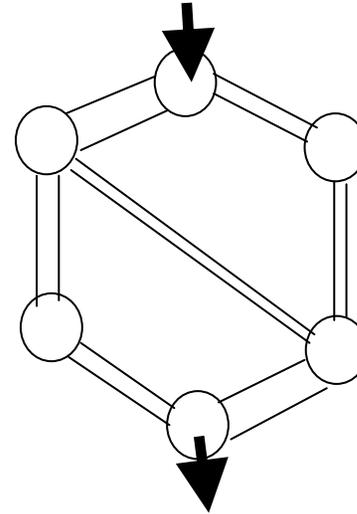
This is called the travelling salesman problem and is used a lot in truck routing and communication networks.

GPS and computer dispatch makes it necessary to resolve these problems while the trucks may be in route.

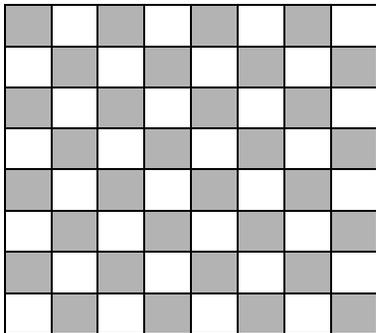
### Maximum Flow

System of pipes - Capacity related to diameter  
What is the maximum flow from top to bottom?

- Oil and gas pipelines
- Traffic flow
- Communication networks
- Water pipes
- Sewers
- Storm sewers
- Drainage ditches/ waterways
- Computer games
- Sorting systems (FedEx)



### 8 Queens Problem



Can you arrange 8 queens on a chessboard so that none are in the same row or column or on the same diagonal?

This can be solved as a maximum flow problem.

**Assessment:** Students will create a diagram for assembling a Key Lime Pie like the Apple Pie diagram.

**Additional Documents:**

NetProjRoutSlides.doc

NetProjRoutSlides.ppt

NetProjRoutSlidesAdv.doc

NetProjRoutHomework1Adv.doc

NetProjRoutHomework2Adv.doc

Key\_Lime\_Pie\_Utensils.doc

Key\_Kime\_Mragarita\_Pie\_Ingredients.doc

Key\_Lime\_Margarita\_Pie\_Recipe.doc

Key\_Lime\_Pie\_Flow\_Chart.doc

# **Projects and Routings Applications of Graphs**

University of Indianapolis  
Department of Mathematics

Leslie Gardner, Ph.D., Professor

## **Indiana Math Standards:**

DM.4.1 Use graphs consisting of vertices and edges to model a problem situation.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

DM.4.2 Use critical path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

# How do you make an apple pie?

## **Crust**

2 cups flour

1 teaspoon salt

½ cup cooking oil

¼ cup + 1 tablespoon water

## **Filling**

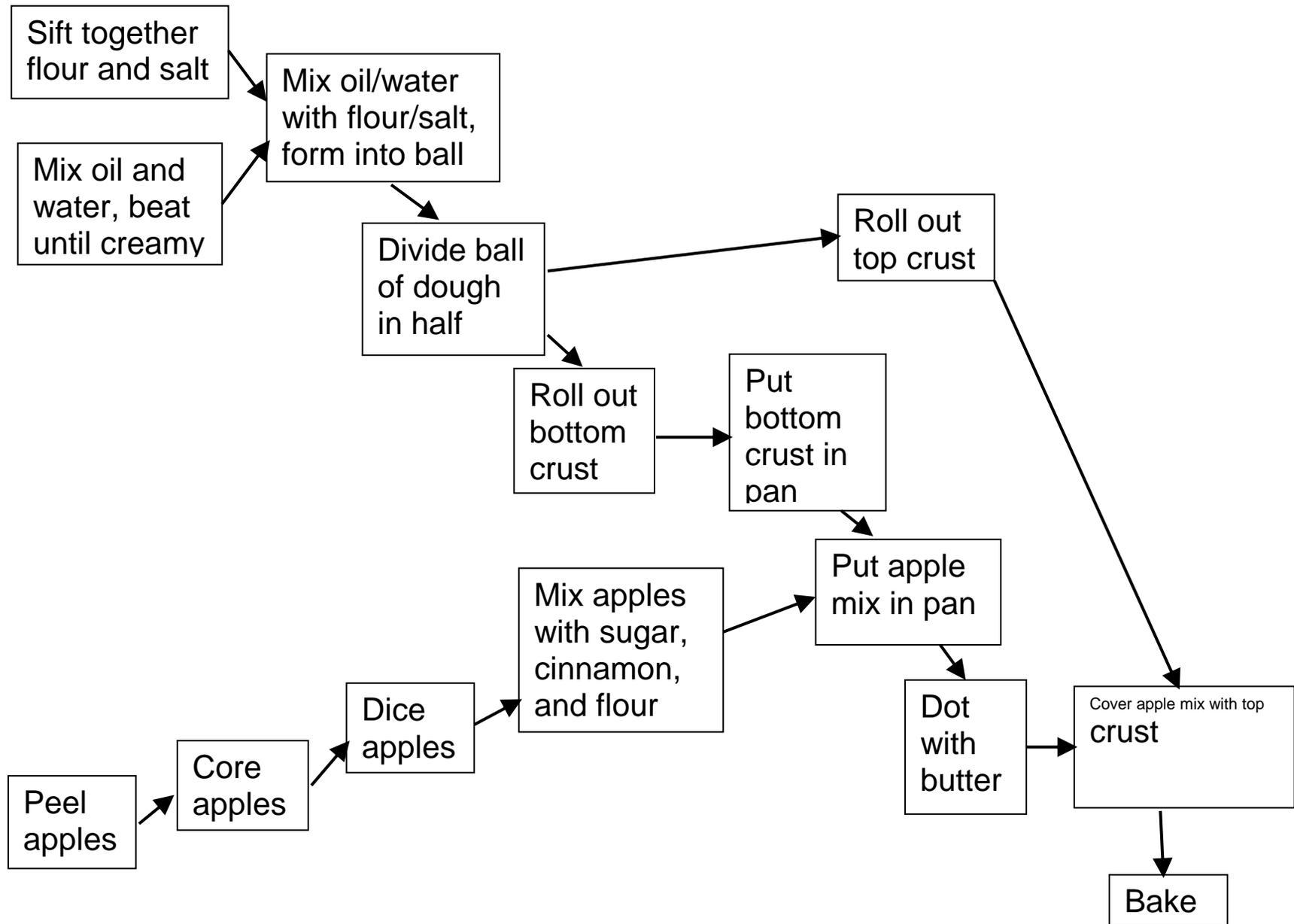
3 cups diced apples

2/3 cup sugar

1 tablespoon flour

½ teaspoon cinnamon

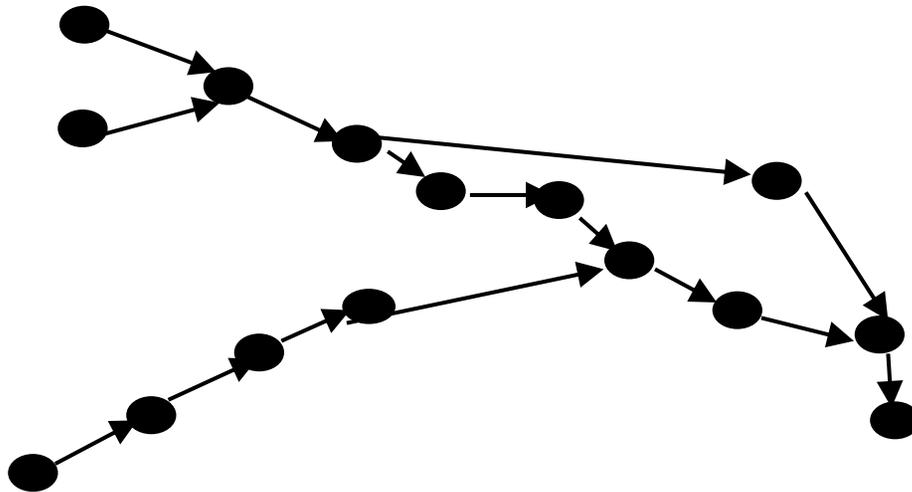
2 tablespoons butter



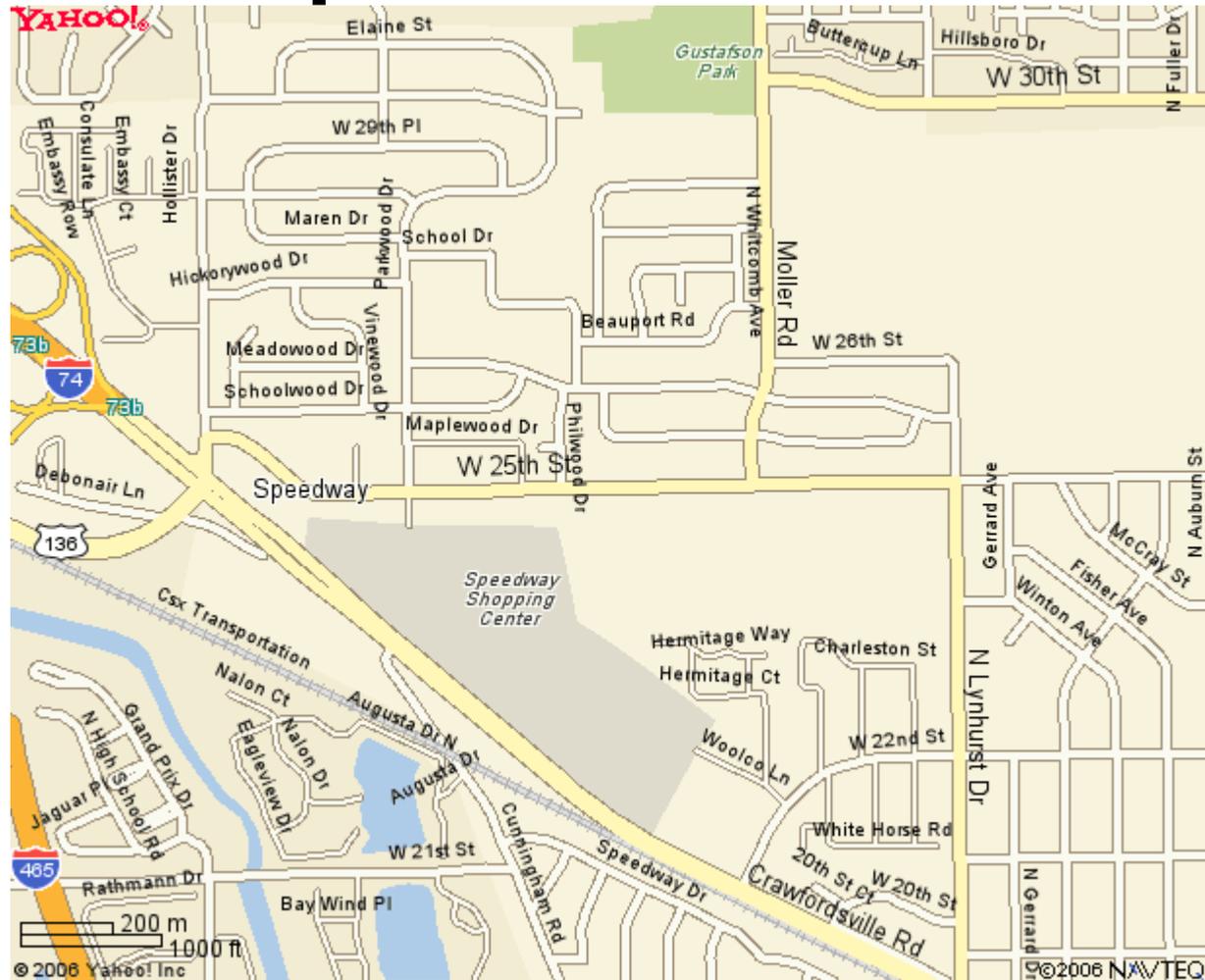


# Design of Assembly Lines And Sorting Systems (FedEx)

- Assign people to tasks so that everyone has the same amount of work.
- Rebalance as workflow changes

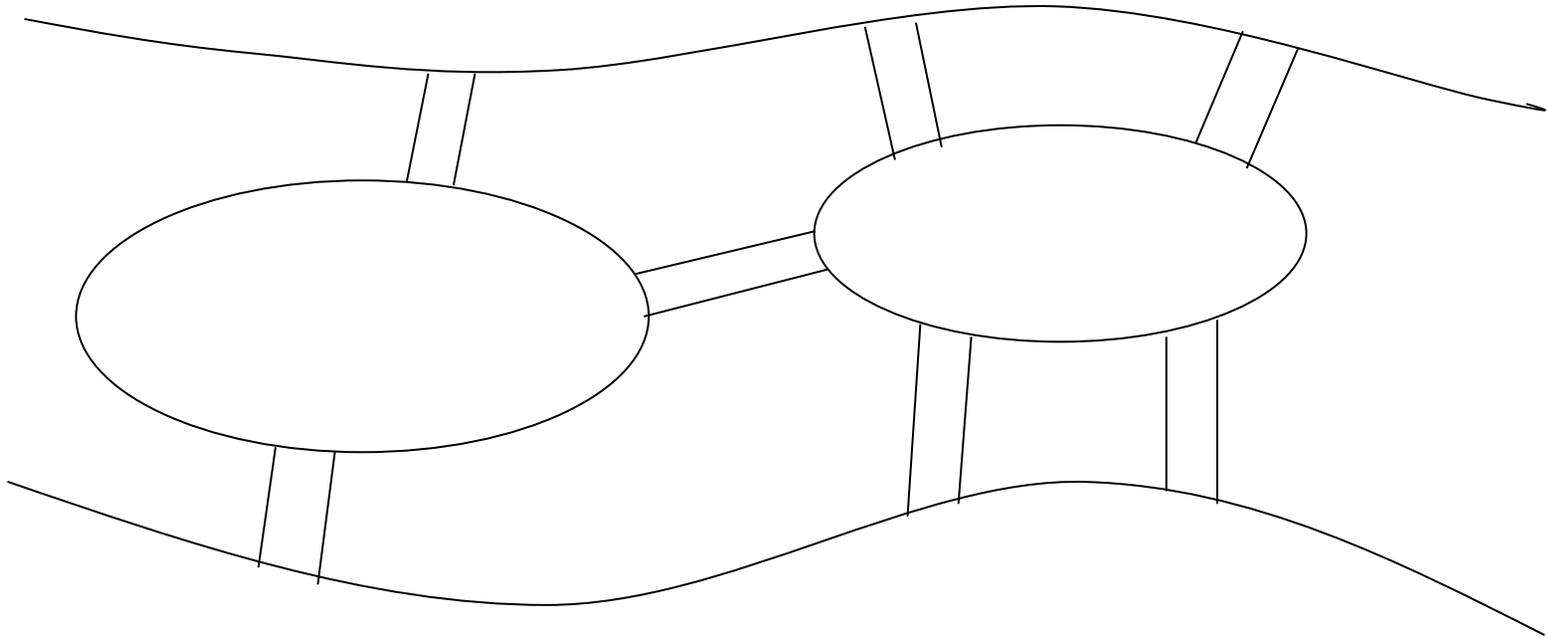


# Trash Pickup and Snow Plow Routing



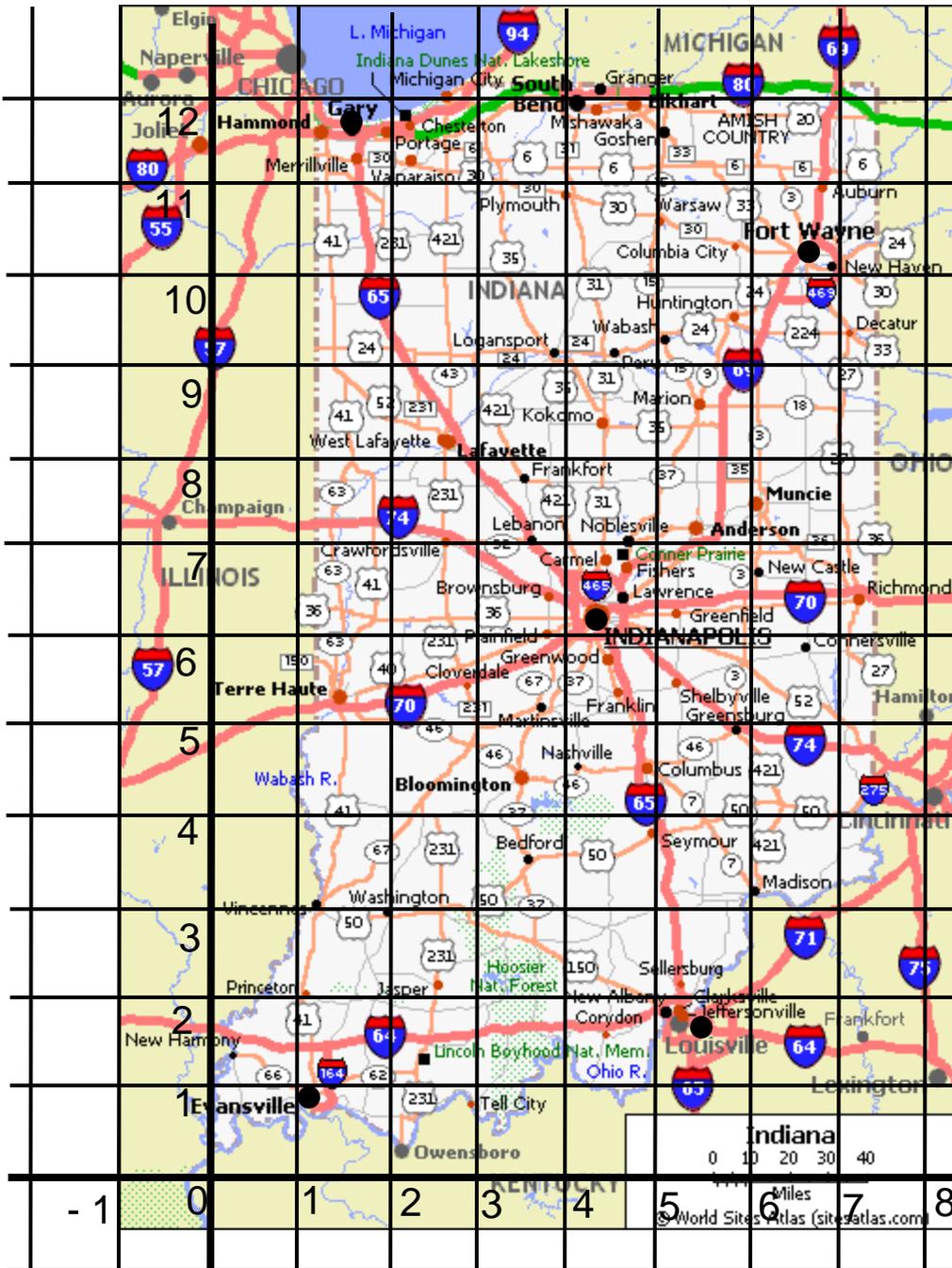
Is it possible to plow every street without repeating?

# Koenigsberg Bridge Problem

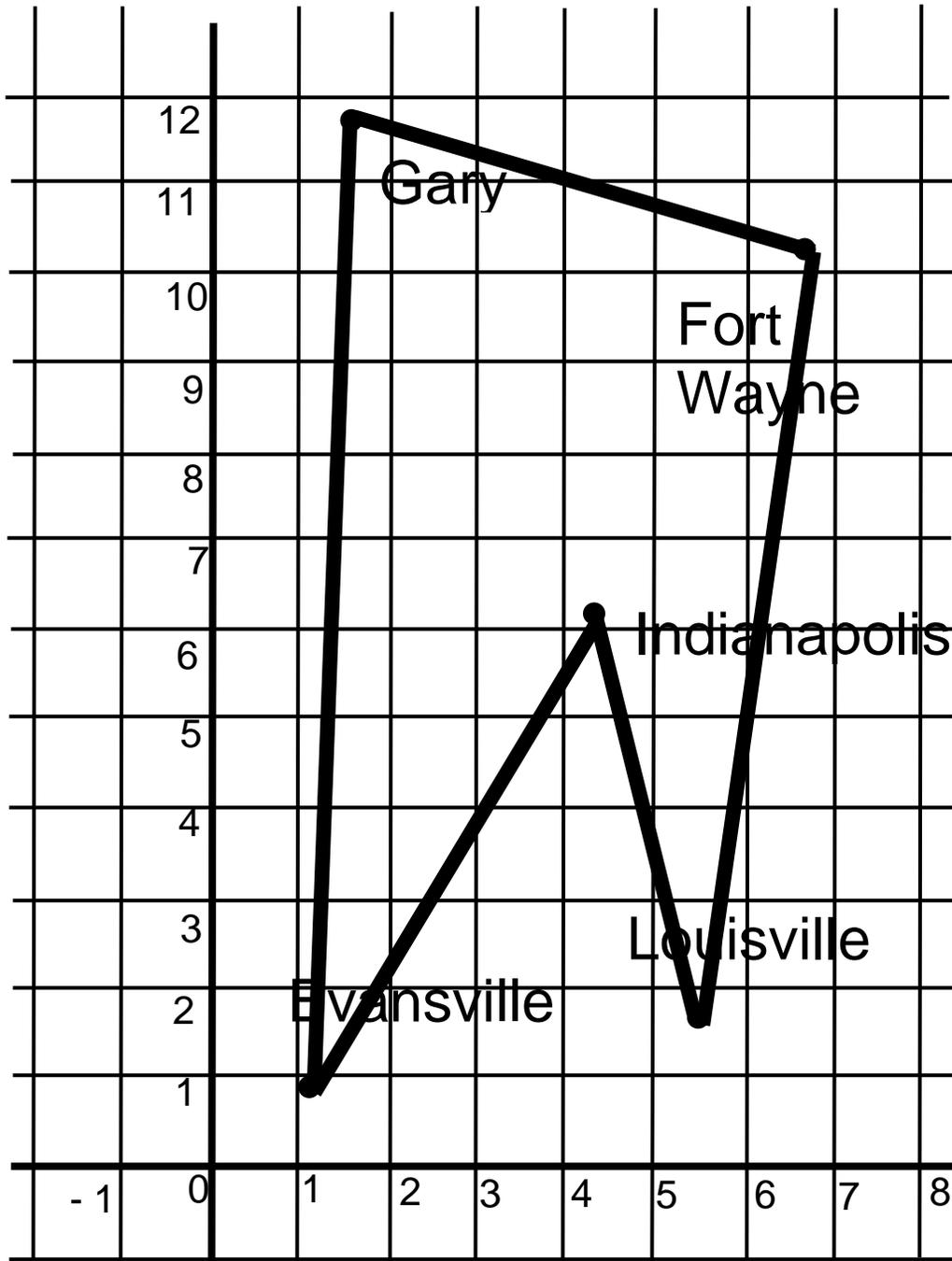


**Can you take a walking tour of the town that crosses every bridge once and only once returning to your starting point?**

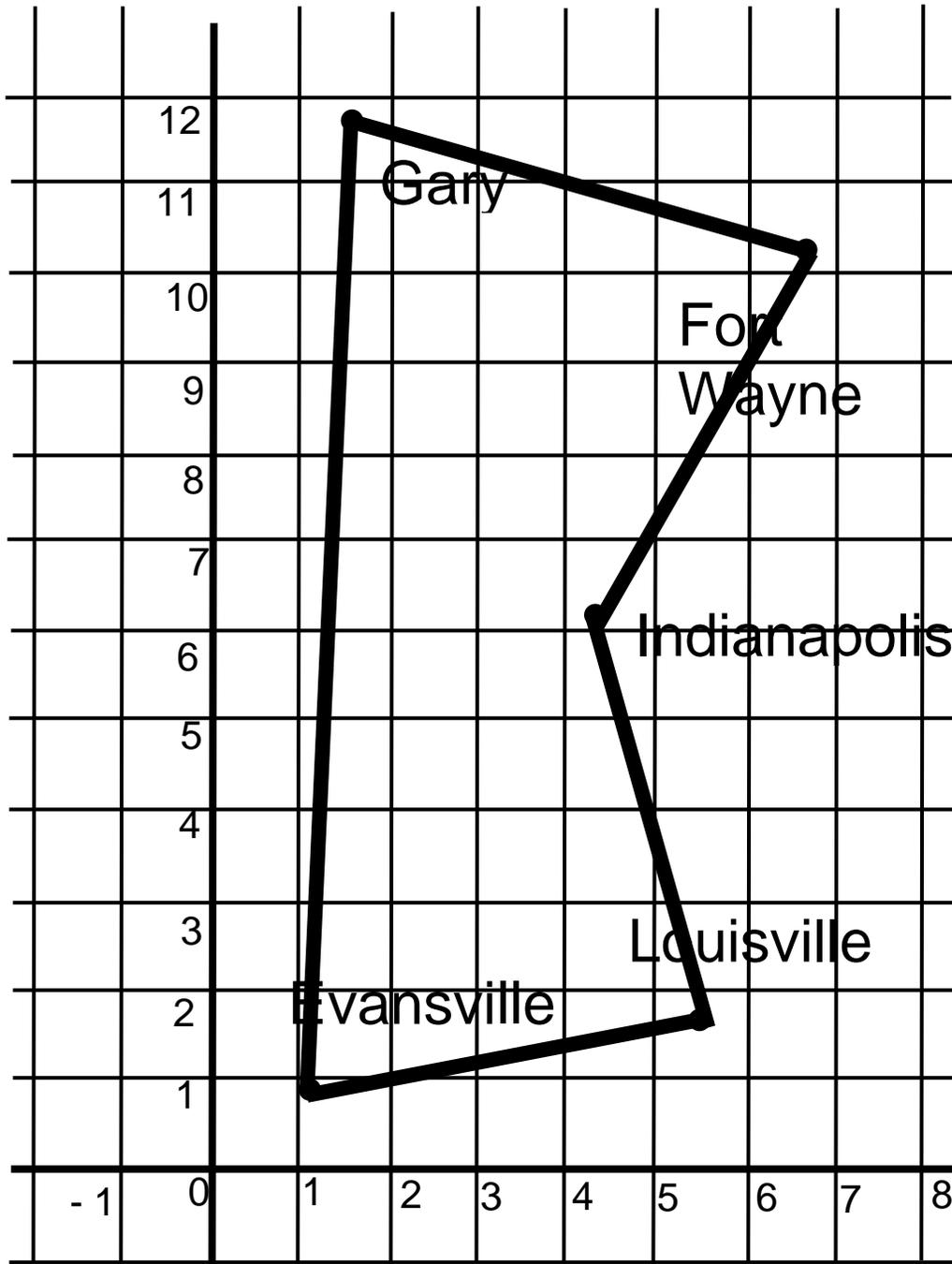
**(called an Euler tour or Euler circuit)**



What is the shortest route passing through Indianapolis, Fort Wayne, Gary, Evansville, and Louisville, returning to the starting point and not passing through any city twice?



Here is a possible route.



Here is a shorter one.

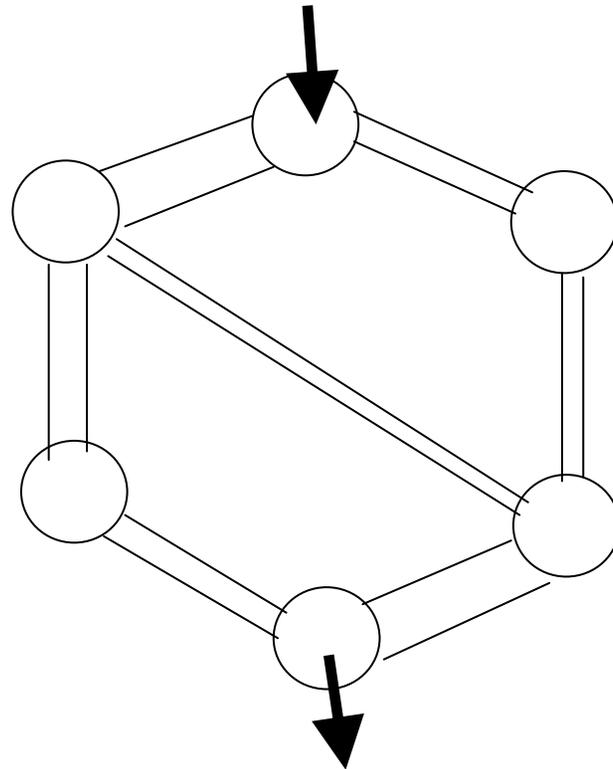
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GPS and computer dispatch makes it necessary to resolve these problems while the trucks may be in route.

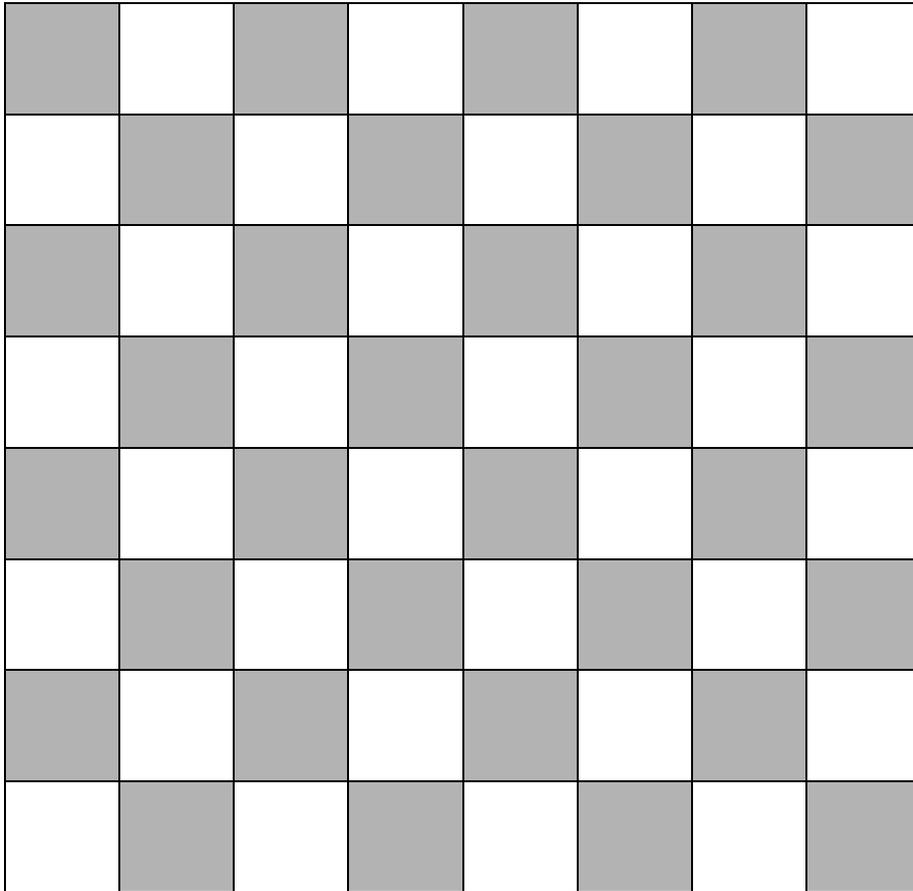
# Maximum Flow

System of pipes – Capacity related to diameter  
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- Sorting systems (FedEx)



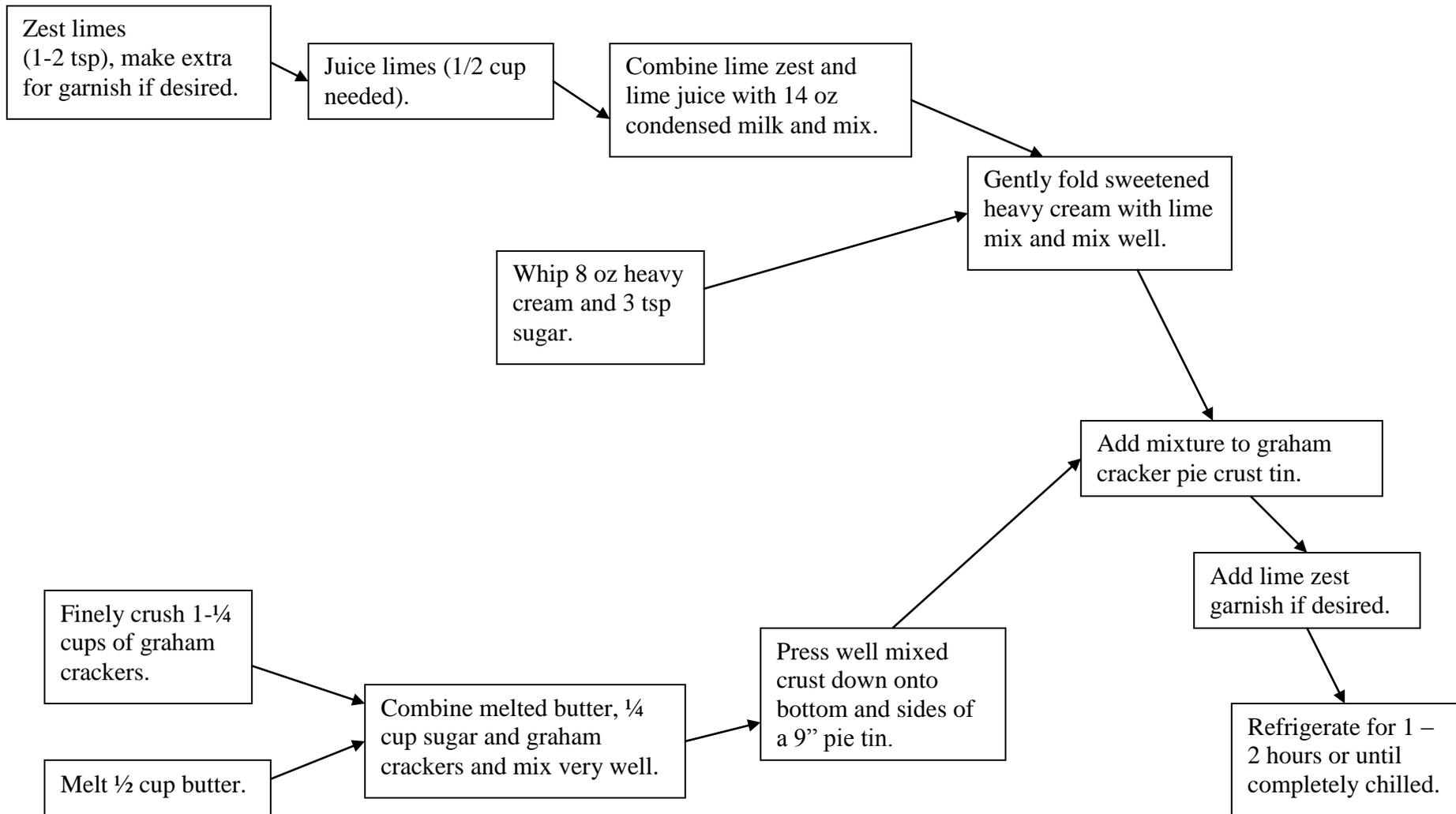
# 8 Queens Problem



Can you arrange 8 queens on a chessboard so that none are in the same row or column or on the same diagonal?

This can be solved as a maximum flow problem.

# No-Bake Key Lime Margarita Pie



## **Crust**

1-1/4 cups finely crushed  
graham crackers  
1/2 cup butter, melted  
1/4 cup sugar

## **Filling**

14 oz of sweetened  
condensed milk  
1 – 2 teaspoons lime zest\*  
1/2 cup Key Lime juice\*\*  
8 oz heavy cream  
3 teaspoons sugar

\* Make extra lime zest for garnish, if desired.

\*\* Approximately 4 limes

## Margarita Key Lime Pie – Required utensils

1. Grater or lime zester
2. Juicer
3. 9 inch pie tin
4. 2 mixing bowls
5. whisk / mixing spoons
6. Measuring spoons / measuring cup
7. Microwave (to melt butter)
8. Refrigerator (to chill after completion)
9. Gallon zip-lock bags (to crush graham crackers in)
10. Rolling pin to crush graham crackers (optional)

## Key Lime Margarita Pie Recipe

This pie can be made with regular lime juice and it is great. Another trick to make this recipe even better, is to use Key Limes. You can find them at most grocery stores year round depending on which state you live in. However, if they are not available year round you can still use the regular limes and it will be a good recipe still.

Ingredients you will need:

Crust:

1 1/4 cups of fine crushed Graham Crackers

1/2 cup of butter, melted

1/4 cup of sugar

Filling:

14 ounces of sweetened condensed milk

1-2 teaspoons of lime zest (grated lime peel) Use a zest-er if you have one.

1/2 cup of Key Lime juice, regular will work if Key Limes are not available to you

8 ounces of sweetened whipped heavy cream (add 3 teaspoons of sugar to the heavy cream), or cool whip topping, or another brand if you prefer. I use sweetened heavy cream and whip it into soft peaks. You can make this by using a whisk, a mixing bowl, and the heavy cream. Using the whisk briskly until you pull out the whisk from the bowl, and there are what appear to be small mountains on it. Be careful not to whip too much, as it will turn into butter.

Pie crust: First things first, we are going to make the crust for the Key Lime Margarita Pie. You will need two mixing bowls for making this recipe. In the first bowl the ingredients for the crusts are going into this bowl. You will combine all of the ingredients in the bowl, and incorporate all of the butter and sugar into the crushed Graham crackers. Make sure that it is mixed very well. You will then take your well-mixed crust and add it to your 9" pie tin, and press down onto the bottom, and up the sides, too. Make sure it is even in the pie tin, to ensure even cooking.

Pie Filling: This is where your second mixing bowl comes in. First you will need to use your zest-er and get 1-2 teaspoons of your lime zest. You can make more zest than needed for garnish after your pie is completed. Second you will cut up your key limes and juice them. Then you will need to add your condensed milk, lime juice, and zest and mix. You will then add your whipped heavy cream or, cool whip topping. You will need to fold this in gently. After it is mixed well you will add all of the mixture into the graham cracker crust pie tin. You will need to refrigerate this pie until it is completely chilled. Depending on your refrigerator it could take 1-2 hours.

# Process Design

University of Indianapolis  
Department of Mathematics

Leslie Gardner, Ph.D., Professor

## **Indiana Math Standards:**

DM.4.2 Use critical path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

# How do you make an apple pie?

## **Crust**

2 cups flour

1 teaspoon salt

½ cup cooking oil

¼ cup + 1 tablespoon water

## **Filling**

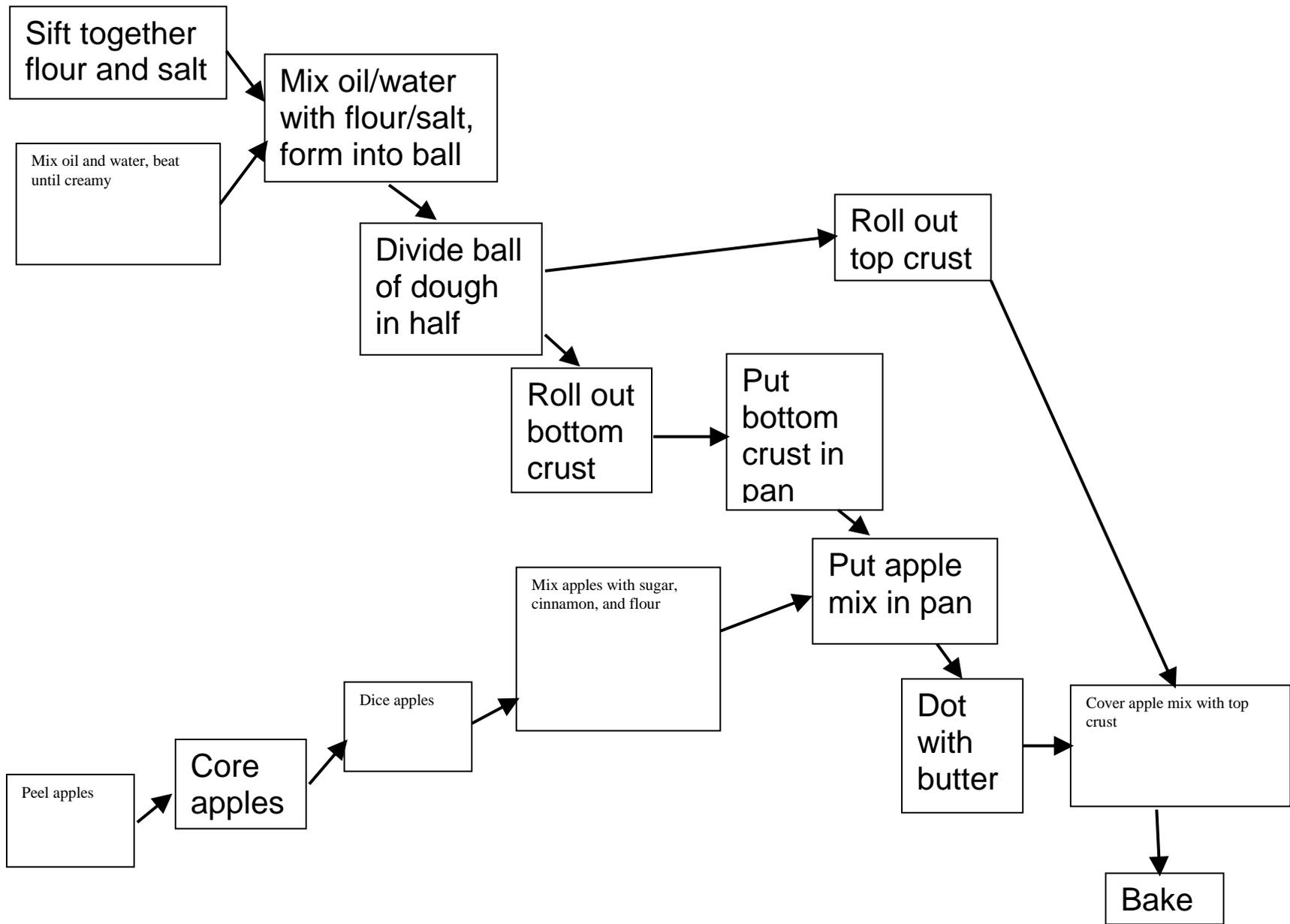
3 cups diced apples

2/3 cup sugar

1 tablespoon flour

½ teaspoon cinnamon

2 tablespoons butter



# How do you make No-Bake Key Lime Margarita Pie?

## *Ingredients (Bill of Materials)*

### **Crust**

1-1/4 cups finely crushed  
graham crackers  
1/2 cup butter, melted  
1/4 cup sugar

### **Filling**

14 oz of sweetened  
condensed milk  
1 – 2 teaspoons lime zest\*  
1/2 cup Key Lime juice\*\*  
8 oz heavy cream  
3 teaspoons sugar

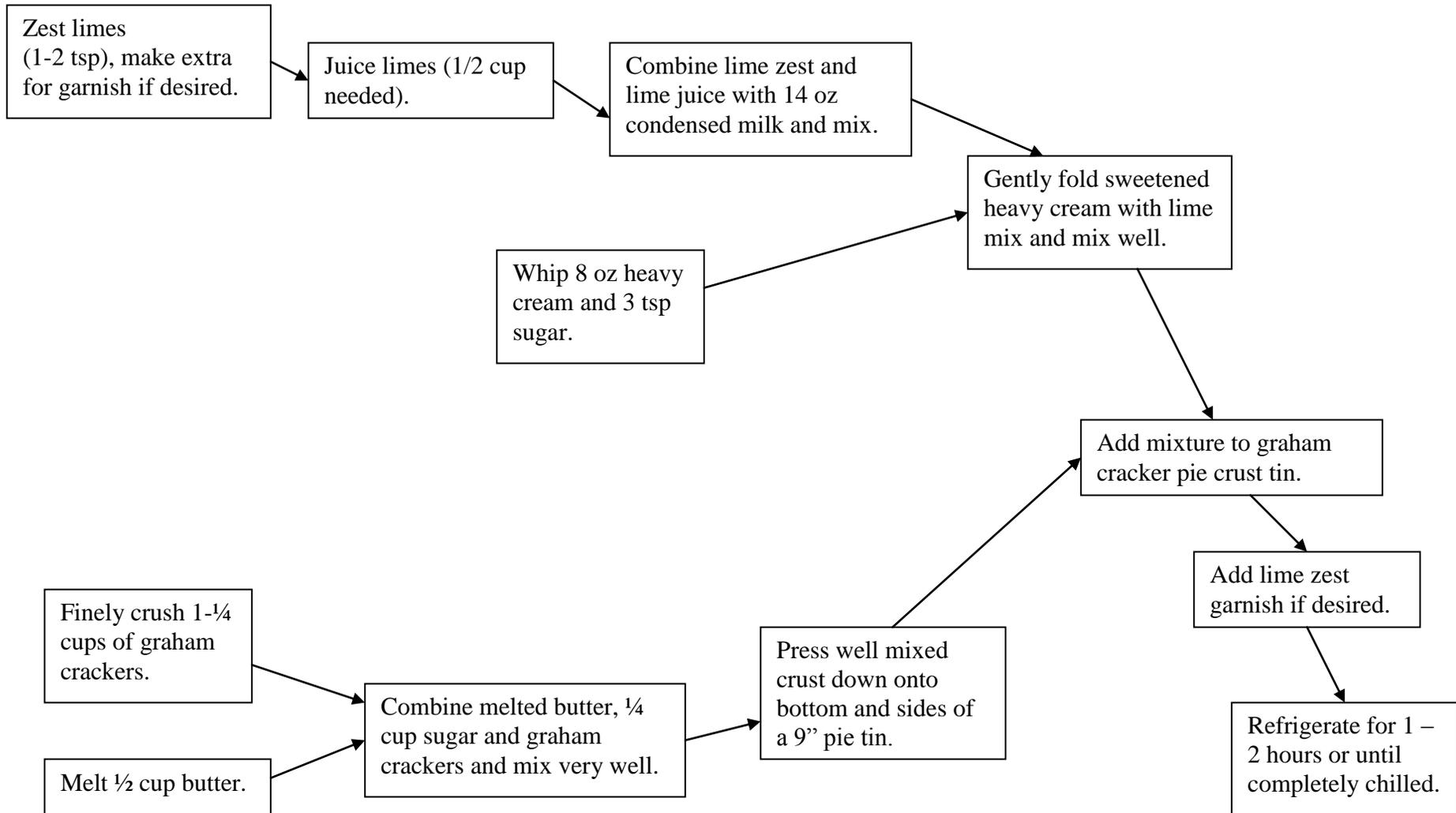
\* Make extra lime zest for garnish, if desired.

\*\* Approximately 4 limes

## Margarita Key Lime Pie – Required utensils (Tools)

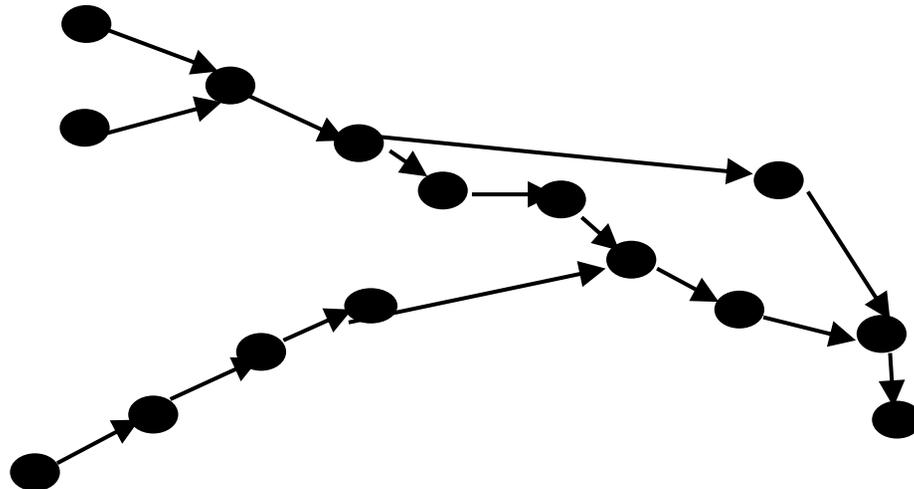
10. Grater or lime zester
11. Juicer
12. 9 inch pie tin
13. 2 mixing bowls
14. whisk / mixing spoons
15. Measuring spoons / measuring cup
16. Microwave (to melt butter)
17. Refrigerator (to chill after completion)
18. Gallon zip-lock bags (to crush graham crackers in)
10. Rolling pin to crush graham crackers (optional)

# Key Lime Margarita Pie (Routing)



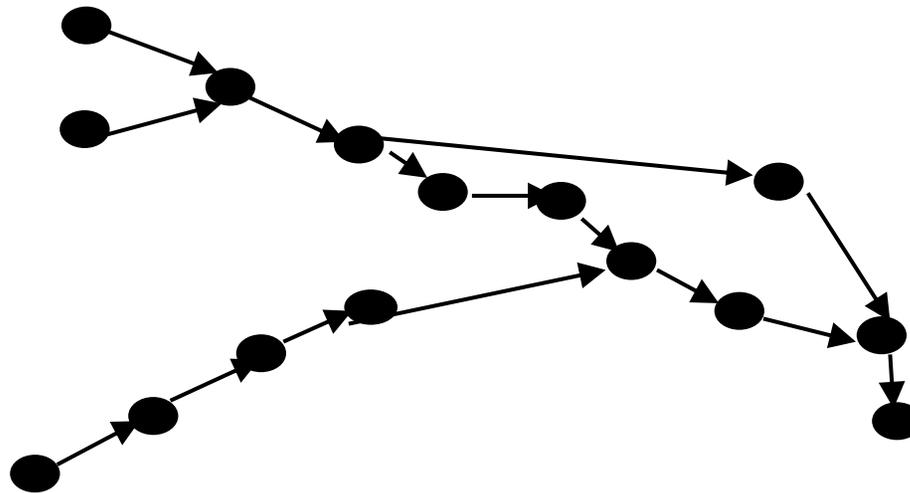
# Project Planning

- Arrows represent precedence
- Dots are tasks
- Add times to dots – figure out
  - how long project will take
  - critical path
  - how to divide up tasks between people
  - how many people needed



# Design of Assembly Lines And Sorting Systems (FedEx)

- Assign people to tasks so that everyone has the same amount of work.
- Rebalance as workflow changes



**Project** - Unique, one-time operations designed to accomplish a specific set of objectives in a limited time frame.

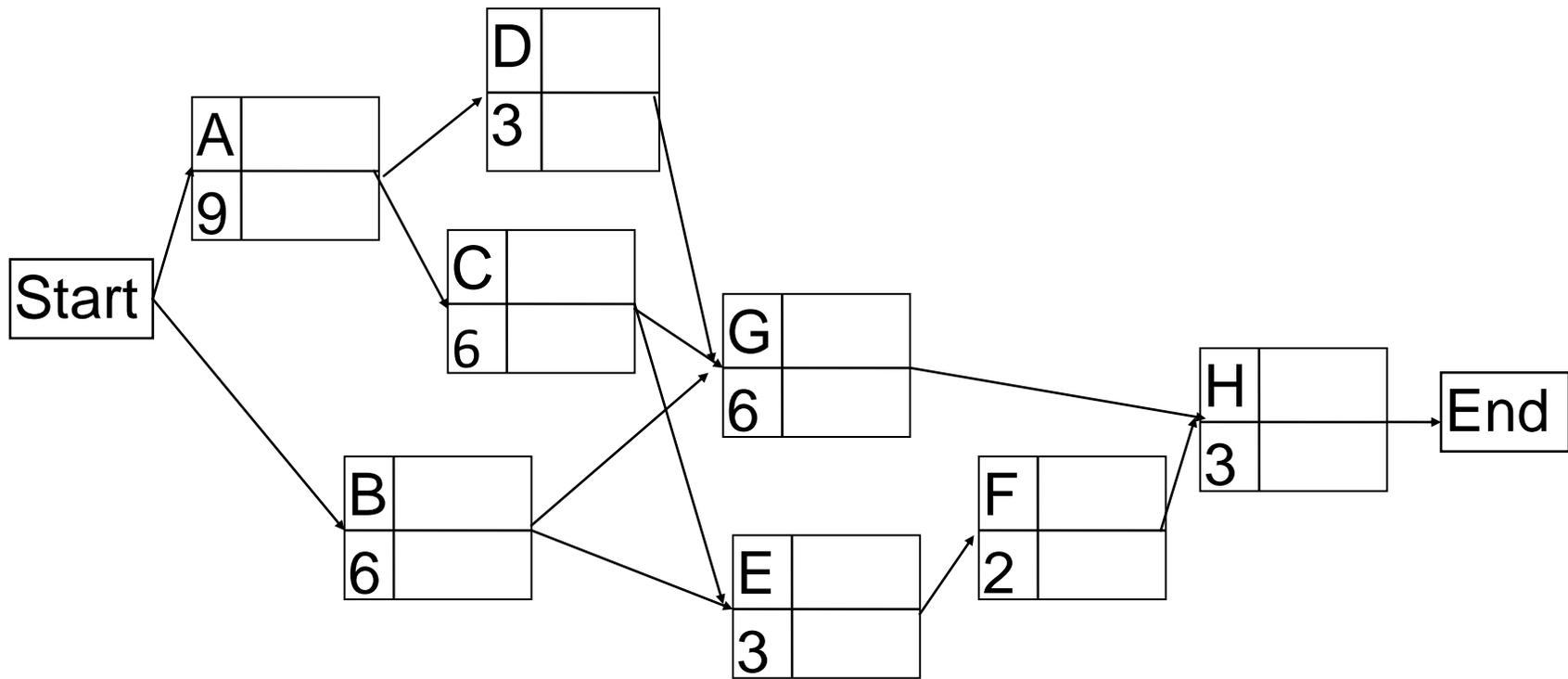
1. Graphical display of project activities (Activity on Arrow or Activity on Node - we will use Activity on Node only).
2. Estimate of how long project will take.
3. Identification of which activities most critical to timely project completion.
4. Indication of how long any activity can be delayed without lengthening the project.

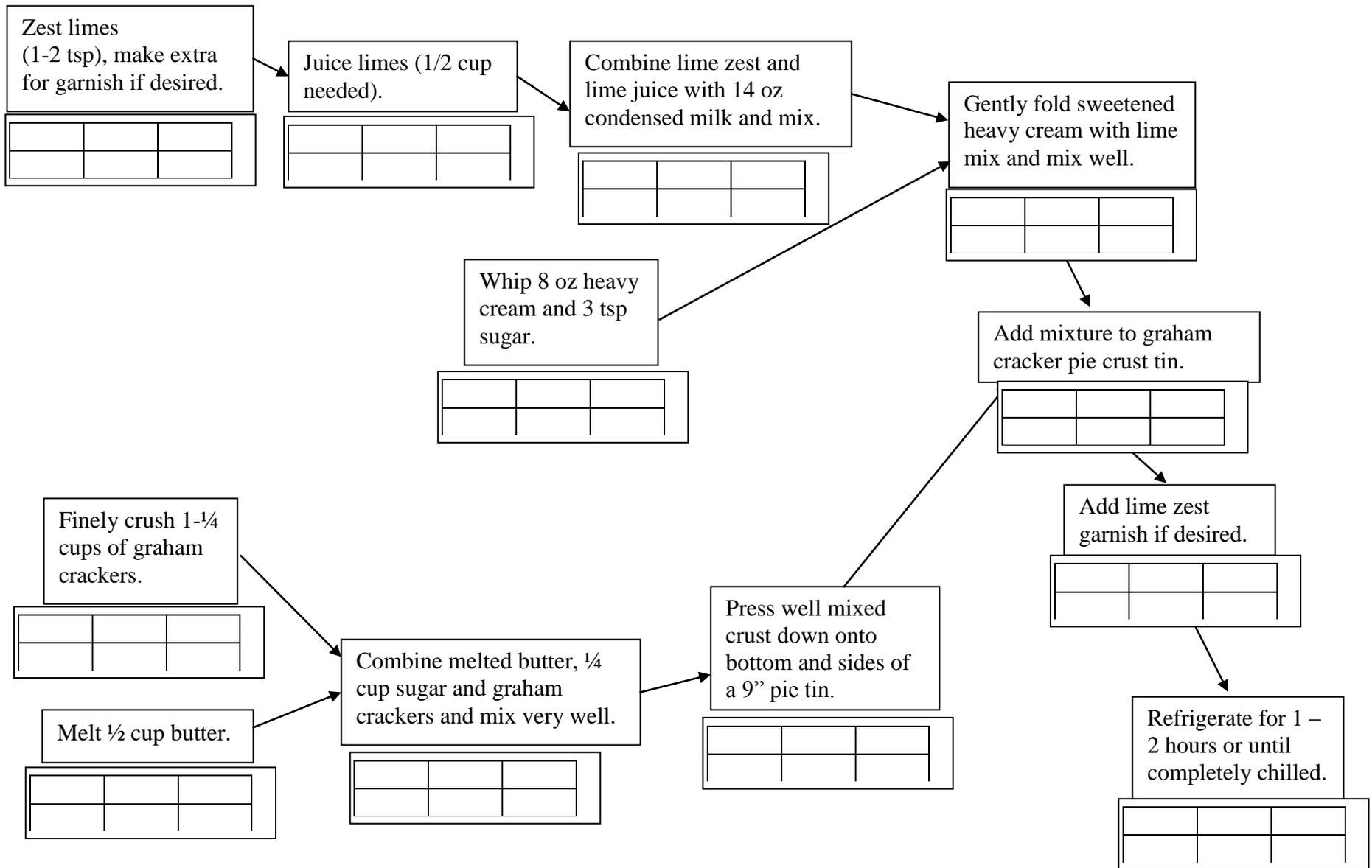
## Finding the Critical Path

1. Make a list of activities that make up the project.
2. Determine a list of immediate predecessors of each activity.
3. Estimate the time needed to complete each activity.
4. Draw **network** (also called precedence diagram).
5. Starting at time 0 and working forward, compute **earliest start(ES)** and **earliest finish(EF)** dates for each activity.
6. Starting at earliest finish of last activity and working backward, compute **latest start (LS)** and **latest finish (LF)** dates for each activity.
7. Find **slack** =  $LS - ES$  or  $LF - EF$  for each activity.  
Find **critical path** which is path having slack = 0 for all activities.

Example:

Activity	Immed. Pred.	Time (weeks)
A	-	9
B	-	6
C	A	6
D	A	3
E	B,C	3
F	E	2
G	B,C,D	6
H	F,G	3





## Layout Types

**Product** - production or assembly lines, repetitive processing

**Process** - job shops, intermittent processing, varied processing requirements

**Fixed position** - projects

**Combination**

**Cellular** - machines grouped into cell that can process items having similar processing requirements

**Group technology** - grouping into part families with similar design or manufacturing characteristics

**Flexible manufacturing systems** - automated, quick setups

## Design of Product Layouts - Line Balancing

- Assign tasks to workstations (or workers)
- Meet desired output requirements
- Minimize number of workstations (or workers)
- Group tasks so they require approximately equal time (cycle time) and idle time is eliminated

$$\text{Cycle Time} = \frac{\text{Operating time per day}}{\text{Desired output rate per day}}$$

### Steps for Line Balancing

1. Calculate cycle time from desired output rate
2. Construct precedence diagram if heuristic requires an ordering of tasks
3. Assign tasks to a station using some heuristic until no task may be added without the sum of the task times exceeding the cycle time
4. Repeat step 3 for each station until all tasks assigned

$$\text{Min. no. workstations (theor.)} = \frac{\text{Output rate} \times \text{Sum of task times}}{\text{Operating time}}$$

## Heuristic Assignment Rules

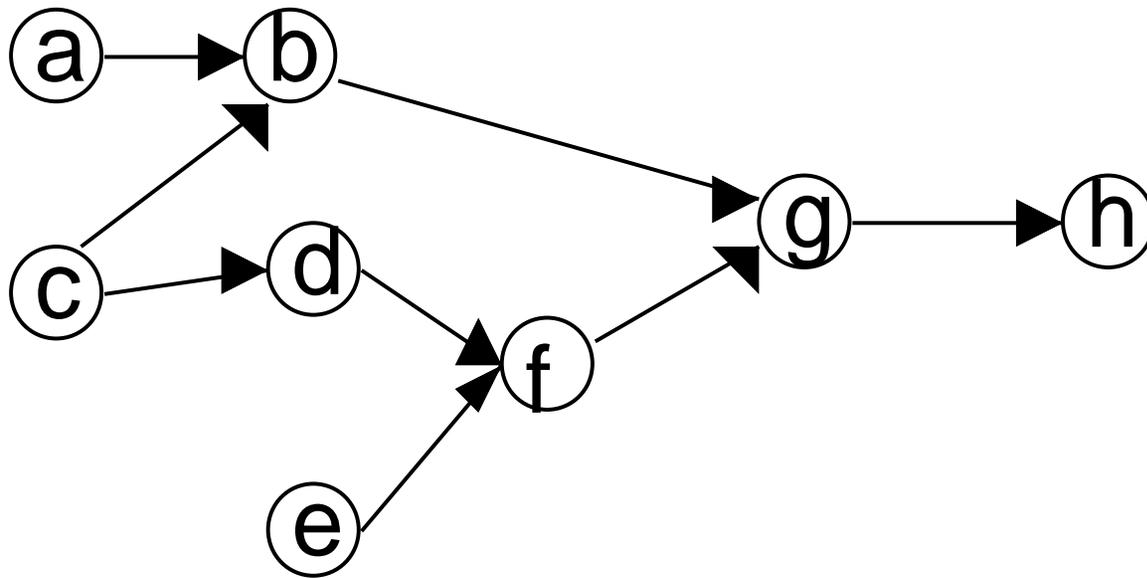
1. Assign tasks to workstations, longest tasks first, and continue until all tasks have been assigned.
2. Assign tasks in order of most number of following tasks.
3. Assign tasks according to positional weight, which is the sum of a task's time and the times of all following tasks.

(There are hundreds of rules.)

Example: Desired output = 200 units per day

<b>Task</b>	<b>Time</b>	<b>Immed. Pred.</b>	1 working day = 8 hours Heuristic = most following tasks
a	1.2	-	Assign tasks to stations. Find theoretical minimum number of stations. Find % idle time and efficiency (1-%idle).
b	1.5	a,c	
c	1.0	-	
d	2.1	c	
e	1.6	-	
f	1.0	d,e	
g	1.8	b,f	
h	2.5	g	

$$\text{Cycle time} = \frac{480 \text{ min / day}}{200 \text{ units / day}} = 2.4 \text{ min / unit}$$



$$\% \text{ idle time} = \frac{\text{idle time per cycle}}{\text{no. station} \times \text{cycle time}}$$

## Stopwatch Time Study

How much time does it take to do some short, repetitive tasks? (Warning: workers hate time studies.)

Observed time (OT) - Average of observations

Performance rating (PR) - An adjustment factor to account for a worker working at a rate different than “normal” due to his or her natural abilities or because he or she is deliberately slowing the pace

Normal time (NT)- Adjusted for worker performance,  $OT \times PR$

Standard time (ST) - Accounts for delays and interruptions  
 $NT \times (1+A)$  where A is an allowance (things that slow a process down like bathroom breaks, fatigue, etc.)

Example: A time study of a manual sort operation at FedEx yielded the following observed times for sorting 100 documents by a worker with a performance rating of 0.9. Using an allowance factor of 20%, find the standard time for sorting 100 documents. Find the standard time.

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Observation	Time (min.)
1	1.34
2	1.57
3	1.44
4	1.38
5	1.50

---

How many observations must be taken to get the desired accuracy for a time study?

$$\text{Number of observations } n = \left( \frac{zs}{ax} \right)^2 \text{ or } \left( \frac{zs}{e} \right)^2$$

where

$z$  = number of normal standard deviations for desired confidence

$s$  = sample standard deviation

$a$  = desired accuracy percentage

$\bar{X}$  = sample mean

$e$  = accuracy or maximum acceptable error

Every time a new job is designed it is not necessary to do an extensive study with lots of people to determine how long it will take. Most companies base time estimates on:

**Standard elemental times** - Time standards from a firm's own historical time data.

**Predetermined time standards** - Published data based on extensive research on many workers to determine standard elemental times.

## Project Planning Homework

1. The project of installing a computer system consists of 11 major activities. The activities, the activity they precede, and time estimates in weeks are shown below.

- a. Develop the PERT/CPM network for this project.
- b. Find the critical path.
- c. How long will it take to complete this project?

<i>Activity</i>	<i>Precedes</i>	<i>Time</i>
a	c,b	5
c	d	8
d	i	2
b	i	7
e	f	3
f	j	6
i	j	10
j	end	8
g	h	1
h	k	2
k	end	17

2. Determine the amount of slack in each of a project's activities (see the list below), and identify those which are on the critical path.

<b>Activity</b>	<u>Precedes</u>	<u>Est. Time (wk)</u>
Start	a,e	0
a	c,d	4
e	f	9
f	g	5
d	g	2
c	end	6
g	end	3

3. Using the information given, do the following:

- (A) Draw the A-O-N network diagram.
- (B) Identify the critical path.
- (C) Determine expected project duration.

<u>Activity</u>	<u>Precedes</u>	<u>Duration</u>
Start	a,b,c,	-
a	d	5
d	f	6
f	g	2
b	e	5
e	f	9
c	h	4
h	g,i	6
g	end	3
i	end	1

4. The project of installing a computer system consists of nine major activities. The activities, their immediate predecessors, and time estimates in weeks are shown below.

- a. Develop the PERT/CPM network for this project.
- b. Find the critical path.
- c. How long will it take to complete this project?

<b>Activity</b>	<b>Predecessor</b>	<b>Time</b>
<b>A</b>	-	3
<b>B</b>	-	6
<b>C</b>	A	3
<b>D</b>	B,C	5
<b>E</b>	D	4
<b>F</b>	E	2
<b>G</b>	B,C	9
<b>H</b>	F,G	4

## Line Balancing Problems

1. Management wants to design an assembly line that will turn out an automotive subassembly 80 units per day. There are 8 hours in a working day. The staff has provided the information below:

Task	Length (minutes)	Follows task
a	5.0	-
b	5.5	a
c	2.5	b
d	2.0	b
e	6.0	a
f	1.5	c,d
g	3.5	f
h	6.5	e
i	1.5	g,h

- a. Determine the minimum cycle time.
- b. Determine the optimum cycle time.
- c. Draw the precedence diagram.
- d. Assign tasks to stations in order of the most following tasks

2. Consider the following tasks to be assigned to workstations:

Task	Task time (Minutes)	Immediate Predecessor
A	0.75	-
B	0.18	A
C	0.15	B
D	0.83	-
E	0.43	D
F	0.18	E
G	0.20	C
H	0.17	C
I	0.15	F, G, H
J	0.17	I

- a. Develop the precedence diagram.
- b. Determine the maximum cycle time for a desired output of 500 units per 7-hour day.
- c. Balance the line (that is, assign tasks to workstations) using the greatest positional weight heuristic. (Positional weight is the task time plus the sum of the task times of the tasks that follow the task.) Break ties using the most following tasks heuristic.

5. Consider the following tasks to be assigned to workstations:

Task	Length (minutes)	Follows task
a	0.12	-
b	0.11	-
c	0.08	a
d	0.02	a,b
e	0.09	e
f	0.10	c
g	0.11	e
h	0.18	f,g

- Determine the minimum cycle time. (2)
- What is the maximum output possible in a 420-minute day. (2)
- Draw the precedence diagram. (5)
- Assign tasks to stations in order of the most following tasks for a 0.29 minute cycle time. (5)

4. Consider the following tasks to be assigned to workstations:

Task	Task time (Minutes)	Immediate Predecessor
A	0.50	-
B	0.70	A
C	0.40	B
D	0.30	C
E	0.30	-
F	0.20	-
G	0.35	E,F
H	0.40	D
I	0.60	G, H
J	0.30	I

- Develop the precedence diagram.
- Determine the maximum cycle time for a desired output of 200 units per 10-hour day.
- Balance the line (that is, assign tasks to workstations) using the most following tasks heuristic.